SSLX100 - 3D mix - Shell - Beam in bending

Summarized:

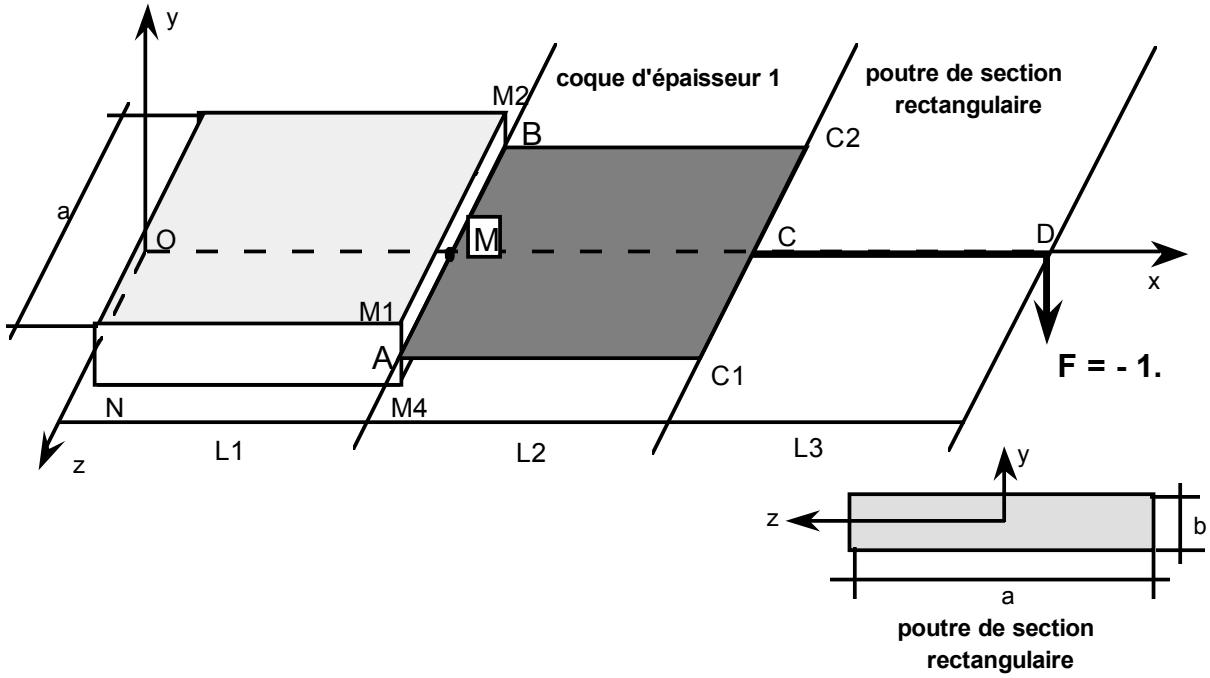
This test makes it possible to validate for a linear elastic design:

- a mixture of various mechanical models: model 3D (element HEXA20), models shell (element DKT) and models beam (elements POU_D_E, or elements COQUE_C_PLAN),
- linear relations between degrees of freedom.

The test leans on the elastic analytical **solution** of a beam in bending, the reduced number of elements for the various **models** leads to a poor solution, which is however improved clearly with employment of boundary conditions appropriate to the theory of the beams.

1 Problem of reference

1.1 Geometry



L1 = L2 = L3 = 10 mm
b = 1 mm
a = 3 mm

1.2 Material properties

$E=200\,000\,MPa$

$\nu=0.3$

$\nu=0.0$ makes it possible to avoid the variation of orthogonal curvature induced by the effect Fish in the plates, which causes a variation between the beam theories and of plates, out of average fiber.

1.3 Boundary conditions and loadings

- forces $F_y=-1$ (load 1) or couples $C_z=1$ (load 2)
- definite or applied to neutral fiber
- fixed support of the section $x=0$
- continuity of displacements of translation to AB
- continuity of displacements of translation in C
- equality of displacements of rotation around z to $C1-C2$
- for the points M of the section ($M1$ $M2$ $M4$) displacements of translation $u(M)$ depend linearly on the displacement of rotation φ_z of the points P of AB

$$u(M)=-\varphi_z(P).y+dx(P)$$

2 Reference solution

2.1 Method of calculating used for the analytical reference solution

Solution, *statically determinate structure*.

