TTNL302 - Infinite wall subjected to a constant flux with variable properties

Abstract:

This test is resulting from the validation independent of version 3 in nonlinear transient thermal.

It is about a problem 1D linear represented by five modelizations, one planes, four the other voluminal ones.

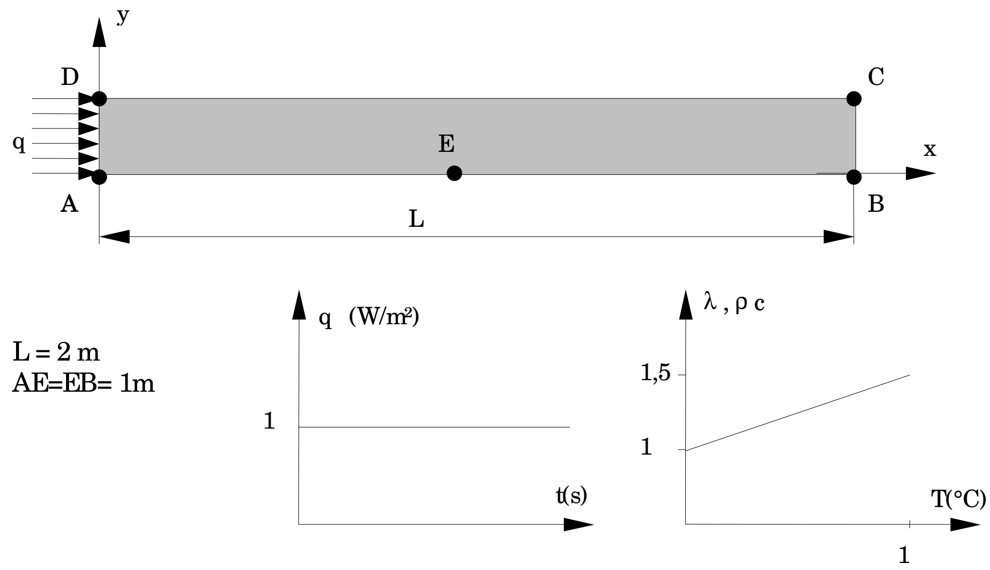
The features tested are the following ones:
- element of thermal plane,
- voluminal element of thermal,
- nonlinear algorithm of transient thermal,
- variable properties,
- limiting condition: imposed flux.

The interest of the test lies in the taking into account of variable properties (thermal conductivity and voluminal heat).

The results are compared with an analytical solution.

1 Problem of reference

1.1 Geometry



1.2 Properties of the thermal

$\lambda=1.0+0.5\,T\;W/m.\,°C$ material Conductivity
$\rho c=1.0+0.5\,T\;J/m^{3.}\,°C$ voluminal Heat

1.3 Boundary conditions and loadings

- with dimensions $[AD]$: flux imposed $q=1\,W/m^2$ for $t>0$,
- dimensioned $[AB]$ $[BC]$, $[CD]$ $\varphi=0.$

1.4 Initial conditions

$T(x,0)=0\,°C$ for any x

2 Reference solution

2.1 Method of calculating used for the semi-analytical reference solution

Solution utilizing functions of error:

$$T(x,t)=2\left\{\sqrt{\left[1+2\sqrt{(t/\pi)}\exp\left(\frac{-x^2}{4\mathrm{t}}\right)+x.erfc\left(\frac{x}{2\sqrt{(t)}}\right)\right]}-1\right\}$$

with $erfc(x)=\frac{2}{\pi}\int_x^\infty e^{-t^2}dt$

where $x=$ X-coordinate
$t=$ time

This formula is valid only for $\lambda(T)=\rho c(T)=1.+0.5\mathrm{T}$

2.2 Results of reference

Temperature to the points A ($x=0$) and E ($x=1$) at following t times: $t=0.1, 0.3, 0.5, 0.7$ and $1\mathrm{s}$

2.3 Uncertainty on the Unknown

solution, due to the evaluating of the functions of error.

