TPLV106 - Steady nonlinear thermal out of Summarized

mobile coordinate system:

This elementary test makes it possible to treat a reducible three-dimensional example with a problem a variable of space in steady nonlinear thermal out of mobile coordinate system (problem of convection-diffusion).

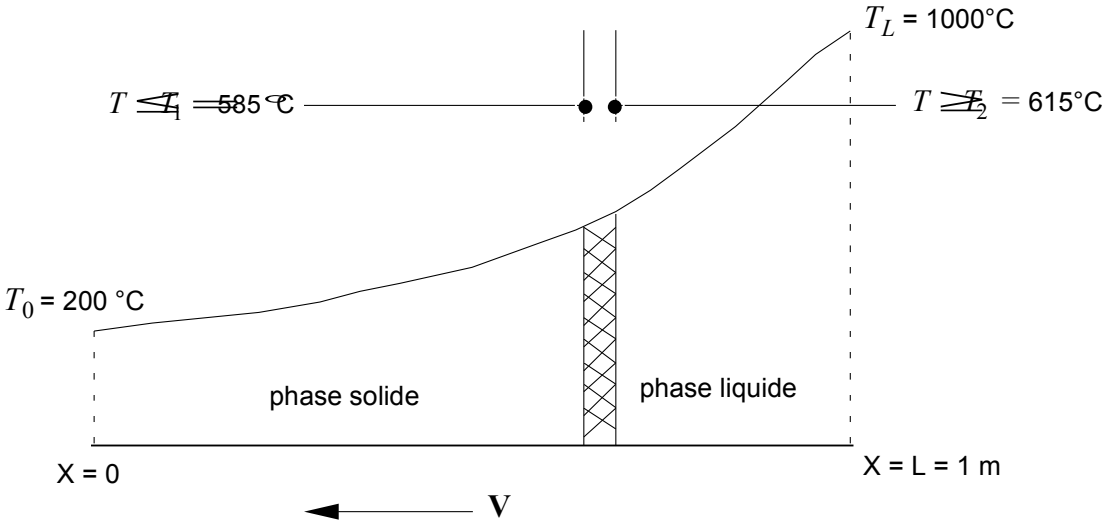
It also makes it possible to check the taking into account of a solid phase change/liquidates by *Code_Aster*.

The reference solution is analytical and the variations with the results got by *Code_Aster* are lower than 1%. The problem is modelled in the plane case.

1 Problem of reference

1.1 Geometry

Is a bar moving, at the speed V , the right of conditions of temperatures imposed in $X=0$ and $X=L$ expressed in a fixed reference frame (compared to the bar moving).



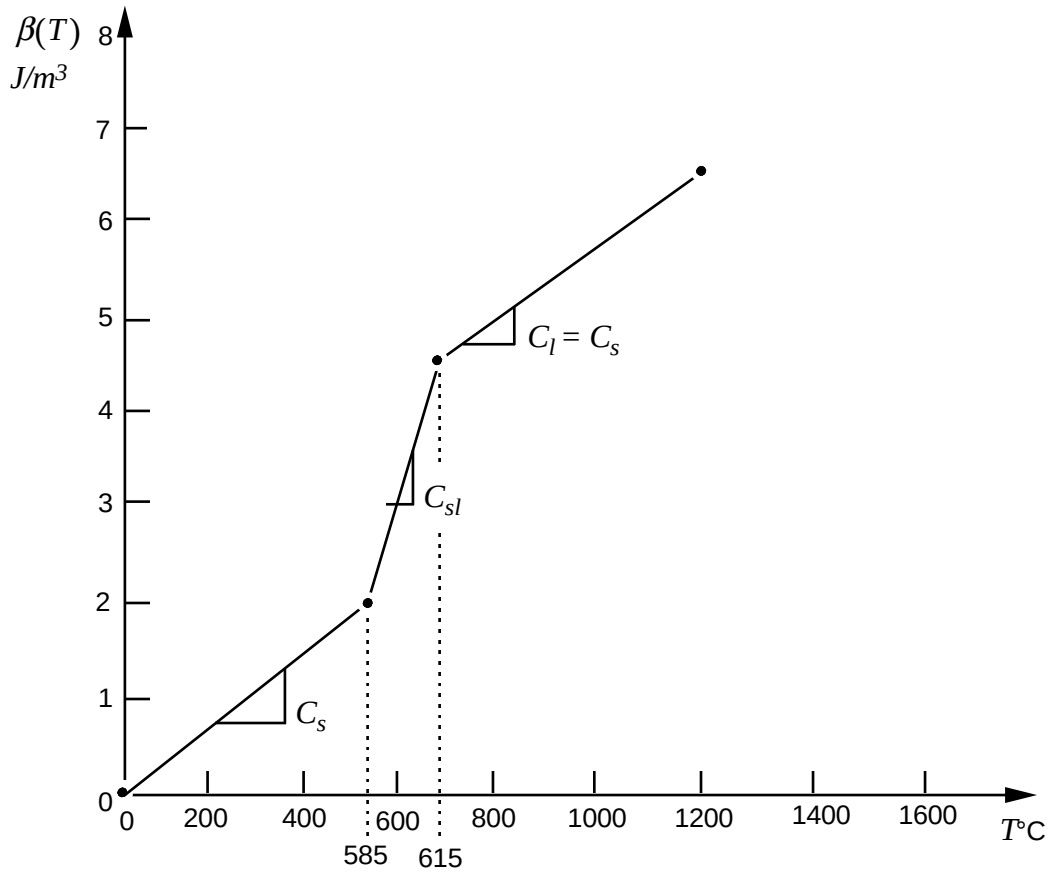
1.2 Properties of the materials

thermal conductivity is constant: $K=150\,W/m°C$
the function enthalpy is such as:

