TPLP305 – Bar with temperatures imposed with adiabatic interface of X-FEM type

Summarized:

This test is the equivalent in steady linear thermal of the test X-FEM mechanics SSNV173 [V6.04.173]. The purpose of this test is to validate two aspects of elementary computation in the frame of the resolution of thermal problems with X-FEM [R7.02.12]:

- the integration of a discontinuous quantity thanks to a under-cutting of the element
- the enrichment of the shape functions by the function Heaviside

One considers a parallelepipedic bar crossed by an adiabatic interface, the two parts of the bar thus defined are thus thermically isolated one compared to the other. Each one of these parts is then subjected to a temperature imposed on one of its edges.

All the elements X-FEM thermals with Heaviside enrichment are tested, the results are compared with an analytical solution.

1 Problem of reference

1.1 Geometry 3D

the structure is a right parallelepiped at square base, its dimensions (see Figure 1.1-1) are the following ones:

$Lx=1\text{m}$, $Ly=1\text{m}$ and $Lz=5\text{m}$

It interface is defined by functions of level (level sets) directly in the command file using operator `DEFI_FISS_XFEM` [U4.82.08]. It is introduced in the middle of structure by a level set LSN (see Figure 1.1-1) equation:

$LSN\ :\ z$

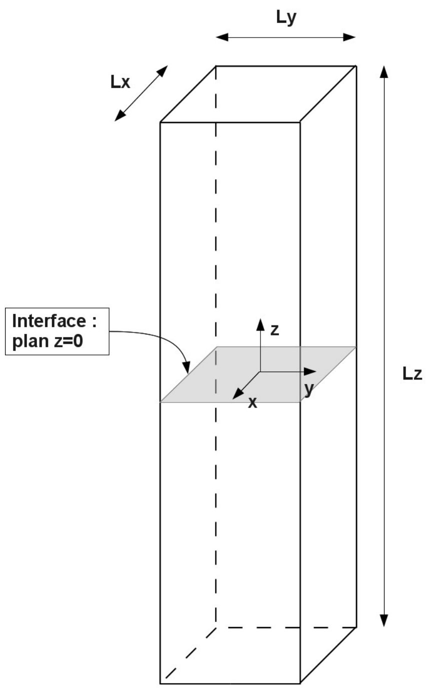

Figure 1.1-1: Geometry of the bar and positioning of the interface

1.2 Geometry 2D

the structure is a rectangle. its dimensions (see Figure 1.2-1) are the following ones:

$Lx=1\text{m}$ and $Ly=5\text{m}$

It interface is defined by functions of level (level sets) directly in the command file using operator `DEFI_FISS_XFEM` [U4.82.08]. It is introduced in the middle of structure by a level set LSN (see Figure 1.2-1) equation:

$LSN\ :\ y$

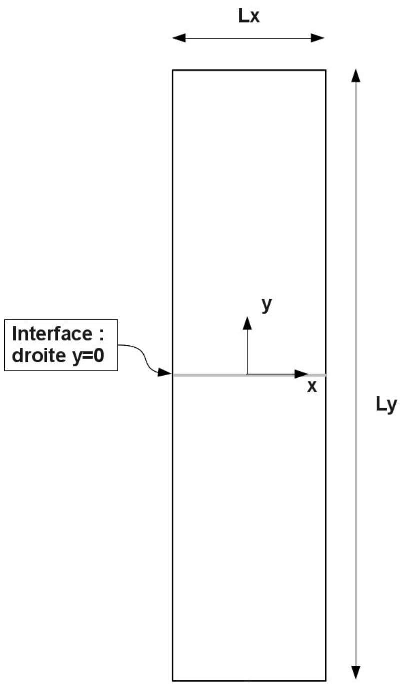

Figure 1.2-1: Geometry of the bar and positioning of the interface

1.3 Properties of the material

- $\lambda = 1\,W.m^{-1}\,°C^{-1}$
- $\rho\,C_p = 2\,J/m^{-3}\,°C^{-1}$

1.4 Boundary conditions and loadings

One imposes a temperature $\bar{T}^{\text{inf}} = 10\,°C$ on the nodes of lower edge of the bar, and $\bar{T}^{\text{sup}} = 20\,°C$ on the nodes of its higher edge.

1.5 Initial conditions

Nothing (the problem is steady)

2 Reference solution

2.1 Method of calculating

It acts of an analytical solution.