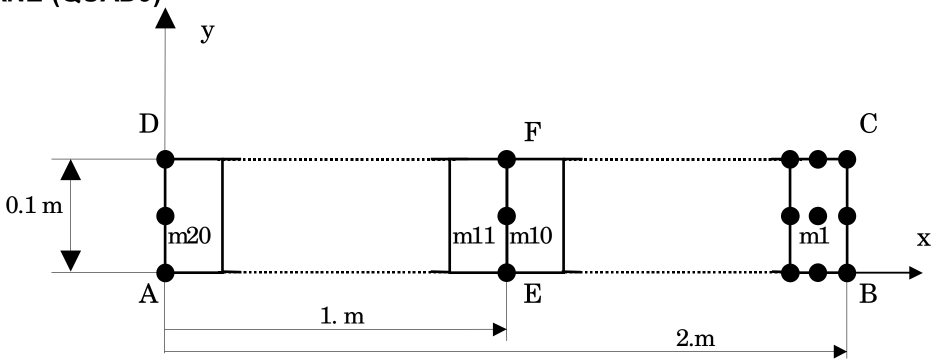
2.4 Ographic references bibli

• Segal, N. Praagman, "A fast implementation of explicit time stepping algorithms with the finite element method for have class of nonlinear evolution problems", Int. J. num. Meth. Engng, flight 23, pp 155-168, 1986.

3 Modelization A

3.1 Characteristic of the modelization

PLANE (QUAD9)



Conditions limites:

- cotés AB, BC, CD   φ = 0
- coté AD   q = 1 W/m²°C

Point	x	y	Noeud
A	0.0	0.0	N1
D	0.0	0.1	N3
E	1.0	0.0	N61
F	1.0	0.1	N63

3.2 Characteristic of the mesh

Many nodes: 123
Number of meshes and types: 20 QUAD9

3.3 Remarks

the discretization in time step are the following one:

10 steps for $[0., 5.D-2]$ either $\Delta t = 5.D^{-3}$
19 step for $[5.D-2, 1.D0]$ or $\Delta t = 5.D^{-2}$

4 Results of the modelization A

4.1 Values tested

Identification	Reference	*Aster*	% difference	tolerance
Temperature $(°C)$				
Node N1 $t=0.1s$	0.330	0.329	0.204%	1%
” “ T = 0.3s	0.544	0.544	0.048%	1%
” “ T = 0.5s	0.682	0.681	0.075%	1%
” “ T = 0.7s	0.789	0.789	0.036%	1%
” “ T = 1.0s	0.918	0.920	0.254%	1%
N3 Node $t=0.1s$	0.330	0.329	0.204%	1%
” “ T = 0.3s	0.544	0.544	0.048%	1%
” “ T = 0.5s	0.682	0.681	0.075%	1%
” “ T = 0.7s	0.789	0.789	0.036%	1%
” “ T = 1.0s	0.918	0.920	0.254%	1%
N61 Node $t=0.1s$	0.004	0.004	**1.161%**	1%
” “ T = 0.3s	0.071	0.071	**0.377%**	1%
” “ T = 0.5s	0.160	0.161	0.573%	1%
” “ T = 0.7s	0.247	0.251	1.616%	1%
” “ T = 1.0s	0.366	0.380	**3.951%**	1%
Node N63 $t=0.1s$	0.004	0.004	**1.161%**	1%
” “ T = 0.3s	0.071	0.071	**0.377%**	1%
” “ T = 0.5s	0.160	0.161	0.573%	1%
” “ T = 0.7s	0.247	0.251	1.616%	1%
” “ T = 1.0s	0.366	0.380	**3.951%**	1%