The elastic deflection, the elastic stresses and axial strains and the bending moment in any point of X-coordinate x are given by:

- Charge n°1: force $F_y=-1$

$M_y(x)=F_y.L(1-x/L)$ (= $E.I_z.u_y''(x)$ in elasticity)
$u_y(x)=F_yL.x^2.(3-x/L)/(6.E.I_z)$ (in elasticity)
$\varepsilon_{xx}(x,y)=-F_y.L(1-x/L).y/(E.I_z)$ (in elasticity)
$\sigma_{xx}(x,y)=-F_y.L(1-x/L).y/I_z$ (in elasticity)

- Load n°2: couples $C_z=1$ or rotation $dr_z=C_{z.L}/(E.I_z)$

$M_y(x)=C_z$ (= $E.I_z.u_y''(x)$ in elasticity)
$u_y(x)=C_z.x^2/(2.E.I_z)$ (in elasticity)
$\sigma_{xx}(x,y)=-C_z.y/I_z$ (in elasticity)
$\varepsilon_{xx}(x,y)=-C_z.y/(E.I_z)$ (in elasticity)

with:

$L=L1+L2+L3=30\,mm$
$I_z=a.h^3/12=0.25\,mm^4$
$dr_z=0.0006$

2.2 Results of reference

Deflections, stresses and strains axial and bending moments in 4 points of the axis of the beam.

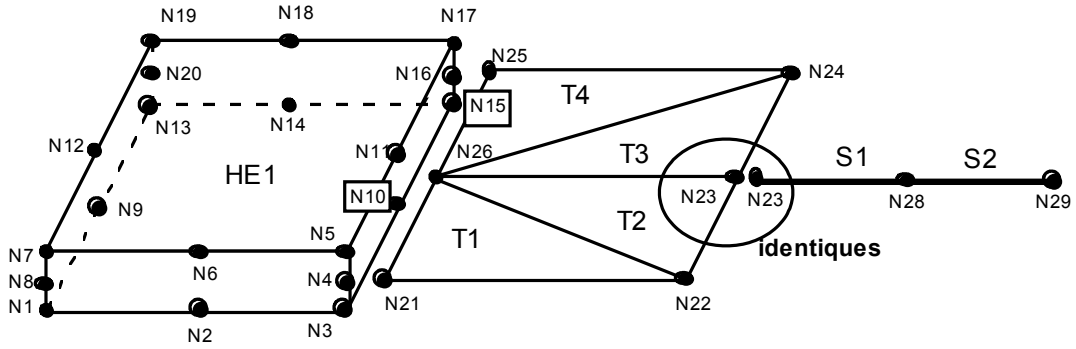
2.3 Uncertainty on the analytical

solution Solution.

3 Modelization A

3.1 Characteristic of the Modélsiation

modelization mixed: 3D, DKT and POU_D_E



Point N --> N1
Point A --> N4
Point C --> N23
Point D --> N29

Charge n°1: force F_y
total Fixed support on the section in O

3.2 Characteristics of the mesh

Many nodes: 28
Number of meshes and types: 1 HEXA20 , 4 TRIA3 / DKT , 2 SEG2/ POU_D_E