The interface being adiabatic, there are on both sides of this one two thermically isolated solids. Each solid sees a temperature imposed on part of its edge, and a null flux on the rest of its edge. The linear and steady problem being, the temperature is thus constant in each part of the bar (and equal to the corresponding imposed temperature), and discontinuous through the interface:

in 3D: $$\begin{cases} T(x,y,z)=\bar{T}^{\text{inf}}=10\,°C,\ \forall(x,y,z)\in\left[\frac{-Lx}{2},\frac{Lx}{2}\right]\times\left[\frac{-Ly}{2},\frac{Ly}{2}\right]\times\left[\frac{-Lz}{2},0\right[\\ T(x,y,z)=\bar{T}^{\text{sup}}=20\,°C,\ \forall(x,y,z)\in\left[\frac{-Lx}{2},\frac{Lx}{2}\right]\times\left[\frac{-Ly}{2},\frac{Ly}{2}\right]\times\left]0,\frac{Lz}{2}\right] \end{cases}$$

in 2D: $$\begin{cases} T(x,y)=\bar{T}^{\text{inf}}=10\,°C,\ \forall(x,y)\in\left[\frac{-Lx}{2},\frac{Lx}{2}\right]\times\left[\frac{-Ly}{2},0\right[\\ T(x,y)=\bar{T}^{\text{sup}}=20\,°C,\ \forall(x,y)\in\left[\frac{-Lx}{2},\frac{Lx}{2}\right]\times\left]0,\frac{Ly}{2}\right] \end{cases}$$

In the modelizations presented to the following paragraphs, the approximation of the field of temperature is enriched by a function by Heaviside in order to represent introduced discontinuity. The nodes whose support is crossed by the interface carry “classical” degrees of freedom and “Heaviside”: one cannot then directly compare their values with the analytical values obtained above.
In order to be able to test the values of these degrees of freedom, one places oneself if the mesh of the bar consists of 5 regular hexahedrons on side 1m, the central mesh is thus crossed in his medium by the interface. The solution being constant in the directions x and y, one can be brought back to consider the unidimensional element are equivalent represented on Figure 2.1-1.

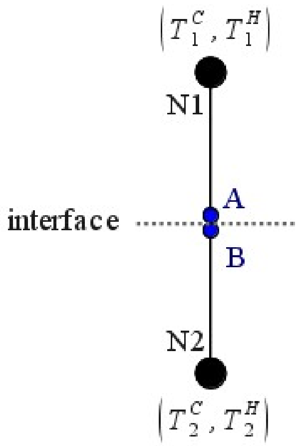

Figure 2.1-1: Linear element 1D are equivalent

One notes N1 and N2 the two nodes of this element, φ_1 and φ_2 the associated shape functions, (T_1^C, T_1^H) and (T_2^C, T_2^H) the couples of associated degrees of freedom. One notes moreover x formula it of space, A the point located formulates $x=0^+$ formulates B the point located in formula $x=0^{-}$. Lastly, it is supposed that the function of Heaviside is worth 1 on the segment $\mathrm{AN1}$ and -1 on the segment $\mathrm{BN2}$.

In any point x element, the field of temperature is expressed by the following relation:

$$T(x)=\varphi_1(x)T_1^C+\varphi_2(x)T_2^C+H(x)\left(\varphi_1(x)T_1^H+\varphi_2(x)T_2^H\right)$$

at the points $\mathrm{N1}$, $\mathrm{N2}$, A and B one a:

$$\begin{cases}\text{En N1 : } H(x)=1,\ \varphi_1(x)=1,\varphi_2(x)=0\\ \text{En A : } H(x)=1,\ \varphi_1(x)=\varphi_2(x)=1/2\\ \text{En B : } H(x)=-1,\varphi_1(x)=\varphi_2(x)=1/2\\ \text{En N2 : } H(x)=-1,\varphi_1(x)=0,\varphi_2(x)=1\end{cases}$$

what leads to the linear system:

$$\begin{cases}T_1^C+T_1^H=\bar{T}^{\sup}\ (\mathrm{N1})\\ \frac{1}{2}T_1^C+\frac{1}{2}T_1^H+\frac{1}{2}T_2^C+\frac{1}{2}T_2^H=\bar{T}^{\sup}\ (\mathrm{A})\\ \frac{1}{2}T_1^C-\frac{1}{2}T_1^H+\frac{1}{2}T_2^C-\frac{1}{2}T_2^H=\bar{T}^{\inf}\ (\mathrm{B})\\ T_2^C-T_2^H=\bar{T}^{\inf}\ (\mathrm{N2})\end{cases}$$

admitting the following solution:

$$\begin{cases}T_1^C=T_2^C=\dfrac{\bar{T}^{\sup}+\bar{T}^{\inf}}{2}\\ T_1^H=T_2^H=\dfrac{\bar{T}^{\sup}-\bar{T}^{\inf}}{2}\end{cases}\text{, numerical application: }\begin{cases}T_1^C=T_2^C=15°C\\ T_1^H=T_2^H=5°C\end{cases}$$

2.2 Quantities and results of reference

One tests initially the values of the degrees of freedom classic TEMP and Heaviside H1 (noted respectively T^C and T^H in paragraph 4) of the field of temperature in output of operator THER_LINEAIRE [U4.54.01], with the nodes located just in lower part and the top of the interface. One ensures oneself to find well the values determined in paragraph 4.

