SSLA100 - Infinite cylinder subjected to a field of volume forces and surface

Abstract:

This test of linear quasi-static mechanics makes it possible to validate the assignment of a loading of field of forces, surface or voluminal.

The studied structure is cylindrical. The fields at nodes of voluminal and surface density of forces are read in a file with the Ideas format. For the voluminal loading, the field read varies quadratically according to the distance to the axis; for the surface loading, the field read corresponds to an internal pressure.

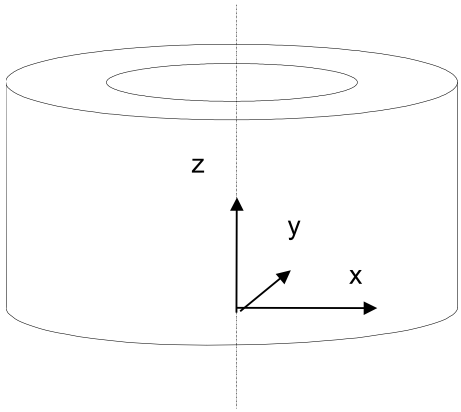
Three modelizations of the same problem are carried out:

- modelization 3D;
- modelization 2D axisymmetric;
- modelization 2D plane strains;

The reference solution is analytical.

1 Problem of reference

1.1 Geometry



selected geometrical dimensions are the following ones:

- height = $0.5\,m$;
- interior radius = $1\,m$;
- external radius = $1.2\,m$.

1.2 Properties of the material

the cylinder consists of a homogeneous material which follows a linear elastic constitutive law:

- $E=10\,Pa$;
- $\rho=1\,kg/m^3$;
- $\nu=0.3$.

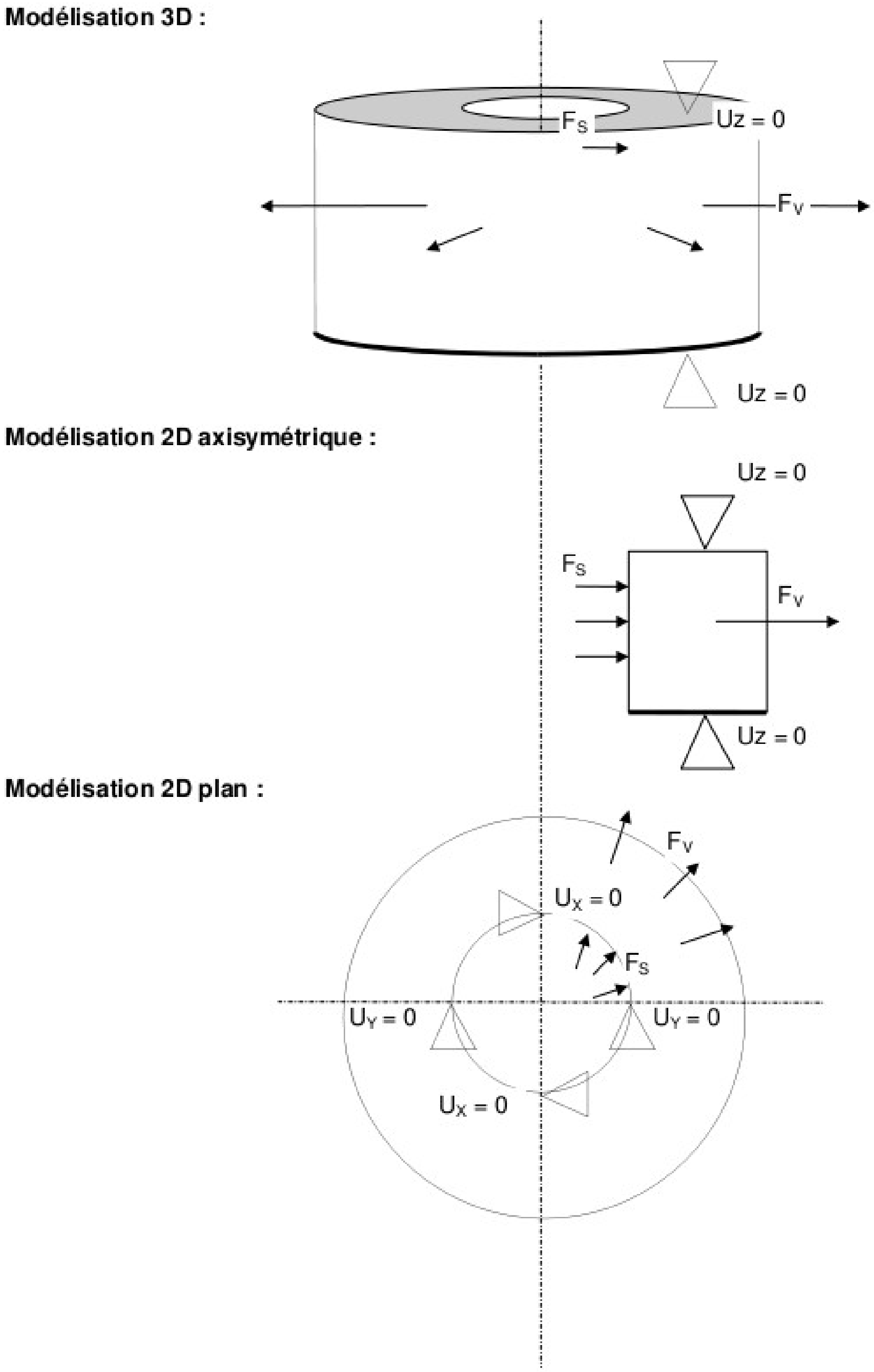
1.3 Boundary conditions and loadings (cf [Figure 1.3-a])