3.3 Quantities tested and results

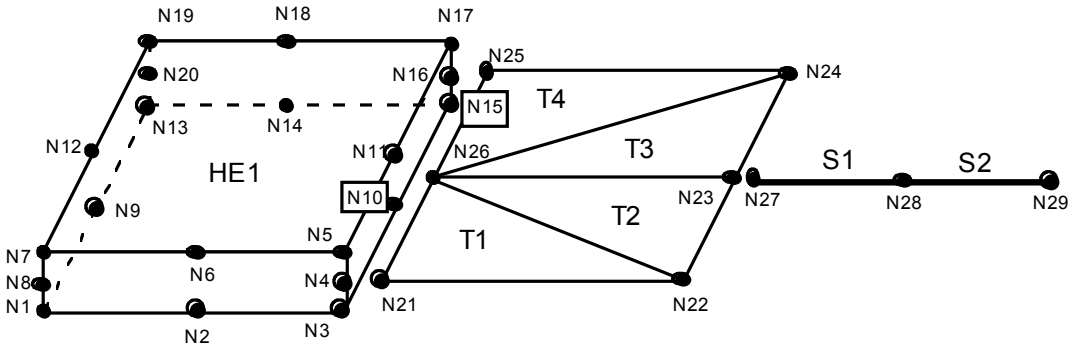
- Charges n°1: force F_y

Standard		Identification of Reference	Reference	Tolerance (%)
uN	(node $N1$)	ANALYTIQUE	0.	1.00E-006
uM	(node $N26$)	ANALYTIQUE	– 0.0267	14.0
uA	(node $N4$)	ANALYTIQUE	– 0.0267	14.0
$uC1$	(node $N22$)	ANALYTIQUE	– 0.0933	8.40
uC	(node $N23$)	ANALYTIQUE	– 0.0933	8.30
uD	(node $N29$)	ANALYTIQUE	– 0.18	6.40

4 Modelization B

4.1 Characteristic of the mixed

modelization Modelization: 3D, DKT, POU_D_E and DIS_TR



Point N --> N1
Point A --> N4
Point C --> N23
Point D --> N29

Charge n°1: force F_y
Fixed support on the section in O realized by a connection 3D_POUTRE between the face $N1\,N13\,N19\,N7$ and a discrete element located on the origin.

Relation additional, compared to the modelization A, enters C_1 C_2 and C, introduced by LIAISON_ELEM: "COQ_POU".

4.2 Characteristics of the mesh

Many nodes: 29

Number of meshes and types: 1 HEXA20 , 4 TRIA3 / DKT , 2 SEG2/ POU_D_E , 1 POI1/DIS_TR , 1 QUAD8 , 2 SEG2/BORD_DKT

4.3 Quantities tested and results

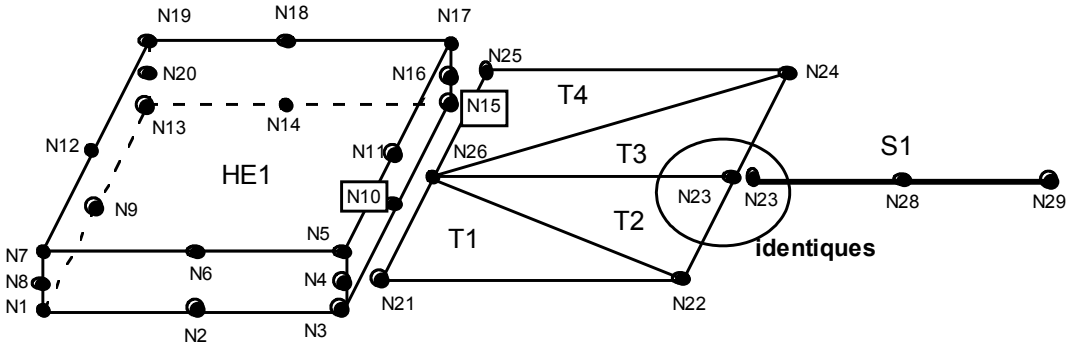
- Charges n°1: force F_y

Standard	Identification of Reference	Reference	Tolerance (%)
uN (node $N1$)	ANALYTIQUE	0.	
uM (node $N26$)	ANALYTIQUE	– 0.0267	3.0
uA (node $N4$)	ANALYTIQUE	– 0.0267	3.0
$uC1$ (node $N22$)	ANALYTIQUE	– 0.0933	1.0
uC (node $N23$)	ANALYTIQUE	– 0.0933	1.0
uD (node $N29$)	ANALYTIQUE	– 0.18	1.0
M_z (node $N27$)	ANALYTIQUE	– 10.	1.0
M_z (node $N29$)	ANALYTIQUE	0.	1.0

5 Modelization C

5.1 Characteristic of the mixed

modelization Modelization: 3D, DKT and COQUE_C_PLAN



Point N –> N1
Point A –> N4
Point C –> N23
Point D –> N29

In $N29$: charge n°1: force F_y , load n°2: couples C_z or rotation dr_z
Displacements DY and DZ on null section 0 on average (command LIAISON_DDL).