Identification	Reference
TEMP for all the nodes located just in dessous/au above of the interface	$15.°C$
H1 for all the nodes located just in dessous/au above of the interface	$5.°C$

operator POST_MAIL_XFEM [U4.82.21] makes it possible to net cracks represented by the method X-FEM. Operator POST_CHAM_XFEM [U4.82.22], then allows to export the X-FEM results on this new mesh. These two operators are to be used only in a posterior way with computation at sights of postprocessing. They make it possible to generate nodes right in lower part and with the top of the interface and to display the field of the nodal unknowns there (here the field of temperature). One then tests the value of the field of temperature TEMP (noted T in paragraph 4) in output of

POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface. One ensures oneself to find well the values determined in paragraph 4.

Identification	Reference
TEMP for all the nodes located just below interface	$10.°C$
TEMP for all the nodes located just at the top of the interface	$20.°C$

Lastly, one tests the value of single component TEMP of field TEMP_ELGA (field of temperature by elements to Gauss points, calculated only on the elements X-FEM by the operator THER_LINEAIRE) on Gauss points of the elements nouveau riches.

Identification	Reference
TEMP on Gauss points located below the interface	$10.°C$
TEMP on Gauss points located at the top of the interface	$20.°C$

Remark: *The elements X-FEM contain a significant number of points of integration (being able to amount to 480 for the thermal elements leaning on mesh HEXA8) because of cutting in subelements. Moreover, as the position of crack conditions result procedure of under-cutting, this number is variable for the same type of element. One thus does not test the value of component TEMP of field TEMP_ELGA on all the points of integration of the elements X-FEM, and one is satisfied to test it:*

- *in only one point of integration for the elements nouveau riches not being crossed by the interface (the value being constant on the element)*
- *in two points of integration for the elements nouveau riches crossed by the interface (the first located below the interface, the second with the top), and this only for the modelizations A, F, and I (structured mesh in HEXA8 or QUAD4)*

3 Modelization A

3.1 Characteristic of the modelization

One uses the modelization `3D` THERMAL `phenomenon`.

3.2 Characteristics of the mesh

The mesh comprises 5 regular hexahedrons of type `HEXA8`.

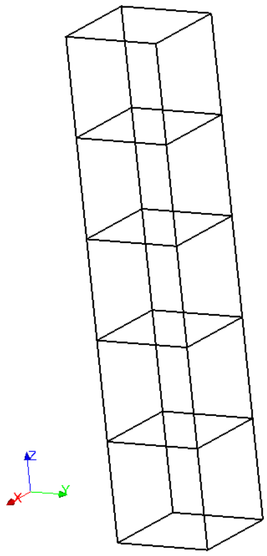

Figure 3.2-1: Mesh A

3.3 Quantities tested and results

One tests initially the values of the degrees of freedom classic `TEMP` and Heaviside `H1` of the field of temperature in output of operator `THER_LINEAIRE`, with the nodes located just in lower part (4 nodes) and at the top of the interface (4 nodes).

Standard	identification of reference	Value of reference	Tolerance
All the nodes located just in lower part/at the top of the interface - $TEMP$	`"ANALYTIQUE"`	15	0,1%
All nodes located just in lower part/at the top of the interface - $H1$	`"ANALYTIQUE"`	5	0,1%

One tests then the value of degree of freedom `TEMP` of the field of temperature in output of `POST_CHAM_XFEM`, with the nodes located just in lower part and the top of the interface.

Standard	identification of reference	Value of reference	Tolerance
All the nodes located just below the interface	`"ANALYTIQUE"`	10	0,1%

- $TEMP$			
All nodes located just at the top of the interface - $TEMP$	`"ANALYTIQUE"`	20	0,1%

One tests finally the value of component `TEMP` of field `TEMP_ELGA` on Gauss points located in lower part and at the top of the interface (*cf* notices page 6).

Standard	**identification of reference**	**Value of reference**	**Tolerance**
On Gauss points located below the interface - $TEMP$	`"ANALYTIQUE"`	10	0,1%
On Gauss points located at the top of the interface - $TEMP$	`"ANALYTIQUE"`	20	0,1%

4 Modelization B

This modelization is exactly the same one as modelization A. the only difference lies in mesh: the HEXA8 of the mesh A are cut out in PENTA6.

4.1 Characteristics of the modelization

One uses the modelization 3D THERMAL phenomenon.

4.2 Characteristics of the mesh

The mesh comprises 10 meshes type PENTA6.

