TTNL101 – Nonlinear thermal source in a Summarized

bar:

This test checks thermal computation in the presence of a loading of nonlinear source, depend on the temperature.

The reference solution is analytical and variable in time and space. The part considered in the two modelizations is a symmetric bar composed of lumped elements:

- quadrangles QUAD4 for a modelization `AXIS_DIAG`
- of hexahedrons HEXA8 for a modelization `3D_DIAG`

the two ends of the bar are subjected to the conditions of adiabadicity per default. The volumic source of heat is a linear function of the temperature.

1 Problem of reference

1.1 Geometry

One considers a unidimensional structure (a bar whose side sides are subjected to adiabatic conditions) length $2L$ occupying the field $[-L;L]$.

The temperature being homogeneous in the normal directions with the bar, computation can be regarded as 1D.

1.2 Properties of the thermal

$\lambda=2$ material conductivity
$\rho C=2$ voluminal heat

1.3 Boundary conditions and loadings

nonlinear Loading of volumic source, function of the temperature:
$s(T)=2-2\times w\times T$ with $w=2$

the boundary conditions are adiabatic on the side sides and of standard temperature imposed null at the end of the bar; a condition of symmetry is put in work as regards symmetry (what is equivalent to an adiabatic boundary condition).

The temporal beach $[0.;1.]$ is discretized into 100 time step (lasted of each time step equalizes with 0.01).

1.4 Initial conditions

an analytical initial state is provided. See the developments in the paragraph detailing the reference solution.

2 Reference solution

2.1 Method of calculating used for the reference solution

the bar is subjected to a heat source $r(T)=r_0-r_1 T$, where $r_1>0$ for questions of thermal stability. Its initial temperature is worth $T_0(x)$ ends of the bar and the are maintained with a temperature null. The evolution of temperature obeys the equation of heat:

$$\rho c \dot{T}=\lambda \nabla^2 T+r(T) \;;\; T(x,0)=T_0(x) \;;\; T(-L,t)=T(L,t)=0$$

By standardization, one can be reduced without loss of generality to the following equation:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}+1-\omega^2 u \;;\; u(x,0)=u_0(x) \;;\; u(-1,t)=u(1,t)=0$$

To solve this equation, one is interested initially in the asymptotic solution $u_\infty(x)$ which checks:

$$0=\frac{\partial^2 u_\infty}{\partial x^2}+1-\omega^2 u_\infty \;;\; u_\infty(-1)=u_\infty(1)=0$$

The solution of this linear differential equation of the second order is worth:

$$u_\infty(x)=\frac{1}{\omega^2}\left(1-\frac{\cosh \omega x}{\cosh \omega}\right)$$

The solution of the transitory equation is then obtained by projection of $v=u-u_\infty$ on the eigenfunctions of the Laplacian on $]-1,1[$. To simplify the analysis, one adopts an initial condition u_0 equal to the first eigen mode, namely:

$$u_0(x)=u_\infty(x)-\cos\frac{\pi x}{2}$$

Only the first mode being activated, one is brought back to the solution of a differential equation in first order time, to obtain the solution finally:

$$u(x,t)=u_\infty(x)-\exp\left(-\omega^2 t-\frac{\pi^2}{4}t\right)\cos\frac{\pi x}{2}$$

Lastly, like previously, one goes up of u with T by adopting a specific set of parameters, without taking account of the units, so that $T=u$. For that, one takes $\lambda=r_0=\rho c \quad r_1=\omega^2 r_0$, $L=1$ and $T_0(x)=u_0(x)$.

2.2 Results of reference

the benchmark is carried out with $\omega=\sqrt{2}$ and one examines the temperature with $t=1$ in a node of the symmetry plane $(x=0)$. The data are the following ones:

Thermal conductivity	`LAMBDA`	2.
Voluminal heat capacity	`RHO_CP`	2.
Initial temperature	T_0	$u_0(x)=\frac{1}{\omega^2}\left(1-\frac{\cosh \omega x}{\cosh \omega}\right)-\cos\frac{\pi x}{2}$ with $\omega=\sqrt{2}$
Heat source	r_0	2. 4.

	r_1	

Quantity tested	$T\ (\ x=0, t=1\ )$
Value of reference	0.258974

3 Modelization A

3.1 Characteristic of the modelization

The mesh of this modelization is made up 40 of the same quadrangles QUAD4 size in modelization AXIS_DIAG . The length of the elements is worth 0.025 because only the half of bar is represented (symmetry).

4 Results of the modelization A

4.1 Values tested

the solution is in conformity with the analytical value with less than 0.05 % for a temporal discretization of 100 time step.

5 Modelization B

5.1 Characteristic of the modelization

The mesh of this modelization is made up 40 of the same hexahedrons HEXA8 cuts in modelization `3D_DIAG` . The length of the elements is worth 0.025 because only the half of bar is represented (symmetry).

6 Results of the modelization B

6.1 Values tested

the solution is in conformity with the analytical value with less than 0.05 % for a temporal discretization of 100 time step.

7 Summary of the results

the results are in conformity with the analytical solution.