

TTLL100 - Thermal shock on a plane wall with condition of Summarized

exchange:

This test of transitory linear thermal consists in imposing a cold thermal shock on an infinite plane wall using a limiting condition of exchange. The shock is modelled by a linear slope $\Delta T = -100^\circ C$ in $10^{-2} s$.

With the problem is dealt out of plane.

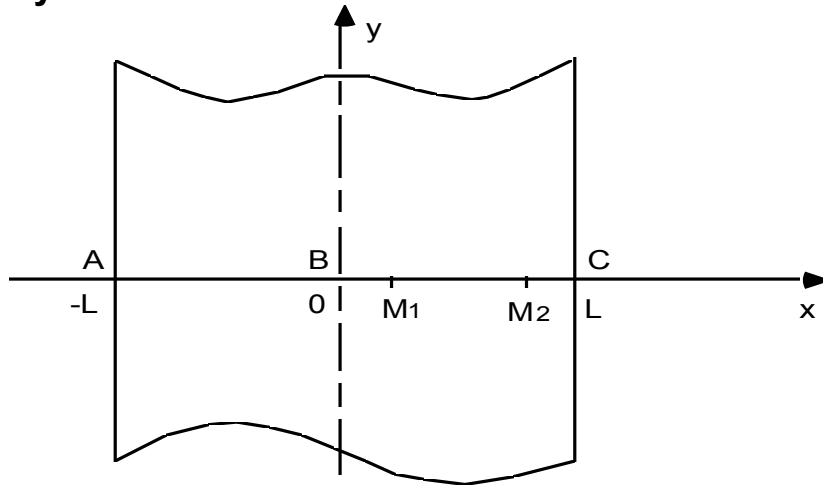
The reference solution is analytical.

The test is carried out on 2 modelizations: (TRIA3, QUAD4) and (TRIA6, QUAD9).

One tests the algorithm of linear thermal transitory when the mass matrix is diagonalized (modelization PLAN_DIAG with "farmhouse lumping").

1 Problem of reference

1.1 Geometry



$$\begin{aligned}\overline{AB} = \overline{BC} = L &= 0.1 \text{ m} \\ x(M1) &= 0.02 \text{ m} \\ x(M2) &= 0.08 \text{ m}\end{aligned}$$

1.2 Material properties

$$\begin{aligned}\lambda &= 1 \text{ W/m}^\circ\text{C} \\ \rho C_p &= 1000 \text{ J/m}^3\text{ }^\circ\text{C}\end{aligned}$$

1.3 Boundary conditions and loadings

$$\text{exchanges } \lambda \left. \frac{\partial T}{\partial n} \right|_{x=\pm L} = h(T_{ext} - T(x, t))$$

$$\text{with: } h = 100. \text{ W/m}^2\text{ }^\circ\text{C}$$

$$\begin{cases} T_{ext}(C)_{t=0} = 100. \\ T_{ext}(C)_{t=10^{-2}} = 0. \end{cases}$$

1.4 Initial conditions

$$T(x, 0) = 100^\circ\text{C} \quad \text{for any } x$$

Discretization in time (T):

10 steps for	[0., 1.D-2]	either	$\Delta t = 10^{-3} \text{ s}$
9 steps for	[1.D-2, 1.D-1]	or	$\Delta t = 10^{-2} \text{ s}$
9 steps for	[1.D-1, 1.]	or	$\Delta t = 10^{-1} \text{ s}$
5 steps for	[1., 2.]	or	$\Delta t = 2. \cdot 10^{-1} \text{ s}$
8 steps for	[2., 10.]	or	$\Delta t = 1 \text{ s}$

2 Reference solution

2.1 Method of calculating used for the reference solution

$$\frac{T(x,t) - T_p}{T_0 - T_p} = \sum_{n=1}^{\infty} A_n \exp\left(-\xi_n^2 \frac{\lambda}{\rho C_p L^2} t\right) \cos\left(\xi_n \frac{x}{L}\right)$$

$x =$ X-coordinate
 $t =$ Time
 $T_0 =$ initial Temperature
 $T_p =$ Temperature imposed
 $n =$ 1,2,3, ...

With ξ_n positive roots of $\xi_n \tan \xi_n = hL/\lambda = 10$.

$$\text{and } A_n = \frac{4 \sin \xi_n}{2 \xi_n + \sin(2 \xi_n)}$$

2.2 Results of reference

Temperatures to the points $M1$ ($x=0.02$) and $M2$ ($x=0.08$),
and to various times ($t=0.1, 0.5, 2.0$ and 10.0).

The values of reference are obtained by calculating the first 30 terms of the series (Mathematica).

2.3 Uncertainty on the analytical

solution Solution.

2.4 Bibliographical references

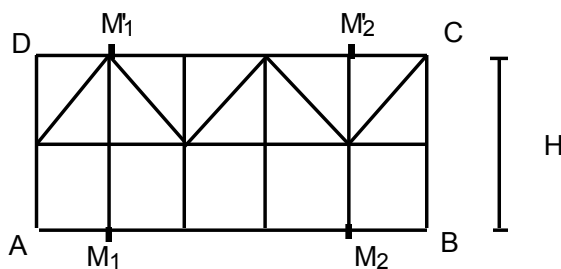
- INCROPERA F.P., OF WITT D.P., Fundamentals of heat and farmhouse transfer. Third Edition. 1990.