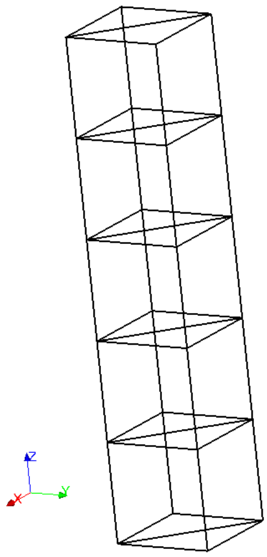

Figure 4.2-1: Mesh B

4.3 Quantities tested and results

One tests initially the values of the degrees of freedom classic TEMP and Heaviside H1 of the field of temperature in output of operator THER_LINEAIRE, with the nodes located just in lower part (4 nodes) and at the top of the interface (4 nodes).

Standard	identification of reference	Value of reference	Tolerance
All the nodes located just in lower part/at the top of the interface - $TEMP$	"ANALYTIQUE"	15	0,1%
All nodes located just in lower part/at the top of the interface - $H1$	"ANALYTIQUE"	5	0,1%

One tests then the value of degree of freedom TEMP of the field of temperature in output of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Standard	identification of	Value of reference	Tolerance

	reference		
All the nodes located just below the interface - *TEMP*	"ANALYTIQUE"	10	0,1%
All nodes located just at the top of the interface - *TEMP*	"ANALYTIQUE"	20	0,1%

One tests finally the value of component TEMP of field TEMP_ELGA on Gauss points located in lower part and at the top of the interface (*cf* notices page 6).

Standard	identification of reference	Value of reference	Tolerance
On Gauss points located below the interface - *TEMP*	"ANALYTIQUE"	10	0,1%
On Gauss points located at the top of the interface - *TEMP*	"ANALYTIQUE"	20	0,1%

5 Modelization C

This modelization is exactly the same one as modelization A. the only difference lies in mesh: the HEXA8 of the mesh A are cut out in TETRA4.

5.1 Characteristics of the modelization

One uses the modelization 3D THERMAL phenomenon.

5.2 Characteristics of the mesh

The mesh comprises 30 meshes type TETRA4.

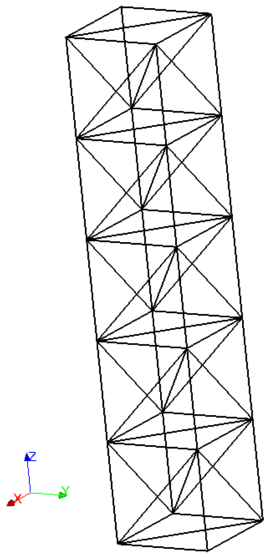

Figure 5.2-1: Mesh C

5.3 Quantities tested and results

One tests initially the values of the degrees of freedom classic TEMP and Heaviside H1 of the field of temperature in output of operator THER_LINEAIRE, with the nodes located just in lower part (4 nodes) and at the top of the interface (4 nodes).

Standard	identification of reference	Value of reference	Tolerance
All the nodes located just in lower part/at the top of the interface - *TEMP*	"ANALYTIQUE"	15	0,1%
All nodes located just in lower part/at the top of the interface - *H1*	"ANALYTIQUE"	5	0,1%

One tests then the value of degree of freedom TEMP of the field of temperature in output of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Standard	identification of	Value of reference	Tolerance

	reference		
All the nodes located just below the interface - $TEMP$	"ANALYTIQUE"	10	0,1%
All nodes located just at the top of the interface - $TEMP$	"ANALYTIQUE"	20	0,1%

One tests finally the value of component TEMP of field TEMP_ELGA on Gauss points located in lower part and at the top of the interface (*cf* notices page 6).

Standard	identification of reference	Value of reference	Tolerance
On Gauss points located below the interface - $TEMP$	"ANALYTIQUE"	10	0,1%
On Gauss points located at the top of the interface - $TEMP$	"ANALYTIQUE"	20	0,1%

6 Modelization D

This modelization is exactly the same one as modelization A. the only difference lies in mesh: one invites MACR_ADAP_MAIL [U7.03.01] to refine some meshes HEXA8, which results in to generate meshes PYRA5 (and TETRA4).

6.1 Characteristics of the modelization

One uses the modelization 3D THERMAL phenomenon.

6.2 Characteristics of the mesh

The mesh comprises 3 meshes type HEXA8, 8 meshes of type TETRA4 and 10 meshes of type PYRA5.

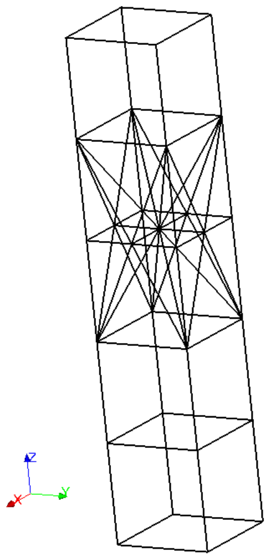

Figure 6.2-1: Mesh D

6.3 Quantities tested and results

One tests initially the values of the degrees of freedom classic TEMP and Heaviside H1 of the field of temperature in output of operator THER_LINEAIRE, with the nodes located just in lower part (4 nodes) and at the top of the interface (4 nodes).