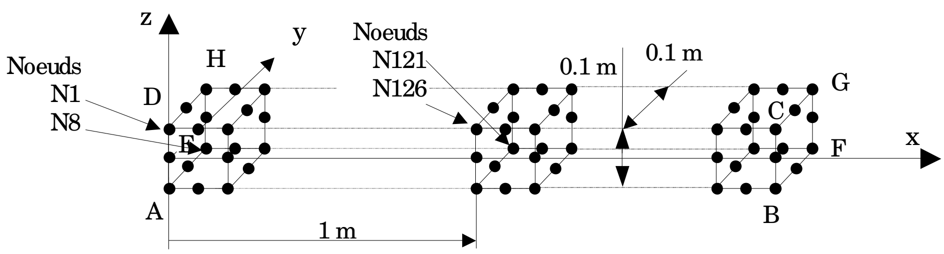
4.2 Remarks

the relative error is to the maximum of 3.9%.

5 Modelization B

5.1 Characteristic of the modelization

3D (HEXA20)



Conditions limites:

- faces ABCD, ABFE φ = 0
- faces EFGH, DCGH φ = 0
- face BFGC φ = 0
- face AEHD q = 1 W/m²

Noeuds	x	y	z
N1	0.0	0.0	0.05
N8	0.0	0.1	-0.05
N121	1.0	0.0	-0.05
N126	1.0	0.1	0.05

5.2 Characteristic of the mesh

Many nodes:	248
Number of meshes and types:	20 HEXA20

5.3 Remarks

the discretization in time step are the following one:

10 steps for $[0., 5.D-2]$ either $\Delta t=5.D^{-3}$

19 step for $[5.D-2, 1.D0]$ or $\Delta t=5.D^{-2}$

6 Results of the modelization B

6.1 Values tested

Identification	Reference	*Aster*	% difference	tolerance
Temperature $(°C)$				
Node N1 T = 0.1 S	0.330	0.330	˜ 0.129	1%
” “ T = 0.3s	0.544	0.543	˜ 0.149	1%
” “ T = 0.5s	0.682	0.681	˜ 0.154	1%
” “ T = 0.7s	0.789	0.788	˜ 0.092	1%
” “ T = 1.0s	0.918	0.920	0.222	1%
Node N8 T = 0.1s	0.330	0.330	˜ 0.129	1%
” “ T = 0.3s	0.544	0.543	˜ 0.149	1%
” “ T = 0.5s	0.682	0.681	˜ 0.154	1%
” “ T = 0.7s	0.789	0.788	˜ 0.092	1%
” “ T = 1.0s	0.918	0.920	0.222	1%
Node N121 T = 0.1s	0.004	0.004	**10.931**	1%
” “ T = 0.3s	0.071	0.071	˜ **0.242**	1%
” “ T = 0.5s	0.160	0.161	0.587	1%
” “ T = 0.7s	0.247	0.251	1.619	1%
” “ T = 1.0s	0.366	0.380	**3.95**	1%
Node N126 T = 0.1s	0.004	0.004	**10.931**	1%
” “ T = 0.3s	0.071	0.071	˜ **0.242**	1%
” “ T = 0.5s	0.160	0.161	0.587	1%
” “ T = 0.7s	0.247	0.251	1.619	1%
” “ T = 1.0s	0.366	0.380	**3.95**	1%

6.2 Remarks

the relative error is to the maximum of 3.95%, except for $x=1$ at time $t=0.1\mathrm{s}$ the error is of 11%. This error was obtained for the smallest value of the temperature ($T=0.004°C$). This variation is explained by the fact why the function of error in this point is of 0.025347 and that uncertainty on the computation of the function of error is unknown.

