TPLV100 - Roll subjected to non axisymmetric boundary conditions

Abstract:

It is about a test in steady thermal with modelization of Fourier.

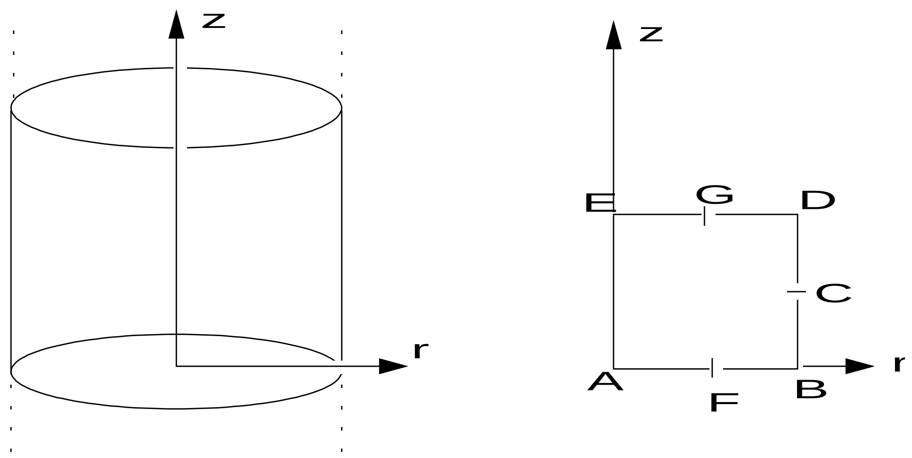
This test validates all the elements of Fourier in thermal (5 different modelizations) with various types of boundary conditions: imposed temperature, exchange, imposed flux, heat source.

The interest of the test, in addition to the validation of the thermal Fourier, lies in the following points:

- comparison between the results and an analytical solution on various harmonics of Fourier (1, 2 and 3),
- homogeneity of the elements between them.

1 Problem of reference

1.1 Geometry



Radius of the cylinder $R=1\ m$.

1.2 Material properties

$\lambda=1\,W/m°C$

1.3 Boundary conditions and loadings

$[EA]$: imposed temperature	$T=T_0=0.°C$
$[BC]$: imposed flux	$\phi=\phi_0=2.\,W/m2°C$
$[CD]$: exchange	$h=2.\,W/m^2°C$ $T_{ext}=2.°C$

2 Reference solution

2.1 Method of calculating used for the reference solution

$$T(r,z,\theta)=R^2\cos l\theta$$

with l number of the harmonic of Fourier

$$-\Delta T=(l^2-4)\cos l\theta=S$$

$$\vec{\phi}=-(\lambda\vec{\nabla}T)=\begin{bmatrix}-2\mathrm{r}\cos l\theta\\0.\\+(lr\sin l\theta)\end{bmatrix}$$

on $[AB]$ and $[ED]$: $\phi_0=\vec{\phi}.\vec{n}=0.$

on $[BC]$: $\phi_0=2R=2.$

on $[CD]$: $\vec{\phi}.\vec{n}=2R=\frac{2}{R}(2R^2-R^2)=h(T_{ext}-T)$

from where $h=\frac{2}{R}=2.$

$T_{ext}=2R^2=2.$

Only the source term varies according to the harmonic $(S^l(r,z)=l^2-4)$

In the following modelizations, one will solve the problem on harmonics 1,2 and 3.

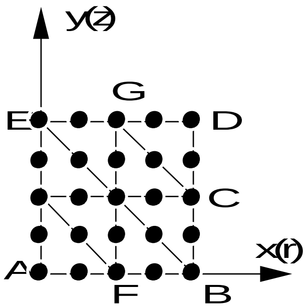
2.2 Results of reference

Temperatures and flux with the points B, C, D, F, G .

3 Modelization A

3.1 Characteristic of modelization

AXIS-FOURIER (TRIA6)



	r	z	noeud
A	0.	0.	N1
B	1.	0.	N7
C	1.	0.5	N8
D	1.	1.	N9
E	0.	1.	N3
F	0.5	0.	N4
G	0.5	1.	N6

the axes of description of the mesh are $x(r)$ and $y(z)$.

Mode - Fourier: 1 $T(A)=0.$

$S=-3.$	on all the field
$[BC]$:	$\phi=2.$
$[CD]$:	$h=2.$ $T_{ext}=2.$

3.2 Characteristics of the mesh

Many nodes: 25.
Number of meshes and types: 8 TRIA6

3.3 Remarks

the number of the mode of Fourier not affecting the loading, key word `MODE_FOURIER` are not necessary in command `CALC_VECT_ELEM`.

Use of the command `CREA_CHAMP/ASSE` is not a recombination of Fourier but a simple validation of this key word.