Standard	identification of reference	Value of reference	Tolerance
All the nodes located just in lower part/at the top of the interface - *TEMP*	"ANALYTIQUE"	15	0,1%
All nodes located just in lower part/at the top of the interface - *H1*	"ANALYTIQUE"	5	0,1%

One tests then the value of degree of freedom TEMP of the field of temperature in output of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Standard	identification of reference	Value of reference	Tolerance
All the nodes located just below the interface - *TEMP*	"ANALYTIQUE"	10	0,1%
All nodes located just at the top of the interface - *TEMP*	"ANALYTIQUE"	20	0,1%

One tests finally the value of component TEMP of field TEMP_ELGA on Gauss points located in lower part and at the top of the interface (*cf* notices page 6).

Standard	identification of reference	Value of reference	Tolerance
On Gauss points located below the interface - *TEMP*	"ANALYTIQUE"	10	0,1%
On Gauss points located at the top of the interface - *TEMP*	"ANALYTIQUE"	20	0,1%

7 Modelization E

For this modelization, one changes the values of dimensions of the bar, the equation of the level-set, as well as the mesh. The boundary conditions remain unchanged.
The geometrical position of the interface is selected of kind to lead to a bad conditioning of the stiffness matrix, in order to validate the algorithm of zero setting of the Heaviside degrees of freedom automatically implemented in this configuration.

7.1 Characteristics of the modelization

For this modelization, the values dimensions of the bar are:

$Lx=1\text{m}$, $Ly=1\text{m}$ and $Lz=7\text{m}$

as well as the equation of the level-set, one considers here a plane interface of norm $n=(-1,1,1)^T$ and passing by the point A of coordinates $(0.5,\ -0.5(1-\delta),\ 0.5)$.

The interface is introduced by a level set LSN (see Figure 7.2-1) equation:

$$LSN\ :\ -x+y+z-(-0.5+\delta)$$

Lastly, one uses the modelization `3D` THERMAL `phenomenon`.

7.2 Characteristics of the mesh

The mesh comprises 7 meshes type `HEXA8`. This choice makes it possible to be ensured to have "classical" elements (not X-FEM) at the two ends of the bar (see Figure 7.2-1), and thus to impose the boundary conditions on nodes not carrying degrees of freedom nouveau riches.

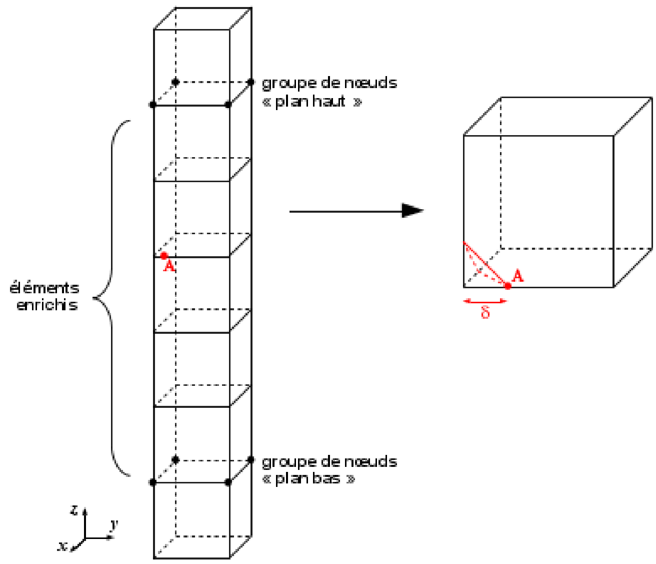

Figure 7.2-1: Mesh E and position of the level-set

7.3 Quantities tested and results

One tests initially the values of classical degrees of freedom `TEMP` and the Heaviside `H1` of the field of temperature in output of operator `THER_LINEAIRE`, on the four nodes which set up plane group the "low" and on the four nodes which set up plane group the "high" (see Figure 7.2-1). These nodes being divided by classical elements and nouveau riches, the Heaviside degrees of freedom `H1` must be null and the classical degrees of freedom TEMP must correspond to the physical temperature ($\bar{T}^{\text{inf}}$ or $\bar{T}^{\text{sup}}$).