7 Modelization C

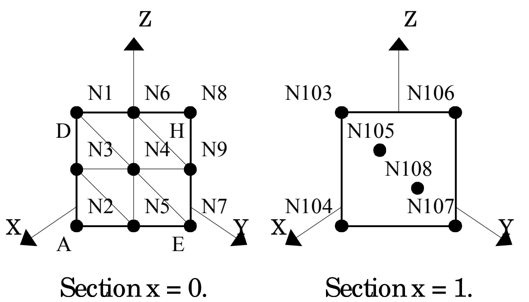
7.1 Characteristic of the modelization

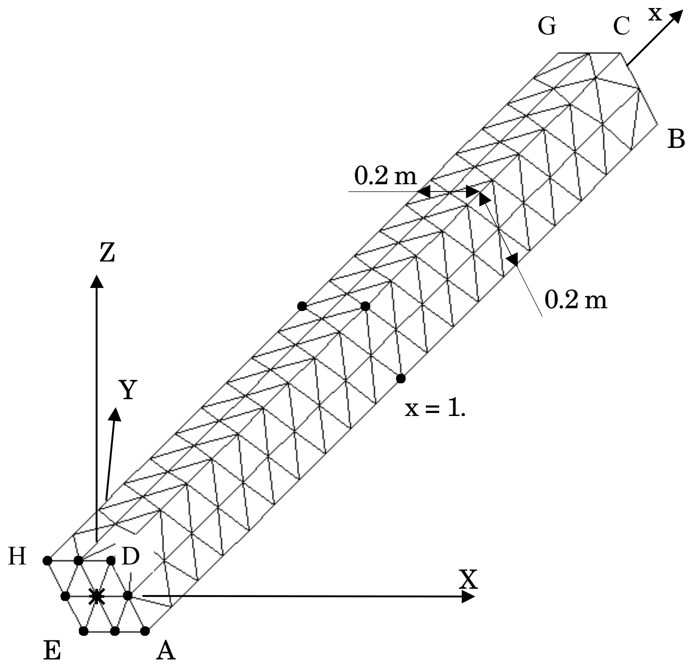
3D (TETRA4)

Axe longitudinal x suivant la trisectrice (X=Y=Z)

Conditions limites:

- faces ABCD, ABFE φ = 0
- faces EFGH, DCGH φ = 0
- face BFGC φ = 0
- face AEHD q = 1 W/m²





7.2 Characteristic of the mesh

Many nodes:	224
Number of meshes and types:	692 TETRA4 (and 8 TRIA3)

7.3 Remarks

the discretization in time step are the following one:

10 steps for $[0., 5.D-2]$ either $\Delta t = 5.D^{-3}$

19 step for $[5.D-2, 1.D0]$ or $\Delta t = 5.D^{-2}$

8 Results of the modelization C

8.1 Values tested

Identification		Reference	*Aster*	relative Variation (%)		absolute Deviation (°C)	
				difference	tolerance	difference	tolerance
Temperatures in $°C$:							
Face	X = 0.m						
Node N7	T = 0.1s	0.330	0.3295	˜ 0.162%	1.%	˜ 0.000536	0.005
” “	T = 0.3s	0.544	0.5425	˜ 0.273%	1.%	˜ 0.00149	0.005
” “	T = 0.5s	0.682	0.6796	˜ 0.351%	1.%	˜ 0.00239	0.005
” “	T = 0.7s	0.789	0.7861	˜ 0.362%	1.%	˜ 0.00285	0.005
” “	T = 1.0s	0.918	0.9165	˜ 0.159%	1.%	˜ 0.00146	0.005
Node N5	T = 0.1s	0.330	0.3279	˜ 0.627%	1.%	˜ 0.00207	0.005
” “	T = 0.3s	0.544	0.5418	˜ 0.406%	1.%	˜ 0.00221	0.005
” “	T = 0.5s	0.682	0.6791	˜ 0.422%	1.%	˜ 0.00288	0.005
” “	T = 0.7s	0.789	0.7858	˜ 0.409%	1.%	˜ 0.00323	0.005
” “	T = 1.0s	0.918	0.9162	˜ 0.192%	1.%	˜ 0.00176	0.005
Section	X = 1.m						
Node N107	T = 0.1s	0.00394	0.004140	**5.085%**	1.%	0.000200	0.005
” “	T = 0.3s	0.0706	0.07013	˜ 0.665%	1.%	˜ 0.000470	0.005
” “	T = 0.5s	0.160	0.1596	˜ 0.228%	1.%	˜ 0.000364	0.005
” “	T = 0.7s	0.247	0.2488	0.730%	1.%	0.00180	0.005
” “	T = 1.0s	0.366	0.3766	**2.889%**	1.%	**0.0106**	0.005
Node N108	T = 0.1s	0.00394	0.004002	**1.577%**	1.%	˜ 0.0000621	0.005
” “	T = 0.3s	0.0706	0.06937	˜ **1.742%**	1.%	˜ 0.00123	0.005
” “	T = 0.5s	0.160	0.1586	˜ 0.895%	1.%	˜ 0.00143	0.005
” “	T = 0.7s	0.247	0.2476	0.238%	1.%	0.000587	0.005
” “	T = 1.0s	0.366	0.3753	**2.534%**	1.%	**0.00928**	0.005