3.4 Values tested

	Identification	Reference
$\theta=0$	$T(B)$	1.
	$T(F)$	0.25
	$\phi_r(B)$	-2
	$\phi_r(F)$	$-1.$
	$\phi_\theta(B)$	1.
	$\phi_\theta(F)$	0.5
	$\phi_z(B)$	0.
	$\phi_z(F)$	0.
$\theta=45$	$T(B)$	0.7071
	$T(F)$	0.177
	$\phi_r(B)$	-1.414
	$\phi_r(F)$	-0.7071

	$\phi_\theta(B)$	– 0.707
	$\phi_\theta(F)$	– 0.3535
	$\phi_z(B)$	0.
	$\phi_z(F)$	0.
$\theta=135$	$T(B)$	– 0.707
	$T(F)$	– 0.177
	$\phi_r(B)$	1.414
	$\phi_r(F)$	0.707
	$\phi_\theta(B)$	– 0.707
	$\phi_\theta(F)$	– 0.3535
	$\phi_z(B)$	0.
	$\phi_z(F)$	0.

3.5 Remarks

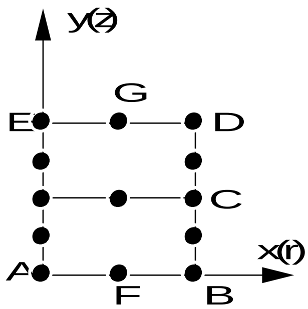
the values of flux to the nodes are realized on the elements containing this node.

It is noticed that the exact solution is not found. This is with the fact that the numerical integration of the thermal stiffness matrix is approximate (formula with 3 Gauss points). If a formula at 6 points were used, one would find the solution exactly.

4 Modelization B

4.1 Characteristic of modelization

`AXIS_FOURIER` **(QUAD8)**



	r	z	noeud
A	0	0	N1
B	1	0	N4
C	1	0.5	N5
D	1	1	N6
E	0	1	N3
F	0.5	0	N7
G	0.5	1	N12

the axes of description of the mesh are $x(r)$ and $y(z)$.

Mode - Fourier: 2 $T(A)=0.$
No source term because $S^l(r,z)=0.$ for $l_. =2$

$[BC]$: $\phi=2.$
$[CD]$: $h=2.$ $T_{ext}=2.$

4.2 Characteristics of the mesh

Many nodes: 13.
Number of meshes and types: 2 QUAD8

4.3 Remarks

the number of the mode of Fourier not affecting the loading, key word `MODE_FOURIER` are not necessary in command `CALC_VECT_ELEM`.

4.4 Values tested

Identification	Reference
$T(B)$	1.
$T(C)$	1.
$T(D)$	1.
$T(F)$	0.25
$T(G)$	0.25
$\phi_r(B)$	$-2.$
$\phi_r(C)$	$-2.$
$\phi_r(D)$	$-2.$
$\phi_r(F)$	$-1.$
$\phi_r(G)$	$-1.$
$\phi_\theta(B)$	2.

$\phi_\theta(C)$	2.
$\phi_\theta(D)$	2.
$\phi_\theta(F)$	1.
$\phi_\theta(G)$	1.
$\phi_z(B)$	0.
$\phi_z(C)$	0.
$\phi_z(D)$	0.
$\phi_z(F)$	0.
$\phi_z(G)$	0.

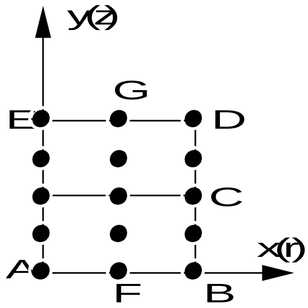
4.5 Remarks

the analytical solution is found exactly.

5 Modelization C

5.1 Characteristic of modelization

`AXIS_FOURIER` (QUAD9)



	r	z	noeud
A	0.	0.	N1
B	1	0.	N4
C	1	0.5	N5
D	1	1	N6
E	0.	1	N3
F	0.5	0.	N7
G	0.5	1	N12

the axes of description of the mesh are $x(r)$ and $y(z)$.

Mode - Fourier: 3 $T(A)=0.$

$S=5.$	on all the field
$[BC]$:	$\phi=2.$
$[CD]$:	$h=2.$ $T_{ext}=2.$

5.2 Characteristics of the mesh

Many nodes: 15.
Number of meshes and types: 2 QUAD9

5.3 Remarks

the number of the mode of Fourier not affecting the loading, key word `MODE_FOURIER` are not necessary in command `CALC_VECT_ELEM`.

5.4 Values tested

Identification	Reference
$T(B)$	1.
$T(C)$	1.
$T(D)$	1.
$T(F)$	0.25
$T(G)$	0.25
$\phi_r(B)$	– 2.
$\phi_r(C)$	– 2.
$\phi_r(D)$	– 2.
$\phi_r(F)$	– 1.
$\phi_r(G)$	– 1.
$\phi_\theta(B)$	3.
$\phi_\theta(C)$	3.

$\phi_\theta(D)$	3.
$\phi_\theta(F)$	1.5.1.5
$\phi_\theta(G)$	
$\phi_z(B)$	0.
$\phi_z(C)$	0.
$\phi_z(D)$	0.
$\phi_z(F)$	0.
$\phi_z(G)$	0.