Note:

As the width of the beam is $a=3\text{mm}$, the Young modulus is multiplied by 3 in the material of the COQUE_C_PLAN.

5.2 Characteristics of the mesh

Many nodes: 29
Number of meshes and types: 1 HEXA20 , 4 TRIA3 / DKT , 1 SEG3/ COQUE_C_PLAN

5.3 Quantities tested and Standard

Identification	results of Reference	Reference	Tolerance (%)
Charges 1: *force* F_y			
uN (node $N8$)	ANALYTIQUE	0.	6.0e-5
uM (node $N26$)	ANALYTIQUE	– 0.0267	5.70
uA (node $N4$)	ANALYTIQUE	– 0.0267	5.70
$uC1$ (node $N22$)	ANALYTIQUE	– 0.0933	2.10
uC (node $N23$)	ANALYTIQUE	– 0.0933	2.00
uD (node $N29$)	ANALYTIQUE	– 0.18	2.0
MyD (node $N28$)	ANALYTIQUE	-5.	0.50
$\sigma_{xx}(x, h/2)$ (node $N28$)	ANALYTIQUE	-30.0000	0.50
$\varepsilon_{xx}(x, h/2)$ (node $N28$)	ANALYTIQUE	– 5.0 10-5	0.50

Load 2: *couples* C_z			
uN (node $N1$)	ANALYTIQUE	0.	3.0e-6
uM (node $N26$)	ANALYTIQUE	– 0.0010	0.50
uA (node $N4$)	ANALYTIQUE	– 0.0010	0.50
$uC1$ (node $N22$)	ANALYTIQUE	– 0.0040	0.20
uC (node $N23$)	ANALYTIQUE	– 0.0040	0.20
uD (node $N29$)	ANALYTIQUE	– 0.0090	0.10
M_{yD} (node $N28$)	ANALYTIQUE	1.0	0.50
$\sigma_{xx}(x, h/2)$ (node $N28$)	ANALYTIQUE	6.0000	0.50
$\varepsilon_{xx}(x, h/2)$ (node $N28$)	ANALYTIQUE	1.0 10-5	0.50

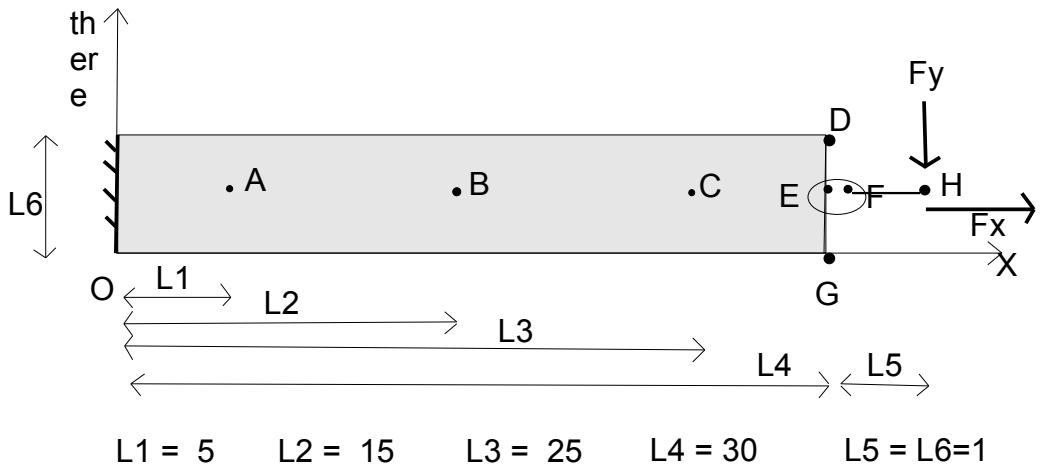
5.4 Remarks

The computation of the forces and moment in element COQUE_C_PLAN is carried out with the medium node $N28$ so that the interpolation is correct; knowing that the Young modulus is triple so that the product $E.I_z$ is identical in all the model, the stresses are it too.