3 Modelization A

3.1 Characteristic of modelization

TRIA3, QUAD4

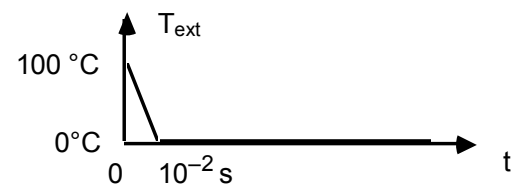
By reason of symmetry, one does not net qu" the half of thickness of the wall. The modelization is made on a height $H=0.1\text{ m}$ with 2 layers of elements.



Conditions limites

sur [AB], [AD] et [CD] : flux nul

sur [BC] : échange h, T_{ext}



Conditions initiales

$T = 100\text{ °C}$

points	nœuds	x	y
M1	N16	0.02	0.0
M2	N6	0.08	0.0
M'1	N14	0.02	0.1
M'2	N4	0.08	0.1

3.2 Characteristics of the mesh

Many nodes: 18

Number of meshes and types: 5 QUAD4, 10 TRIA3

3.3 Values tested

Identification	Reference	Aster	% difference
<i>M1(x=0.02)N16</i>			
$t=0.1$	100.00	99.998	+0.00
$t=0.5$	99.408	99.042	-0.37
$t=2.0$	79.859	79.794	-0.08
$t=10.0$	15.717	16.138	+2.68
<i>M2(x=0.08)N6</i>			
$t=0.1$	93.666	93.380	-0.31
$t=0.5$	63.500	63.813	+0.49
$t=2.0$	35.717	35.667	-0.14
$t=10.0$	6.7948	6.9326	+2.03
<i>M'1(x=0.02)N14</i>			
$t=0.1$	100.00	99.998	+0.00

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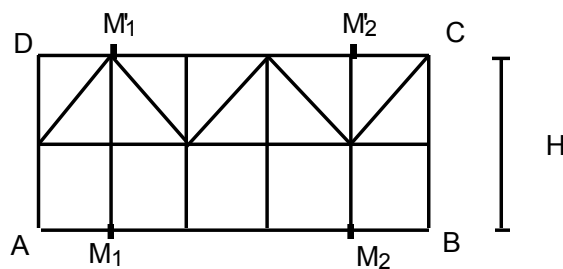
$t=0.5$	99.408	99.077	- 0.33
$t=2.0$	79.859	80.002	+0.18
$t=10.0$	15.717	16.211	+3.14
<hr/>			
$M'2(x=0.08) N4$			
$t=0.1$	93.666	92.895	- 0.82
$t=0.5$	63.500	61.882	- 2.55
$t=2.0$	35.717	35.331	- 1.08
$t=10.0$	6.7948	6.8885	+1.38

4 Modelization B

4.1 Characteristic of modelization

TRIA6, QUAD9

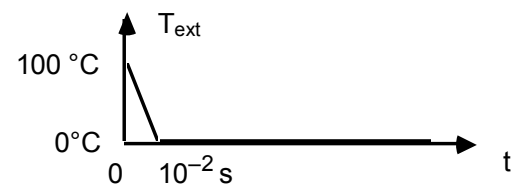
By reason of symmetry, one nets only the half of thickness of the wall. The modelization is made on a height $H = 0.1 m$ with 2 layers of elements.



Conditions limites

sur [AB], [AD] et [CD] : flux nul

sur [BC] : échange h, T_{ext}



Conditions initiales

$T = 100\text{ °C}$

points	nœuds	x	y
M1	N16	0.02	0.0
M2	N6	0.08	0.0
M'1	N14	0.02	0.1
M'2	N4	0.08	0.1

4.2 Characteristics of the mesh

Many nodes: 55

Number of meshes and types: 5 QUAD9, 10 TRIA6

4.3 Values tested

Identification	Reference	Aster	% difference
<i>M1(x=0.02) N18</i>			
$t=0.1$	100.00	100.00	+0.00
$t=0.5$	99.408	99.278	- 0.13
$t=2.0$	79.859	79.898	+0.05
$t=10.0$	15.717	16.043	+2.07
<i>M2(x=0.08) N49</i>			
$t=0.1$	93.666	94.077	+0.44
$t=0.5$	63.500	63.979	+0.75
$t=2.0$	35.717	35.825	+0.30
$t=10.0$	6.7948	6.9321	+2.02
<i>M'1(x=0.02) N12</i>			
$t=0.1$	100.00	100.00	+0.00

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$t=0.5$	99.408	99.311	- 0.10
$t=2.0$	79.859	80.101	+0.30
$t=10.0$	15.717	16.093	+2.39
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$M'2(x=0.08) N30$			
$t=0.1$	93.666	93.469	- 0.21
$t=0.5$	63.500	62.860	- 1.01
$t=2.0$	35.717	35.641	- 0.21
$t=10.0$	6.7948	6.9068	+1.65

5 Summary of results

The modelization `PLAN_DIAG` gives rather satisfactory results. Although the mesh comprises few elements in the thickness, the variation on the temperatures remains lower than 3.2 %.

Although the thermal shock is brutal, the diagonalization of the mass matrix makes it possible to obtain a solution in temperature which does not oscillate during the transient.