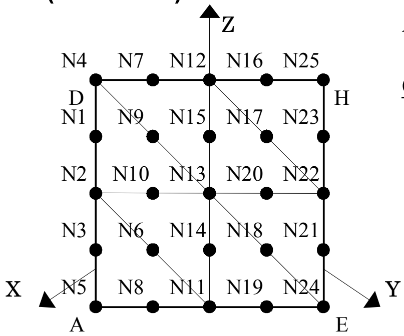
8.2 Remarks

the selected nodes correspond to the extreme results on the same section.

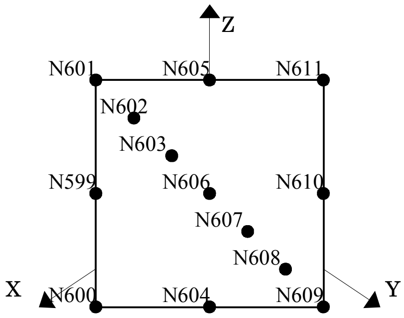
9 Modelization D

9.1 Characteristic of the modelization

3D (TETRA10)



Section x = 0.

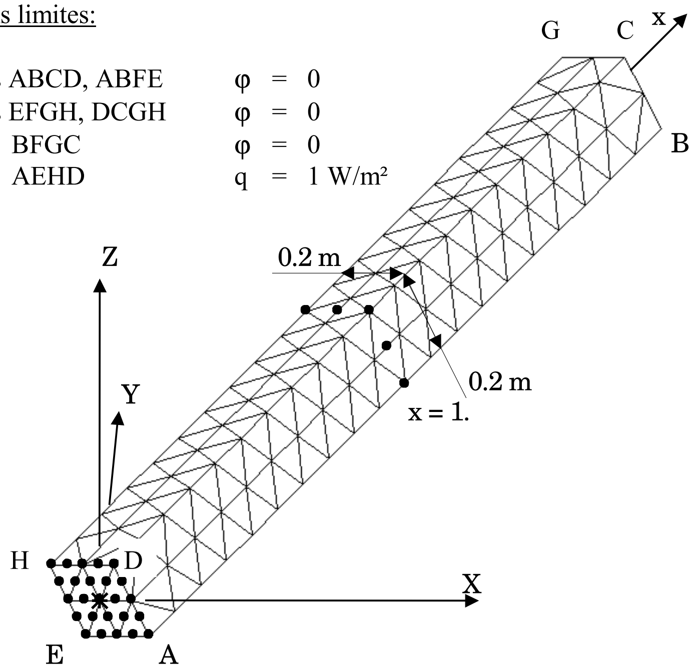


Section x = 1

Axe longitudinal x suivant la trisectrice (X=Y=Z)

Conditions limites:

- faces ABCD, ABFE φ = 0
- faces EFGH, DCGH φ = 0
- face BFGC φ = 0
- face AEHD q = 1 W/m²



9.2 Characteristic of the mesh

Many nodes:	1310
Number of meshes and types:	697 TETRA10 (and 8 TRIA6)

9.3 Remarks

the discretization in time step are the following one:

