

## TTLP304 - Heat transfer in an orthotropic plate: summarized imposed

---

### flux:

This test is resulting from the validation independent of version 3 in linear transient thermal.

It is about a problem 2D plane represented by a modelization (plane).

The features tested are the following ones:

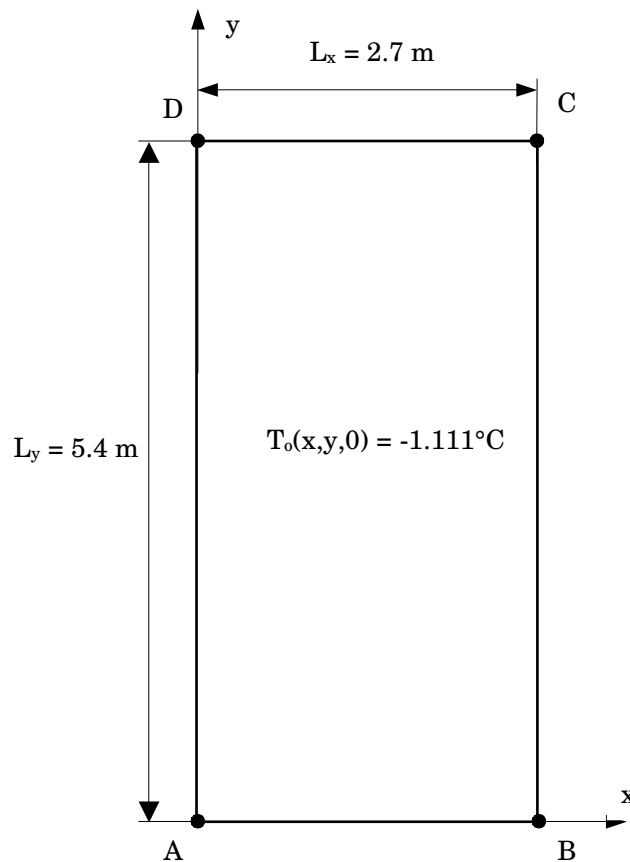
- plane thermal element,
- orthotropic material,
- algorithm of transient thermal,
- conditions limiting: imposed flux.

The interest of the test lies in the taking into account of an orthotropic material.

The results are compared with an analytical solution.

## 1 Problem of reference

### 1.1 Geometry



### 1.2 Properties of the thermal

$\lambda_x = 2.638 \text{ W/m}^\circ\text{C}$	material conductivity along the thermal $x$
$\lambda_y = 0.633 \text{ W/m}^\circ\text{C}$	axis conductivity along the voluminal $y$
$\rho C = 1899.1 \text{ J/m}^3^\circ\text{C}$	axis heat

### 1.3 Boundary conditions and loadings

Contour  $[AB]$   $[BC]$   $[CD]$  :  $T = -17.778^\circ\text{C}$   
Side  $[AD]$  :  $\varphi = 0$

### 1.4 Initial conditions

$T(t=0) = -1.111^\circ\text{C}$

## 2 Reference solution

### 2.1 Method of calculating used for the reference solution

$$T(x, y, t) = \sum_{n=1}^{\infty} \sum_{j=1}^{\infty} B_n \cos \frac{(2n-1)\pi x}{2L_x} \sin \frac{j\pi y}{L_y} \exp \left[ - \left( \frac{\lambda_x (2n-1)^2 \pi^2}{4L_x^2} + \frac{\lambda_y j^2 \pi^2}{L_y^2} \right) t \right]$$

$$\text{where } B_n = \left[ \frac{8T_i}{\pi^2 j(2n-1)} (-1)^{n+2} [(-1)^j - 1] - 32 \right] \frac{5}{9} \quad T_i = \frac{5}{9} T_0 + 32$$

Temperature in °F to $t = 1.2 \text{ hr} (4320\text{s})$										
2.7	-	-15,6480	-15,7455	-15,9049	-16,1211	-16,3876	-16,6964	-17,0381	-17,4022	-
	15,6151									17,7778
2.4	-	-15,6786	-15,7748	-15,9318	-16,1449	-16,4076	-16,7120	-17,0487	-17,4076	-
	15,6462									17,7778
2.1	-	-15,7700	-15,8620	-16,0122	-16,2160	-16,4673	-16,7584	-17,0805	-17,4238	-
	15,7391									17,7778
1.8	-	-15,9208	-16,0058	-16,1447	-16,3333	-16,5657	-16,8349	-17,1328	-17,4503	-
	15,8921									17,7778
1.5	-	-16,1279	-16,2035	-16,3269	-16,4944	-16,7009	-16,9401	-17,2048	-17,4869	-
	16,1025									17,7778
1.2	-	-16,3869	-16,4506	-16,5547	-16,6959	-16,8700	-17,0716	-17,2947	-17,5325	-
	16,3655									17,7778
0.9	-	-16,6911	-16,7409	-16,8222	-16,9325	-17,0685	-17,2261	-17,4004	-17,5862	-
	16,6744									17,7778
0.6	-	-17,0318	-17,0660	-17,1218	-17,1975	-17,2909	-17,3991	-17,5187	-17,6462	-
	17,0203									17,7778
0.3	-	-17,3982	-17,4156	-17,4440	-17,4825	-17,5300	-17,5851	-17,6459	-17,7108	-
	17,3923									17,7778
0.0	-	-17,7778	-17,7778	-17,7778	-17,7778	-17,7778	-17,7778	-17,7778	-17,7778	-
	17,7778									17,7778
Y ↑										
X →	0.0.0.3.0.			0.9.1.2.1.			1.8.2.1.2.			2.7
	6			5			4			

values of reference are obtained with  $n = j = 1000$

### 2.2 Results of reference

$t = 1.2 \text{ hr} (4320\text{s})$  : temperature at the following points:

- in  $x = 0.0$  : for  $y = 0.6, 1.5, 2.7$ ,
- in  $x = 0.9$  : for  $y = 0.6, 1.5, 2.7$ ,
- in  $x = 1.8$  : for  $y = 0.6, 1.5, 2.7$ .

### 2.3 Uncertainty on the analytical

solution Solution.

### 2.4 Bibliographical references

- J.C. Bruch Jr., G. Zyrolski, "Transient two-dimensional heat conduction problems solved by the finite element method", Int. J. num. Meth. Engng, flight 8, n°3, pp 481-494, 1974.

## 3 Modelization A

### 3.1 Characteristic of the modelization

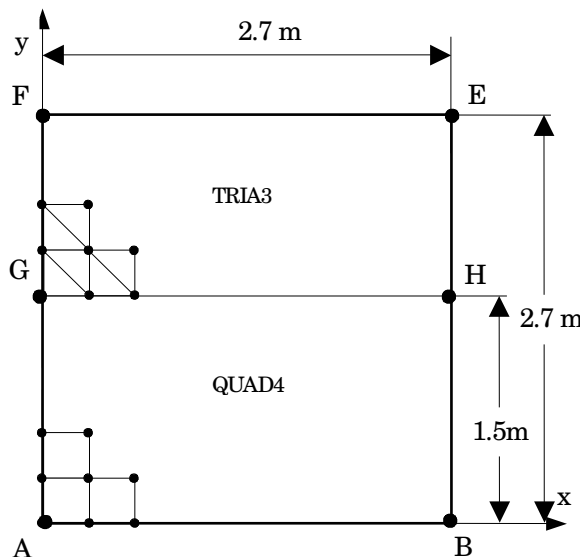
PLANE (QUAD4, TRIA3)

#### Conditions limites

- cotés AB,BH,HE:  $T = -1.111^{\circ}\text{C}$
- cotés EF, FG, GA:  $\varphi = 0$

#### Découpage:

- AB,GH,FE 9 éléments
- AG,BH 5 éléments
- GF,HE 4 éléments



### 3.2 Characteristic of the mesh

Many nodes: 100  
Number of meshes and types: 117 (45 QUAD4, 72 TRIA3)

### 3.3 Remarks

the discretization in time step is the following one:

10 steps for	$[0., 5.00\text{D}0]$	either $\Delta t = 0.5$
9 steps for	$[5.00\text{D}0, 5.00\text{D}1]$	or $\Delta t = 5.$
9 steps for	$[5.00\text{D}1, 5.00\text{D}2]$	or $\Delta t = 50.$
38 steps for	$[5.00\text{D}2, 4.30\text{D}3]$	or $\Delta t = 100.$
1 step for	$[4.30\text{D}3, 4.32\text{D}3]$	or $\Delta t = 20.$

## 4 Results of the modelization A

### 4.1 Values tested

Identification	Reference	Aster	relative Variation %		Absolute Deviation	
			difference	tolerance	difference	tolerance
Temperature in °F						
<i>x</i> = 0.0						
<i>N3</i> ( <i>y</i> = 0.6)	17.0203	17.0146	0.033	1%	0.006	0.05
<i>N6</i> ( <i>y</i> = 1.5)	16.1025	16.0957	0.042	1%	0.007	0.05
<i>N10</i> ( <i>y</i> = 2.7)	15.6151	15.5784	0.235	1%	0.037	0.05
<i>x</i> = 0.9						
<i>N33</i> ( <i>y</i> = 0.6)	17.1218	17.1167	0.029	1%	0.005	0.05
<i>N36</i> ( <i>y</i> = 1.5)	16.3269	16.3127	-0.087	1%	0.014	0.05
<i>N40</i> ( <i>y</i> = 2.7)	15.9049	15.8905	0.091	1%	0.014	0.05
<i>x</i> = 1.8						
<i>N63</i> ( <i>y</i> = 0.6)	17.3991	17.3961	0.017	1%	0.003	0.05
<i>N66</i> ( <i>y</i> = 1.5)	16.9401	16.9297	0.061	1%	0.010	0.05
<i>N70</i> ( <i>y</i> = 2.7)	16.6964	16.6930	0.020	1%	0.003	0.05

## 5 Synthesis results

---

the got results are satisfactory, the maximum change obtained is of 0.235%.

For the points of observation selected, the variations are more important with the nodes belonging to meshes the TRIA3.