the volume force considered is radial, it varies in a quadratic way with the radius: $F_V=\alpha.r^2$ with $\alpha=1\,N/m^3$.
The surface force considered is applied to the internal wall of the cylinder, perpendicular to the wall (is equivalent to an internal pressure imposed on the cylinder): $F_S(r=R_{\text{int}})=1\text{N}/m^2$.

The boundary conditions make it possible to be placed on the assumption of the plane strains on a section of the cylinder: vertical displacements blocked on the sections high and low of the cylinder.

Note:

> *For the modelization 3D, the suppression of the eigen modes is ensured by the conditions of 2D plane applied to the low section of the cylinder. This kind of boundary conditions makes it possible to obtain an axisymmetric solution in displacement, directly comparable to the analytical solution.*



Appear 1.3-a: Boundary conditions and loadings

2 Reference solution

2.1 Method of calculating used for the reference solution

the problem of linear static mechanics axisymmetric considered can be solved in an analytical way. One solves independently the response with the request volume force and surface force to add them then.

Quadratic volume force $F_V(r)=\alpha r^2$

One considers the balance equations in cylindrical coordinates:

$$\begin{cases} \frac{\partial\sigma_r}{\partial r}+\frac{1}{r}\frac{\partial\sigma_r}{\partial\theta}+\frac{\partial\sigma_{rz}}{\partial z}+\frac{\sigma_r-\sigma_\theta}{r}+f_r=0 \\ \frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{\partial\sigma_{\theta z}}{\partial z}+\frac{\partial\sigma_{\theta r}}{\partial r}+2\frac{\sigma_{r\theta}}{r}+f_\theta=0 \\ \frac{\partial\sigma_{zz}}{\partial z}+\frac{\partial\sigma_{rz}}{\partial r}+\frac{\partial\sigma_{\theta z}}{r\partial\theta}+2\frac{\sigma_{r\theta}}{r}+f_z=0 \end{cases}$$

who are simplified being given axial symmetry in: $\frac{\partial\sigma_r}{\partial r}+\frac{\sigma_r-\sigma_\theta}{r}+f_r=0$