10 steps for $[0., 5.D-2]$ either $\Delta t = 5.D^{-3}$

19 step for $[5.D-2, 1.D0]$ or $\Delta t = 5.D^{-2}$

10 Results of the modelization D

10.1 Values tested

Identification		Reference	*Aster*	relative Variation (%)		absolute Deviation (°C)	
				difference	tolerance	difference	tolerance
Temperatures in $°C$:							
Face	X = 0.m						
Node N4	T = 0.1s	0.330	0.3291	~ 0.281%	1.%	~ 0.000926	0.005
” “	T = 0.3s	0.544	0.5423	~ 0.318%	1.%	~ 0.00173	0.005
” “	T = 0.5s	0.682	0.6794	~ 0.383%	1.%	~ 0.00261	0.005
” “	T = 0.7s	0.789	0.7860	~ 0.384%	1.%	~ 0.00303	0.005
” “	T = 1.0s	0.918	0.9164	~ 0.180%	1.%	~ 0.00165	0.005
Node N25	T = 0.1s	0.330	0.3292	~ 0.255%	1.%	~ 0.000843	0.005
” “	T = 0.3s	0.544	0.5423	~ 0.314%	1.%	~ 0.00171	0.005
” “	T = 0.5s	0.682	0.6794	~ 0.382%	1.%	~ 0.00261	0.005
” “	T = 0.7s	0.789	0.7860	~ 0.383%	1.%	~ 0.00303	0.005
” “	T = 1.0s	0.918	0.9163	~ 0.180%	1.%	~ 0.00165	0.005
Section	X = 1.m						
Node N606	T = 0.1s	0.00394	0.004331	**9.913%**	1.%	0.000391	0.005
” “	T = 0.3s	0.0706	0.07021	~ 0.551%	1.%	~ 0.000389	0.005
” “	T = 0.5s	0.160	0.1596	~ 0.251%	1.%	~ 0.000402	0.005
” “	T = 0.7s	0.247	0.2488	0.710%	1.%	0.00175	0.005
” “	T = 1.0s	0.366	0.3764	**2.855%**	1.%	**0.0104**	0.005
Node N611	T = 0.1s	0.00394	0.004332	**9.944%**	1.%	0.000392	0.005
” “	T = 0.3s	0.0706	0.07021	~ 0.550%	1.%	~ 0.000388	0.005
” “	T = 0.5s	0.160	0.1596	~ 0.251%	1.%	~ 0.000402	0.005
” “	T = 0.7s	0.247	0.2488	0.710%	1.%	0.00175	0.005
” “	T = 1.0s	0.366	0.3764	**2.855%**	1.%	**0.0104**	0.005

10.2 Remarks

the calculated results are almost identical on the nodes of the same section.