Standard	identification of reference	Value of reference	Tolerance
All the nodes of plane group the "low" - $TEMP$	ANALYTIQUE formulates"	10	0,1%
All the nodes of plane group the "low" - $H1$	"ANALYTIQUE"	0	0,1%
All the nodes of plane group the "high" - $TEMP$	ANALYTIQUE formulates"	20	0,1%
All the nodes of plane group the "high" - $H1$	"ANALYTIQUE"	0	0,1%

One tests then the value of degree of freedom TEMP of the field of temperature in output of POST_CHAM_XFEM, on the four nodes which set up plane group the "low" and on the four nodes which set up plane group the "high" (see Figure 7.2-1).

Standard	identification of reference	Value of reference	Tolerance
All the nodes of plane group the "low" - $TEMP$	ANALYTIQUE formulates"	10	0,1%
All the nodes of plane group the "high" - $TEMP$	"ANALYTIQUE"	20	0,1%

One tests finally the value of component TEMP of field TEMP_ELGA on Gauss points located in lower part and at the top of the interface (*cf* notices page 6).

Standard	identification of reference	Value of reference	Tolerance
On Gauss points located below the interface - $TEMP$	"ANALYTIQUE"	10	0,1%
On Gauss points located at the top of the interface - $TEMP$	"ANALYTIQUE"	20	0,1%

8 Modelization F

This modelization corresponds to the problem of reference 2D described on page 2.

8.1 Characteristics of the modelization

One uses the modelization PLANE of the THERMAL phenomenon.

8.2 Characteristics of the mesh

The mesh comprises 5 meshes type QUAD4.

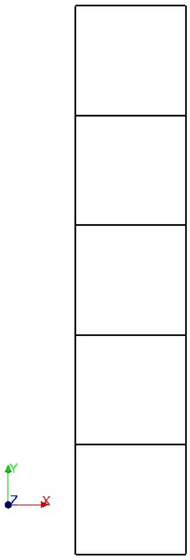

Figure 8.2-1: Mesh F

8.3 Quantities tested and results

One tests initially the values of the degrees of freedom classic TEMP and Heaviside H1 of the field of temperature in output of operator THER_LINEAIRE, with the nodes located just in lower part (2 nodes) and at the top of the interface (2 nodes).

Standard	identification of reference	Value of reference	Tolerance
All the nodes located just in lower part/at the top of the interface - $TEMP$	"ANALYTIQUE"	15	0,1%
All nodes located just in lower part/at the top of the interface - $H1$	"ANALYTIQUE"	5	0,1%

One tests then the value of degree of freedom TEMP of the field of temperature in output of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Standard	identification of reference	Value of reference	Tolerance

All the nodes located just below the interface - *TEMP*	"ANALYTIQUE"	10	0,1%
All nodes located just at the top of the interfaceformule *TEMP* -	"ANALYTIQUE"	20	0,1%

One tests finally the value of component TEMP of field TEMP_ELGA on Gauss points located in lower part and at the top of the interface (*cf* notices page 6).

Standard	identification of reference	Value of reference	Tolerance
On Gauss points located below the interface - *TEMP*	"ANALYTIQUE"	10	0,1%
On Gauss points located at the top of the interface - *TEMP*	"ANALYTIQUE"	20	0,1%

9 Modelization G

This modelization is exactly the same one as the modelization F. the only difference lies in mesh: the QUAD4 of the mesh F are cut out in TRIA3.

9.1 Characteristics of the modelization

One uses the modelization PLANE of the THERMAL phenomenon.

9.2 Characteristics of the mesh

The mesh comprises 10 meshes type TRIA3.

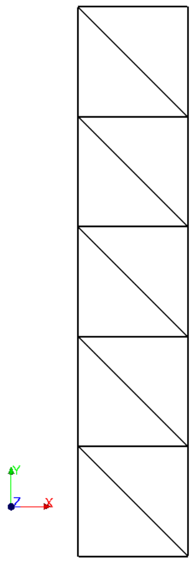

Figure 9.2-1: Mesh G

9.3 Quantities tested and results

One tests initially the values of the degrees of freedom classic TEMP and Heaviside H1 of the field of temperature in output of operator THER_LINEAIRE, with the nodes located just in lower part (2 nodes) and at the top of the interface (2 nodes).

Standard	identification of reference	Value of reference	Tolerance
All the nodes located just in lower part/at the top of the interface - *TEMP*	"ANALYTIQUE"	15	0,1%
All nodes located just in lower part/at the top of the interface - *H1*	"ANALYTIQUE"	5	0,1%

One tests then the value of degree of freedom TEMP of the field of temperature in output of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Standard	identification of	Value of reference	Tolerance

	reference		
All the nodes located just below the interface - $TEMP$	"ANALYTIQUE"	10	0,1%
All nodes located just at the top of the interfaceformule $TEMP$ -	"ANALYTIQUE"	20	0,1%

One tests finally the value of component TEMP of field TEMP_ELGA on Gauss points located in lower part and at the top of the interface (*cf* notices page 6).