By means of the constitutive law then the relations strain-displacements, one leads to the following differential equation: $u''+\frac{u'}{r}-\frac{u}{r^2}+\frac{f_V}{\frac{E(1-\nu)}{(1+\nu)(1-2\nu)}}=0$

The volume force applied is of the type: fV= α .r ²

the solution of the differential equation is written then:

$$u=\frac{-c_1}{2r}-\frac{\alpha(1+\nu)(1-2\nu)r^4}{15E(1-\nu)}+c_2 r$$ éq the 2.1-1

two constants of integrations c_1 and c_2 are given thanks to the boundary conditions: $\begin{cases}\sigma(R_{\text{int}})=0\\ \sigma(R_{ext})=0\end{cases}$

One obtains: $$\begin{cases} c_1=\frac{4-3\nu}{1-\nu}\cdot\frac{2\alpha}{15}\cdot\frac{1+\nu}{E}\cdot\frac{R_{\text{int}}^2R_{ext}^2(R_{\text{int}}^3-R_{ext}^3)}{R_{ext}^2-R_{\text{int}}^2} \\ c_2=\frac{(1+\nu)(1-2\nu)}{E}\cdot\frac{4-3\nu}{1-\nu}\cdot\frac{\alpha}{15}\cdot(R_{ext}^3-R_{\text{int}}^2\cdot\frac{R_{\text{int}}^3-R_{ext}^3}{R_{ext}^2-R_{\text{int}}^2}) \end{cases}$$

Surface force standard pressure $F_S(R_{\text{int}})=P$

the problem to be solved is of comparable nature, but with a volume force applied null: $f_V=0$ that is to say $\alpha=0$.

The solution in displacement [éq 2.1-1] is written then: $u=\frac{-c_1}{2r}+c_2 r$, having to observe the conditions: $\begin{cases}\sigma(R_{\text{int}})=-P\\ \sigma(R_{ext})=0\end{cases}$

What gives:

$$u=P\cdot\frac{1+\nu}{E}\cdot\frac{R_{\text{int}}^2}{R_{ext}^2-R_{\text{int}}^2}\left[\frac{R_{ext}^2}{r}+(1-2\nu)\cdot r\right]$$ éq 2.1-2

2.2 Results of Numerical

reference Application:

- height = $0.5\,m$;
- interior radius = $1\,m$;
- external radius = $1.4\,m$;
- E = $10\,Pa$;
- ρ = $1\,kg/m^3$;
- ν = 0.3 ;
- α = $1\,N/m^5$;
- P = $1\,N/m^2$.

by injecting the numerical values in the solutions [éq 2.1-1] and [éq 2.1-2] one finds after summation:

$$\begin{cases}u(1.0)=0.52130982\,m\\ u(1.4)=0.44203108\,m\end{cases}$$

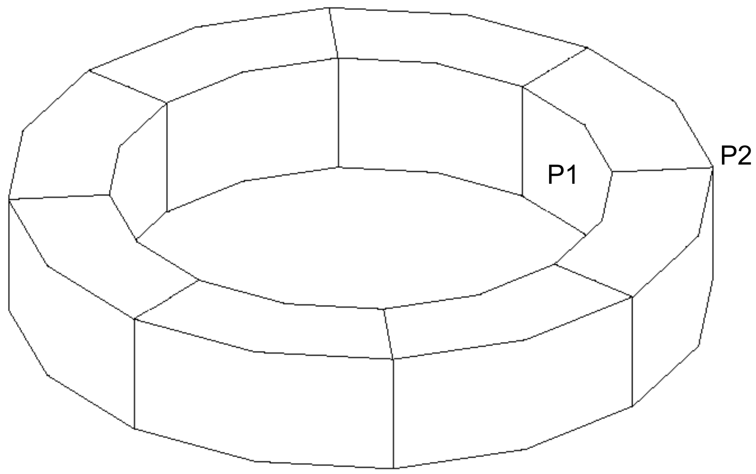
2.3 Uncertainties on the solution

Null (analytical reference solution).

