TTNL02 - Thermal transient with Summarized

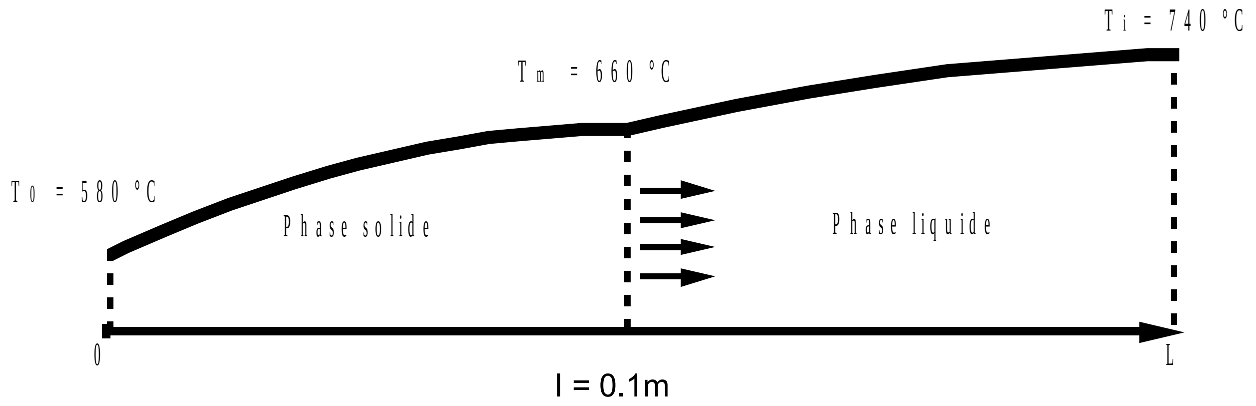
phase change:

This elementary test makes it possible to deal with an one-way problem in nonlinear transient thermal and to check the taking into account of a liquid/solid phase change by *Code_Aster* by introducing via the voluminal enthalpy the latent heat of fusion. The solution is analytical and utilizes the functions of error erf and $erfc$. With the problem is dealt in the plane and voluminal cases.

For the modelizations presented here, the variations of the results got by *Code_Aster* range between 1 and 4% of the analytically calculated reference.

1 Problem of reference

1.1 Geometry



1.2 Material properties

They are the subscripted characteristics of aluminum by S for the solid phase and L for the liquid phase. They are supposed to be constant within each phase.

Density	$\rho_S=2550\,kg/m^3$	$\rho_1=2390\,kg/m^3$
voluminal Heat	$c_S=3.\,10^6\,J/m^3\,°C$	$c_1=2.58\,10^6\,J/m^3\,°C$
thermal Conductivity	$k_S=210\,W/m\,°C$	$k_1=95\,W/m\,°C$
Latent heat of fusion	$L=437.44\,10^{-3}\,J.kg$	
Melting point	$T_m=660.\,°C$	
voluminal Variation of enthalpy	$\Delta H=1.08048\,10^9\,J/m^3$	

1.3 Boundary conditions and loadings

Temperature imposed at the ends.

$T_0=580\,°C$ in $x=0$
$T_i=740\,°C$ of $x=l$

1.4 Initial conditions

initial Temperature uniform

$$T_{init}=T_i=740\,°C$$

2 Reference solution

2.1 Method of calculating used for the reference solution

One has an semi-analytical solution utilizing the functions of errors:

$$erf(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt \text{ and } erfc(x)=\frac{2}{\sqrt{\pi}}\int_x^{+\infty} e^{-t^2}\,dt$$

This solution is valid for a semi-infinite medium, it could thus be used only in one restricted field of variation of the variable of time.

That is to say x_t the position of the solid interface/liquidates. Are $s_t=\frac{L}{\sqrt{t_{total}}}$ and $\lambda=\frac{s_t}{2\sqrt{d_s}}$ where

d_s and d_l indicate the solid diffusivity of the mediums and liquidates $\left(d_s=\frac{k_s}{c_s}, d_l=\frac{k_l}{c_l}\right)$. The solution of the equation of heat is form:

$$T_s(x,t)=T_0+\frac{T_m-T_0}{erf(\lambda)}\,erf\left(\frac{x}{2\sqrt{d_s t}}\right) \text{ if } x\leq x_t$$

$$T_l(x,t)=T_i+\frac{T_m-T_i}{erfc\left(\lambda\sqrt{\frac{d_s}{d_l}}\right)}\,erfc\left(\frac{x}{2\sqrt{d_l t}}\right) \quad x\geq x_t$$

the data of t_{total} is enough to define the solution, one thus fixes $t_{total}=420$.