Standard	identification of reference	Value of reference	Tolerance
On Gauss points located below the interface - $TEMP$	"ANALYTIQUE"	10	0,1%
On Gauss points located at the top of the interface - $TEMP$	"ANALYTIQUE"	20	0,1%

10 Modelization H

For this modelization, one changes the values of dimensions of the bar, the equation of the level-set, as well as the mesh. The boundary conditions remain unchanged.
The geometrical position of the interface is selected of kind to lead to a bad conditioning of the stiffness matrix, in order to validate the algorithm of zero setting of the Heaviside degrees of freedom automatically implemented in this configuration.

10.1 Characteristics of the modelization

For this modelization, the values dimensions of the bar are:

$Lx=1\text{m}$ and $Ly=7\text{m}$

as well as the equation of the level-set, one considers here a plane interface of norm $n=(1,1)^T$ and passing by the point A coordinates $(-0.5(1-\delta),\ 0.5)$.

The interface is introduced by a level set LSN (see Figure 10.2-1) equation:

$LSN\ :\ x+y-\delta$

Lastly, one uses the modelization PLANE of the THERMAL phenomenon.

10.2 Characteristics of the mesh

The mesh comprises 7 meshes type QUAD4. This choice makes it possible to be ensured to have “classical” elements (not X-FEM) at the two ends of the bar (see Figure 10.2-1), and thus to impose the boundary conditions on nodes not carrying degrees of freedom nouveau riches.

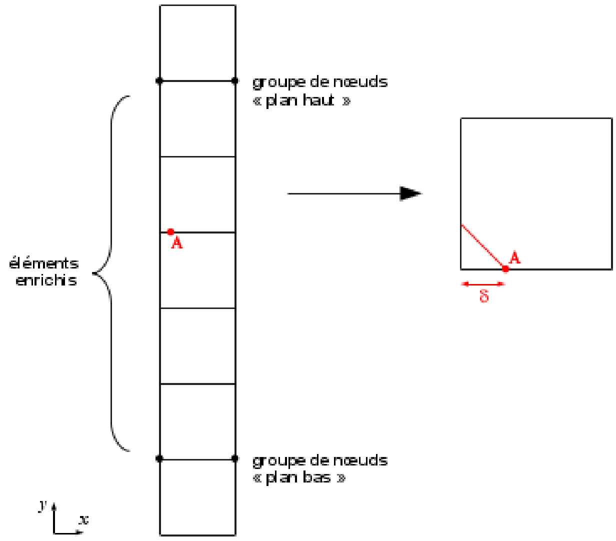

Figure 10.2-1: mesh H and position of the level-set

10.3 Quantities tested and results

One tests initially the values of the degrees of freedom classic TEMP and Heaviside H1 of the field of temperature in output of operator THER_LINEAIRE, on the two nodes which set up plane group the “low” and on the two nodes which set up plane group the “high” (see Figure 10.2-1). These nodes being divided by classical elements and nouveau riches, the Heaviside degrees of freedom H1 must be null and the classical degrees of freedom TEMP must correspond to the physical temperature ($\bar{T}^{\mathrm{inf}}$ or $\bar{T}^{\mathrm{sup}}$).

Standard	identification of reference	Value of reference	Tolerance
All the nodes of plane group the "low" - *TEMP*	ANALYTIQUE formulates"	10	0,1%
All the nodes of plane group the "low" - *H1*	"ANALYTIQUE the "	high	"
" 0 0,1% All the nodes of plane group - *TEMP*	ANALYTIQUE formulates"	20	0,1%
All the nodes of plane group the "high" - *H1*	"ANALYTIQUE"	0	0,1%

One tests then the value of degree of freedom TEMP of the field of temperature in output of POST_CHAM_XFEM, on the two nodes which set up plane group the "low" and on the two nodes which set up plane group the "high" (see Figure 10.2-1).

Standard	identification of reference	Value of reference	Tolerance
All the nodes of plane group the "low" - *TEMP*	ANALYTIQUE formulates"	10	0,1%
All the nodes of plane group the "high" - *TEMP*	"ANALYTIQUE"	20	0,1%

One tests finally the value of component TEMP of field TEMP_ELGA on Gauss points located in lower part and at the top of the interface (*cf* notices page 6).

Standard	identification of reference	Value of reference	Tolerance
On Gauss points located below the interface - *TEMP*	"ANALYTIQUE"	10	0,1%
On Gauss points located at the top of the interface - *TEMP*	"ANALYTIQUE"	20	0,1%

11 Modelization I

This modelization leans on all the characteristics of the problem of reference 2D described on page 2. , except that an axisymmetric modelization is considered. It is thus about a cylindrical bar, but the analytical solution described on page 4 remains valid for this problem.