3 Modelization A

3.1 Characteristic of the modelization

It cylinder is modelled in elements 3D voluminal:



3.2 Characteristics of the mesh

the cylinder is represented by a regular mesh of quadratic elements with 20 nodes containing:

- 8 elements;
- 96 nodes.

The mesh 1 only element in the radial and vertical meaning and 8 cuttings contains on the circumference.

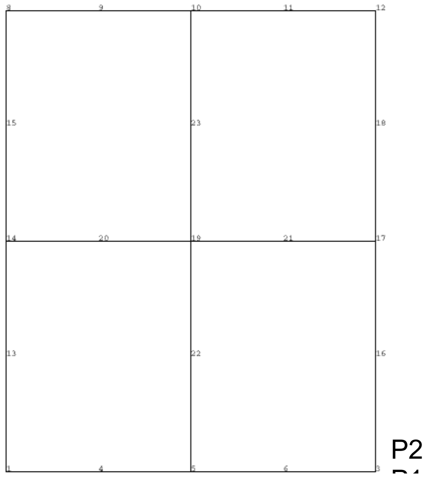
3.3 Values tested

Identification	Times	Reference
U_X into $P1$	1	0.52130982
U_X into $P2$	1	0.44203108

4 Modelization B

4.1 Characteristic of the modelization

a longitudinal section of the cylinder is modelled in elements 2D voluminal, by considering the assumption of axisymetry.



4.2 Characteristic of the mesh

the cylinder is represented by a regular mesh of quadratic elements with 8 nodes containing:

- 4 elements;
- 21 nodes.

The mesh contains 2 cuttings in the radial meaning and 2 cuttings in the vertical meaning.

4.3 Values tested

Identification	Times	Reference
U_X into $P1$	1	0.52130982
U_X into $P2$	1	0.44203108

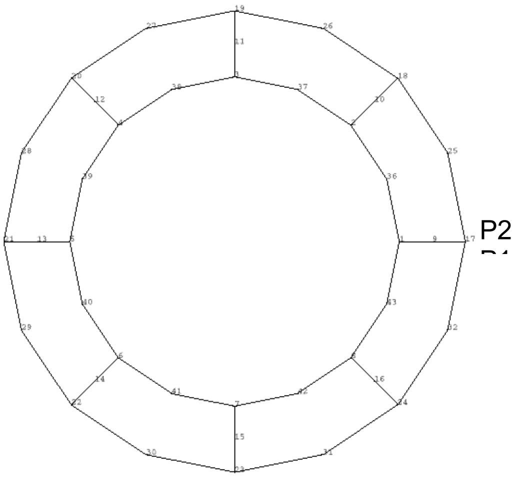
4.4 Remark

Modelization more powerful than 3D because 2 cuttings in the radial meaning and not of circumferential discretization.

5 Modelization C

5.1 Characteristic of the modelization

a transverse section of the cylinder is modelled in elements 2D voluminal, by considering the assumption of the plane strains.



5.2 Characteristic of the mesh

the cylinder is represented by a regular mesh of quadratic elements with 8 nodes containing:

- 8 elements;
- 40 nodes.

The mesh 1 only cutting in the radial meaning and 8 cuttings in the vertical meaning contains (like 3D).

5.3 Values tested

Identification	Times	Reference
U_X into $P1$	1	0.52130982
U_X into $P2$	1	0.44203108

5.4 Remarks

Modelization of performance very close to 3D because same discretizations circumferential and radial.

6 Summary of the results

the results got by *code_Aster* are very close to the analytical solution, in spite of very coarse meshes.

The modelizations 3D and 2D plane give further information very close because they present the same discretizations circumferential and radial. The modelization 2D axisymmetric is more powerful because it presents 2 cuttings in the radial meaning and not of circumferential discretization.