TPLP107 – Method of the solutions manufactured in thermal 2D

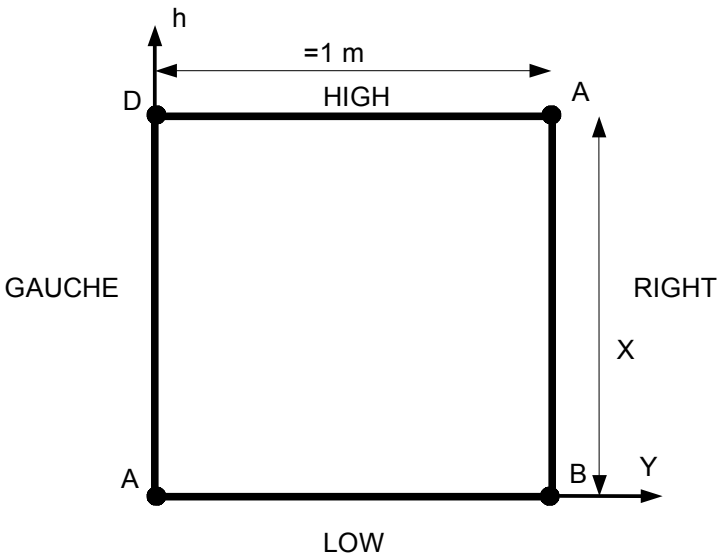
Summarized:

The purpose of this test is to 2D check the thermal modelization thanks to the method of the manufactured solutions [bib1].

1 Problem of reference

1.1 Geometry

One considers a square of with dimensions $1\,m$.



1.2 Properties of the thermal

$\lambda = 15.\,W/m.\,°C$ material Conductivity

1.3 Boundary conditions and loadings

On edges GAUCHE , BAS and HAUT , one forces a temperature (see paragraph 3.
On edge DROITE , one forces a flux (see paragraph 3.
In all the field, one forces a source (see paragraph 3.

1.4 Initial conditions

Nothing

2 Reference solution

2.1 Method of calculating

the analytical reference solution is given by:

$$T = 100 \times (X^6 + Y^6) \quad (1)$$

the conditions of Dirichlet, Neumann and the source term are obtained by the method of the manufactured solutions [bib1].

2.2 Quantities and results of reference

the value of the difference between solutions analytical and calculated on mesh: $\sum^{\text{noeuds } n} |T_{\text{n}}^{\text{calc}} - T_{\text{n}}^{\text{ref}}|$.

In the case of the modelizations which carry out an analysis of convergence with the smoothness of the mesh, the velocity of convergence with the smoothness of the mesh of the solution calculated towards the analytical solution in norm L_2, is the greatest reality $\alpha > 0$ such as $\|T^{\text{calc}} - T^{\text{ref}}\|_{0,\Omega} < C \times h^{\alpha}$ where C is independent of h .

2.3 Uncertainties on the solution

No

2.4 bibliographical References

1 Document U2.08.08, Use of the Method of the Solutions Manufactured for the software validation, Documentation U2 of Code_Aster

3 Modelization A

3.1 Characteristic of the modelization

One uses a modelization PLANE.

3.2 Characteristics of the mesh

The mesh contains 56 elements of the type SEG3 and 392 elements of the type TRIA6.

3.3 Quantities tested and results

One tests the sum of the absolute values of the difference between the calculated solution and the analytical solution.

Standard	identification of reference	Value of reference
$\sum_{n}^{\text{noeuds}} \lvert T_{\text{n}}^{\text{calc}} - T_{\text{n}}^{\text{ref}} \rvert$	"NON_REGRESSION"	0.59734630434863

4 Modelization B

4.1 Characteristic of the modelization

One uses a modelization PLANE.

4.2 Characteristics of the mesh

One carries out a study of convergence with the smoothness of the mesh of the solution calculated towards the analytical solution. A succession of meshes obtained by uniform refinement using command MACR_ADAP_MAIL is used:

- mesh 0: 4 SEG3, 2 TRIA6
- mesh 1: 8 SEG3, 8 TRIA6
- mesh 2: 16 SEG3, 32 TRIA6
- mesh 3: 32 SEG3, 128 TRIA6

4.3 Quantities tested and results

One tests the velocity of convergence with the smoothness of the mesh of the solution calculated towards the analytical solution in norm L_2, that is to say the greatest reality $\alpha>0$ such as $\|T^{\text{calc}}-T^{\text{ref}}\|_{0,\Omega}<C\times h^{\alpha}$ where C is independent of h.
One tests also the sum of the absolute values of the difference between the calculated solution and the analytical solution.

Standard	identification of reference	Value of reference
$\sum_{\text{noeuds } n} \lvert T_{\text{n}}^{\text{calc}}-T_{\text{n}}^{\text{ref}}\rvert$	"NON_REGRESSION"	0.59734630434863
α	"ANALYTIQUE"	3.0

5 Summary of the results

the results are in very good agreement with the theory.