$$\beta(T)=\begin{cases} C_S T & ;T\leq T_1 \\ C_S T + C_{Sl}(T-T_1) & ;T_1\leq T\leq T_2 \\ C_S T + C_{Sl}(T_2-T_1)+C_l(T-T_2) & ;T_1\leq T\leq T_2 \end{cases}$$

with the following values:

$$C_S=C_l=1/3.10^7\,J/m^3°C$$
$$C_{Sl}=8.33310^7\,J/m^3°C$$
$$T_1=585.°C$$
$$T_2=615°C$$



1.3 Boundary conditions and loadings

Temperatures imposed at the ends

$$T_0 = 200\,°C \quad \text{for} \quad x = 0$$

$$T_L = 1000\,°C \qquad x = L = 1\text{m}$$

Rate of travel of solid: $V = 10^{-4}\,m/s$

2 Reference solution

2.1 Method of calculating used for the reference solution

result of reference is of the semi-analytical type. The equation 1D to be solved is the following one:

$$\begin{cases} V\,\beta(T)_{,x} - K\,T_{,xx} = 0 \\ \text{avec } T_{(x=0)} = T_0 \text{ et } T_{(x=L)} = T_L \end{cases} \qquad \text{éq 2.1-1}$$

by integrating the equation [éq 2.1-1] one obtains:

$$\frac{V}{K}\beta(T) - \frac{dT}{dx} = A \qquad \text{éq 2.1-2}$$

where A is a constant depending on the boundary conditions, the ratio $\frac{V}{K}$ and the function enthalpy $\beta(T)$.

This constant will be analytically given.

The equation [éq 2.1-2] led to:

$$x = \int_{T_0}^{T(x)} \frac{dT}{A + \frac{V}{K}\beta(T)} \qquad \text{éq 2.1-3}$$

which must check:

$$L = \int_{T_0}^{T_L} \frac{dT}{A + \frac{V}{K}\beta(T)} \qquad \text{éq 2.1-4}$$

Knowing $T_{0,} T_L, L, V, t$ and $\beta(T)$, the equation [éq 2.1-4] must give the value of the constant of integration A .

However, it is difficult (even impossible) to determine this constant analytically, from where the recourse to a numerical resolution of the equation [éq 2.1-4] to determine A .

With the facts of the case $(T_{0,} T_L, T_{1,} T_{2,} C_S = C_l, C_{Sl} \ldots)$, we obtained the solution (physical) A which takes the value $A = -294.9117$.

From this constant, the analytical solution of the problem [éq 2.1-1] is analytical.

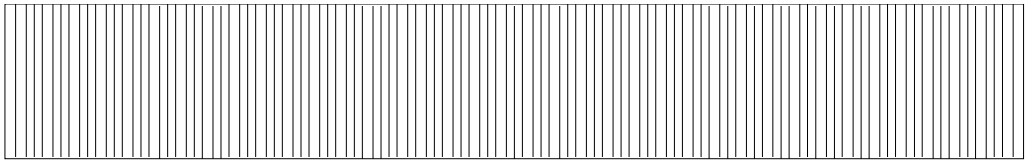
2.2 Results of reference

X-coordinate	Temperature
0.6	387.98514
0.7	451.51001
0.725	469.72232
0.750	488.97505
0.775	509.32766
0.80	530.84296
0.825	553.58738
0.85	577.63114
0.9	683.71269
0.9125	719.51615
0.925	756.32221
0.9375	794.16795
0.95	833.07971
0.9625	873.08751
0.9750	914.22222
0.9875	956.51557

3 Modelization A

3.1 Characteristic of the modelization

Modelization 2D



3.2 Characteristic of the mesh

80 QUAD8

3.3 Values tested

Identification Temperature	Reference
$N80\,(X=0.9875)$	956.515
$N79\,(X=0.9750)$	914.222
$N78\,(X=0.9625)$	873.087
$N77\,(X=0.9500)$	833.079
$N76\,(X=0.9375)$	794.167
$N75\,(X=0.9250)$	756.322
$N74\,(X=0.9125)$	719.516
$N73\,(X=0.9000)$	683.712
$N69\,(X=0.8500)$	577.631
$N67\,(X=0.8250)$	553.587

$N65(X=0.8000)$	530.842
$N63(X=0.7750)$	509.327
$N61(X=0.7500)$	488.975
$N59(X=0.7250)$	469.722
$N57(X=0.7000)$	451.510
$N44(X=0.6000)$	387.985

4 Summary of the results

the results are very satisfactory with variations with the reference solution lower than 1%.