2.2 Results of reference

TEMPS: **X-coordinate**	**0.5.1.0.1.5**			**2.0.2.5.3.0**		
.000	580.	580.	580.	580.	580.	580.
.005	682.43	661.33	647.50	638.74	632.69	628.20
.010	726.05	705.75	692.06	682.43	675.24	669.63
.015	738.11	728.70	718.44	709.60	702.23	696.06
.020	739.86	737.22	731.99	726.05	720.27	714.94
.025	740.	739.50	737.56	734.47	730.81	727.00
.030	740.	739.93	739.39	738.11	736.20	733.88
.035	740.	739.99	739.88	739.45	738.61	737.40
.040	740.	740.	739.98	739.86	739.55	739.00
.045	740.	740.	740.	739.97	739.87	739.65
.050	740.	740.	740.	740.	739.97	739.89
.055	740.	740.	740.	740.	740.	739.97
.060	740.	740.	740.	740.	740.	740.
.065	740.	740.	740.	740.	740.	740.
.070	740.	740.	740.	740.	740.	740.
.075	740.	740.	740.	740.	740.	740.
.080	740.	740.	740.	740.	740.	740.
.085	740.	740.	740.	740.	740.	740.
.090	740.	740.	740.	740.	740.	740.
.095	740.	740.	740.	740.	740.	740.
.100	740.	740.	740.	740.	740.	740.

TEMPS: X-coordinate	**3.5.4.0.4.5**			**5.0.5.5.6.0**		
.000	580.	580.	580.	580.	580.	580.
.005	624.68	621.84	619.48	617.48	615.25	614.25
.010	665.09	661.33	657.43	653.65	650.37	647.49
.015	690.83	686.33	682.43	678.99	675.95	673.22
.020	710.11	705.75	701.81	698.25	709.92	692.06
.025	723.23	719.60	716.17	712.95	720.89	707.09
.030	731.34	728.70	726.05	723.43	728.48	718.44
.035	735.89	734.18	732.34	730.43	733.42	726.53
.040	738.21	737.22	736.07	734.79	736.44	731.99
.045	739.29	738.77	738.11	737.33	738.18	735.47
.050	739.74	739.50	739.15	738.71	739.12	737.56
.055	739.91	739.81	739.65	739.42	739.60	738.75
.060	739.97	739.93	739.86	739.75	739.83	739.39
.065	739.99	739.98	739.95	739.90	739.93	739.72
.070	740.	739.99	739.98	739.96	739.97	739.88
.075	740.	740.	740.	739.99	739.99	739.95
.080	740.	740.	740.	740.	740.	739.98
.085	740.	740.	740.	740.	740.	739.99
.090	740.	740.	740.	740.	740.	740.
.095	740.	740.	740.	740.	740.	740.
.100	740.	740.	740.	740.	740.	740.

(In $°C$, according to the X-coordinate in meter and of time in seconds).

Note:

> *One limits oneself to the variations during the 6 first second, beyond 10 seconds the boundary condition at the end* $x=1$ *is not more assured.*

2.3 Uncertainty on the Unknown

solution, due to the evaluating of the functions of error.

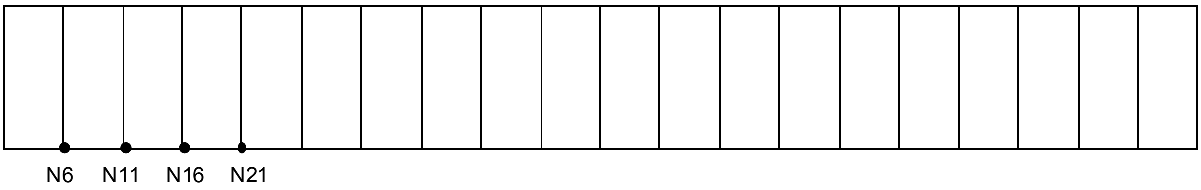
2.4 Bibliographical references

- Mr. Necati Özisik - Heat Conduction - Chapter 10: Phase-change problems example 10-3 - John Wiley & Sounds.

3 Modelization A

3.1 Characteristic of the modelization

Modelization 2D:



3.2 Characteristics of the mesh

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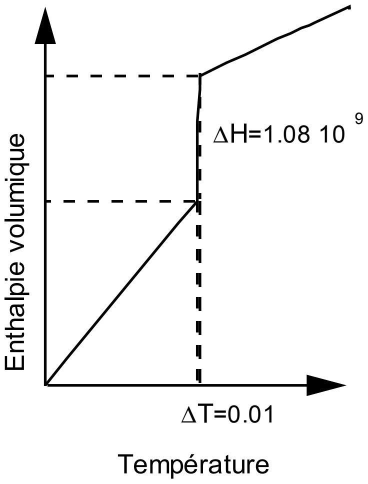
3.3 Notice

the latent heat of fusion is provided via the enthalpy on an interval of $0.01\,°C$.