11 Modelization E

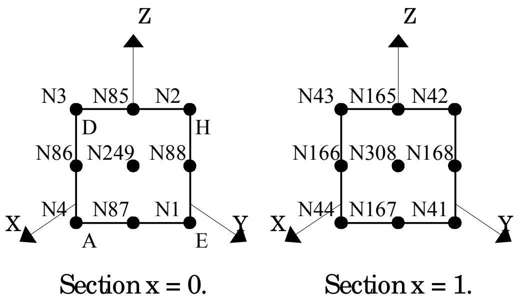
11.1 Characteristic of the modelization

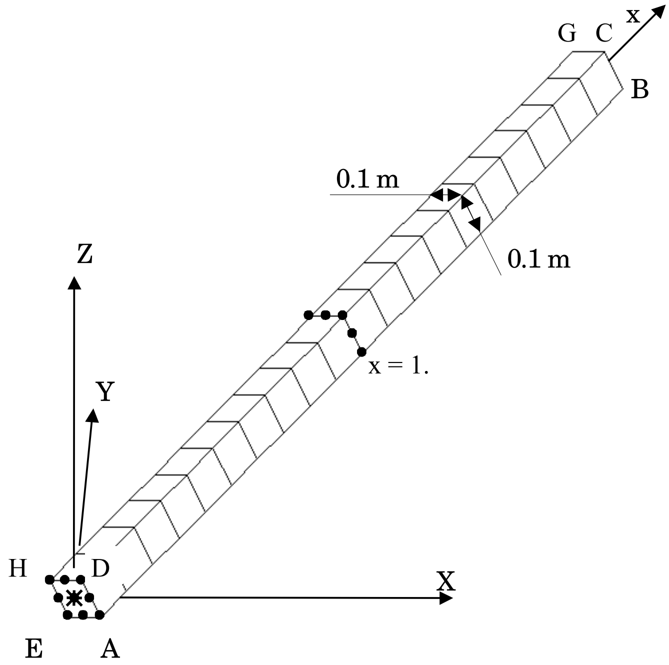
3D (HEXA27)

Axe longitudinal x suivant la trisectrice (X=Y=Z)

Conditions limites:

- faces ABCD, ABFE $\varphi = 0$
- faces EFGH, DCGH $\varphi = 0$
- face BFGC $\varphi = 0$
- face AEHD $q = 1\ W/m^2$





11.2 Characteristic of the mesh

Many nodes:	369
Number of meshes and types:	20 HEXA27 (and 1 QUAD9)

11.3 Remarks

the discretization in time step is the following one:

10 steps for $[0., 5.D-2]$ either $\Delta t=5.D^{-3}$

19 step for $[5.D-2, 1.D0]$ or $\Delta t=5.D^{-2}$

12 Results of the modelization E

12.1 Values tested

Identification		**Reference**	***Aster***	**relative Variation (%)**		**absolute Deviation (°C)**	
				difference	**tolerance**	**difference**	**tolerance**
Temperatures in $°C$:							
Face	X = 0.m						
Node N249	T = 0.1s	0.330	0.3291	~ 0.283%	1.%	~ 0.000933	0.005
” “	T = 0.3s	0.544	0.5423	~ 0.317%	1.%	~ 0.00173	0.005
” “	T = 0.5s	0.682	0.6794	~ 0.376%	1.%	~ 0.00256	0.005
” “	T = 0.7s	0.789	0.7860	~ 0.378%	1.%	~ 0.00298	0.005
” “	T = 1.0s	0.918	0.9165	~ 0.168%	1.%	~ 0.00154	0.005
Section	X = 1.m						
Node N308	T = 0.1s	0.00394	0.004331	**9.926%**	1.%	0.000391	0.005
” “	T = 0.3s	0.0706	0.07021	~ 0.554%	1.%	~ 0.000391	0.005
” “	T = 0.5s	0.160	0.1596	~ 0.227%	1.%	~ 0.000363	0.005
” “	T = 0.7s	0.247	0.2488	0.726%	1.%	0.00179	0.005
” “	T = 1.0s	0.366	0.3766	**2.886%**	1.%	**0.0106**	0.005

12.2 Remarks

the calculated results are identical (with 10^{-7} near) on the nodes of the same section.

13 Summary of the results

the five modelizations carried out, have same cutting in the meaning of propagation of the temperature, they are different only by their type of meshes.

The five modelizations give results whose certain values exceed the tolerance fixed initially (1%). The maximum change is of 9.9%. It appears for the smallest value of reference located in the middle of the wall and at the beginning of the transient.

A mesh finer associate with a finer temporal discretization should improve quality of the results.

Moreover, the reference solution utilizes a function of error whose accuracy is unknown.

The results are regarded as acceptable taking into account the points evoked above.

This test made it possible to test meshes hexahedral and tetrahedral ones in transitory nonlinear thermal as well as the principal following thermal commands:

- DEFI_MATERIAU associated with key word THER_NL, making it possible to define the characteristics of a material whose characteristics vary according to the temperature (conductivity and enthalpy),
- THER_NON_LINE orders allowing the resolution of a steady thermal nonlinear problem or not.