11.1 Characteristics of the modelization

For this modelization, the bar is cylindrical and its dimensions are the following ones:

- radius $Lx=0{,}5\,m$
- height $Ly=5\mathrm{m}$

the interface is introduced with middle height of the bar by a level set LSN equation:

$$LSN\ :\ y$$

One uses modelization AXIS of the THERMAL phenomenon.

11.2 Characteristics of the mesh

The mesh comprises 5 meshes type QUAD4.

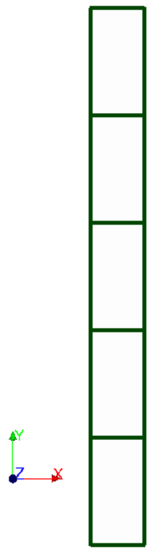

Figure 11.2-1: Mesh I

11.3 Quantities tested and results

One tests initially the values of the degrees of freedom classic TEMP and Heaviside H1 of the field of temperature in output of operator THER_LINEAIRE, with the nodes located just in lower part (2 nodes) and at the top of the interface (2 nodes).

Standard	identification of reference	Value of reference	Tolerance
All the nodes located just in lower part/at the	"ANALYTIQUE"	15	0,1%

top of the interface - $TEMP$			
All nodes located just in lower part/at the top of the interface - $H1$	"ANALYTIQUE"	5	0,1%

One tests then the value of degree of freedom TEMP of the field of temperature in output of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Standard	identification of reference	Value of reference	Tolerance
All the nodes located just below the interface - $TEMP$	"ANALYTIQUE"	10	0,1%
All nodes located just at the top of the interfaceformule $TEMP$ -	"ANALYTIQUE"	20	0,1%

One tests finally the value of component TEMP of field TEMP_ELGA on Gauss points located in lower part and at the top of the interface (*cf* notices page 6).

Standard	identification of reference	Value of reference	Tolerance
On Gauss points located below the interface - $TEMP$	"ANALYTIQUE"	10	0,1%
On Gauss points located at the top of the interface - $TEMP$	"ANALYTIQUE"	20	0,1%

12 Modelization J

This modelization leans on all the characteristics of the problem of reference 2D described on page 2. , except that an axisymmetric modelization is considered. It is thus about a cylindrical bar, but the analytical solution described on page 4 remains valid for this problem.

12.1 Characteristics of the modelization

For this modelization, the bar is cylindrical and its dimensions are the following ones:

- radius $Lx=0,5\,m$
- height $Ly=5\mathrm{m}$

the interface is introduced with middle height of the bar by a level set LSN equation:

$$LSN \; : \; y$$

One uses modelization AXIS of the THERMAL phenomenon.

12.2 Characteristics of the mesh

The mesh comprises 5 meshes type TRIA3.

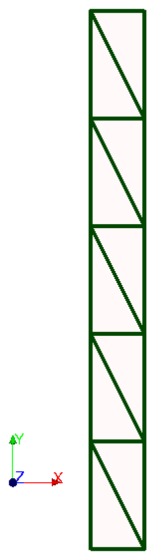

Figure 12.2-1: Mesh J

12.3 Quantities tested and results

One tests initially the values of the degrees of freedom classic TEMP and Heaviside H1 of the field of temperature in output of operator THER_LINEAIRE, with the nodes located just in lower part (2 nodes) and at the top of the interface (2 nodes).

Standard	identification of reference	Value of reference	Tolerance
All the nodes located just in lower part/at the	"ANALYTIQUE"	15	0,1%

top of the interface - $TEMP$			
All nodes located just in lower part/at the top of the interface - $H1$	"ANALYTIQUE"	5	0,1%

One tests then the value of degree of freedom TEMP of the field of temperature in output of POST_CHAM_XFEM, with the nodes located just in lower part and the top of the interface.

Standard	identification of reference	Value of reference	Tolerance
All the nodes located just below the interface - $TEMP$	"ANALYTIQUE"	10	0,1%
All nodes located just at the top of the interfaceformule $TEMP$ -	"ANALYTIQUE"	20	0,1%

One tests finally the value of component TEMP of field TEMP_ELGA on Gauss points located in lower part and at the top of the interface (*cf* notices page 6).

Standard	identification of reference	Value of reference	Tolerance
On Gauss points located below the interface - $TEMP$	"ANALYTIQUE"	10	0,1%
On Gauss points located at the top of the interface - $TEMP$	"ANALYTIQUE"	20	0,1%

13 Summary of the results

the purposes of this test are reached:

- To validate the taking into account of enrichment by the Heaviside function of the classical shape functions.
- Moreover, the modelization E and H made it possible to wrongly validate the suppression of the degrees of freedom nouveau riches (which is made at the assembly time terms of the stiffness matrix), as well as the algorithm of suppression of the Heaviside degrees of freedom implemented for reasons of conditioning of matrix