3.4 Values tested

the nodes observed have as a coordinate $y=0.0$

Identification temperature	Reference
T = 0.5 S N6 (X = 0.005)	682.43
T = 1.0 S N6 (X = 0.005)	661.33
T = 3.0 S N6 (X = 0.005)	628.20
T = 6.0 S N6 (X = 0.005)	614.25
T = 0.5 S N11 (X = 0.010)	726.05
T = 1.0 S N11 (X = 0.010)	705.75
T = 3.0 S N11 (X = 0.010)	669.63

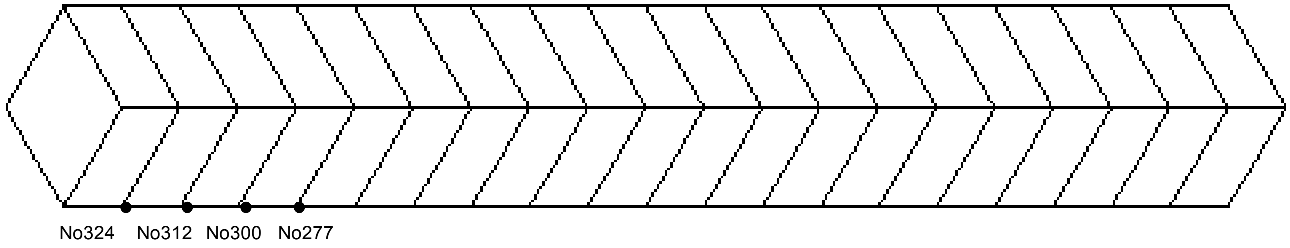
T = 6.0 S	N11	(X = 0.010)	647.49
T = 0.5 S	N16	(X = 0.015)	738.11
T = 1.0 S	N16	(X = 0.015)	728.70
T = 3.0 S	N16	(X = 0.015)	696.06
T = 6.0 S	N16	(X = 0.015)	673.22
T = 0.5 S	N21	(X = 0.020)	739.86
T = 1.0 S	N21	(X = 0.020)	737.22
T = 3.0 S	N21	(X = 0.020)	714.94
T = 6.0 S	N21	(X = 0.020)	692.06

The computation by finite elements require a discretization in times of $\Delta t = 5.10^{-4} s$ at least for the first steps. The boundary condition imposed at the origin making abruptly pass the temperature of $740.\,°C$ to $580.\,°C$. One observes on the level of the first time step some oscillations which are stabilized then rather quickly, despite everything the maximum temperature is exceeded, it does not have respect of the discrete maximum there. This phenomenon is observed during the thermal shocks, only a particular digital processing on the level of the mass matrix can cure this last.

4 Modelization B

4.1 Characteristic of the modelization

Modelization 3D:



4.2 Characteristics of the mesh

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4.3 Values tested

the nodes observed have as coordinates: $x=y=0.005$

Identification Temperature	Reference
T = 0.5 S No324 (Z = 0.005)	682.43
T = 1.0 S No324 (Z = 0.005)	661.33
T = 3.0 S No324 (Z = 0.005)	628.20
T = 6.0 S No324 (Z = 0.005)	614.25
T = 0.5 S No312 (Z = 0.010)	726.05
T = 1.0 S No312 (Z = 0.010)	705.75
T = 3.0 S No312 (Z = 0.010)	669.63
T = 6.0 S No312 (Z = 0.010)	647.49
T = 0.5 S No300 (Z = 0.015)	738.11
T = 1.0 S No300 (Z = 0.015)	728.70
T = 3.0 S No300 (Z = 0.015)	696.06
T = 6.0 S No300 (Z = 0.015)	673.22
T = 0.5 S No277 (Z = 0.020)	739.86
T = 1.0 S No277 (Z = 0.020)	737.22
T = 3.0 S No277 (Z = 0.020)	714.94
T = 6.0 S No277 (Z = 0.020)	692.06

5 Summaries of the results

the error obtained compared to the analytical solution remain reasonable for the points of observation listed in the tables. Let us announce however that the thermal shock imposed on the beginning of the transient causes oscillations (when one observes the variation in the temperature in a point in the course of time) which diminish quickly and which disappeared at time $t=0.5\,s$.