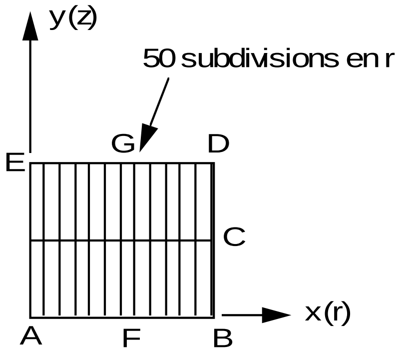
5.5 Remarks

the analytical solution is found exactly.

6 Modelization D

6.1 Characteristic of modelization

`AXIS_FOURIER` (QUAD4)



	r	z	noeud
A	0.	0.	N1
B	1.	0.	N151
C	1.	0.5	N152
D	1.	1.	N153
E	0.	1.	N3
F	0.5	0.	N76
G	0.5	1.	N78

the axes of description of the mesh are $x(r)$ and $y(z)$.

Mode - Fourier: 2 $T(A)=0.$

$S=0.$	on all the field
$[BC]$:	$\phi=2.$
$[CD]$:	$h=2.$ $T_{ext}=2.$

6.2 Characteristics of the mesh

Many nodes: 153
Number of meshes and types: 100 QUAD4

6.3 Remarks

the number of the mode of Fourier not affecting the loading, key word `MODE_FOURIER` are not necessary in command `CALC_VECT_ELEM`.

6.4 Values tested

Identification	Reference
$T(B)$	1.
$T(C)$	1.
$T(D)$	1.
$T(F)$	0.25
$T(G)$	0.25
$\phi_r(B)$	$-2.$
$\phi_r(C)$	$-2.$
$\phi_r(D)$	$-2.$
$\phi_r(F)$	$-1.$
$\phi_r(G)$	$-1.$
$\phi_\theta(B)$	2.
$\phi_\theta(C)$	2.

$\phi_\theta(D)$	2.
$\phi_\theta(F)$	1.
$\phi_\theta(G)$	1.
$\phi_z(B)$	0.
$\phi_z(C)$	0.
$\phi_z(D)$	0.
$\phi_z(F)$	0.
$\phi_z(G)$	0.

6.5 Remarks

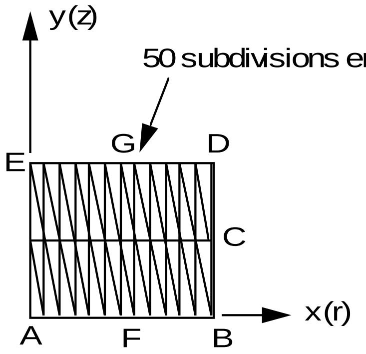
the bad accuracy recorded on $\phi_r(B)$ $\phi_r(C)$, $\phi_r(D)$ is explained by the fact why B , C and D are nodes of edge, therefore the flux are not realized on adjacent elements in the direction of the variation in temperature (direction R).

This phenomenon is not found on ϕ_θ , because of being ϕ_θ balanced par. $1/r$

7 Modelization E

7.1 Characteristic of modelization

`AXIS_FOURIER` (TRIA3)



	r	z	noeud
A	0.	0.	N1
B	1.	0.	N151
C	1.	0.5	N152
D	1.	1.	N153
E	0.	1.	N3
F	0.5	0.	N76
G	0.5	1.	N78

the axes of description of the mesh are $x(r)$ and $y(z)$.

Mode - Fourier: 2 $T(A)=0.$

$S=0.$ on all the field
$[BC]$: $\phi=2.$
$[CD]$: $h=2.$ $T_{ext}=2.$

7.2 Characteristics of the mesh

Many nodes: 153
Number of meshes and types: 200 TRIA3

7.3 Remarks

the number of the mode of Fourier not affecting the loading, key word `MODE_FOURIER` are not necessary in command `CALC_VECT_ELEM`.

7.4 Values tested

Identification	Reference
$T(B)$	1.
$T(C)$	1.
$T(D)$	1.
$T(F)$	0.25
$T(G)$	0.25
$\phi_r(B)$	– 2.
$\phi_r(C)$	– 2.
$\phi_r(D)$	– 2.
$\phi_r(F)$	– 1.
$\phi_r(G)$	– 1.
$\phi_\theta(B)$	2.

$\phi_\theta(C)$	2.
$\phi_\theta(D)$	2.
$\phi_\theta(F)$	1.
$\phi_\theta(G)$	1.
$\phi_z(B)$	0.
$\phi_z(C)$	0.
$\phi_z(D)$	0.
$\phi_z(F)$	0.
$\phi_z(G)$	0.

7.5 Remarks

the bad accuracy recorded on $\phi_r(B)$ $\phi_r(C)$, $\phi_r(D)$ is explained by the fact why B , C and D are nodes of edge, therefore the flux are not realized on adjacent elements in the direction of the variation in temperature (direction R).

This phenomenon is not found on ϕ_θ , because of being ϕ_θ balanced par. $1/r$

8 Summary of the results

This problem is correctly solved:

- whatever the number of harmonic of Fourier,
- by different element types (degree 1 or 2).