6 Modelization F

6.1 Characteristic of the mixed

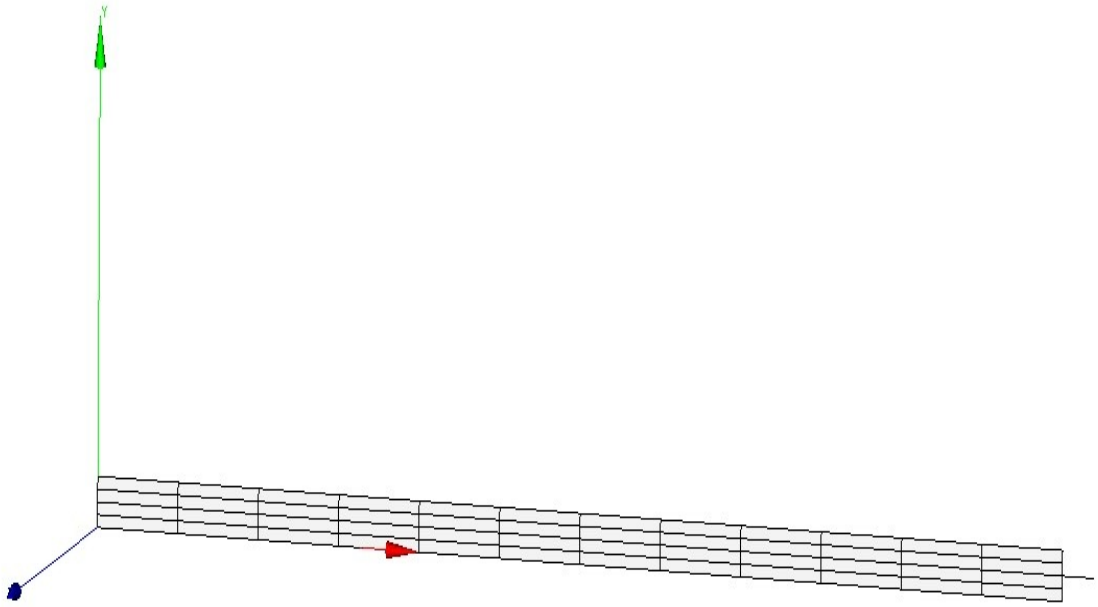
modelization Modelization: C_PLAN , 2D_DIS_TR



In H : charge n°1: force F_x , load n°2: F_y
Displacements u_x and u_y null on left edge of the part 2D.

6.2 Characteristics of the mesh

Many nodes: 179
Number of meshes and types : 8 SEG3, 1 SEG2, 48 QUAD8 (=12 X 4)



6.3 Quantities tested and Standard

Identification	results of Reference	Reference	Tolerance (%)
Charge 1: *force* F_x			
$u_x(A)$ (node $N13$)	ANALYTIQUE	2.50	1.0e-5
$u_x(B)$ (node $N75$)	ANALYTIQUE	7.50	1.0e-5
$u_x(C)$ (node $N131$)	ANALYTIQUE	12.50	1.0e-5
$u_x(D)$ (node $N141$)	ANALYTIQUE	15.00	1.0e-5
$u_x(E)$ (node $N133$)	ANALYTIQUE	15.00	1.0e-5
$u_x(F)$ (node $N178$)	ANALYTIQUE	15.00	1.0e-5
$u_x(G)$ (node $N125$)	ANALYTIQUE	15.00	1.0e-5
$u_x(H)$ (node $N179$)	ANALYTIQUE	15.25	1.0e-5
Load 2: *force* F_y			
$u_y(A)$ (node $N13$)	ANALYTIQUE	2.125e-2	1.0e-2
$u_y(B)$ (node $N75$)	ANALYTIQUE	-1.6875e-1	1.0e-2
$u_y(C)$ (node $N131$)	ANALYTIQUE	-4.0625	1.0e-2
$u_y(D)$ (node $N141$)	ANALYTIQUE	-0.54	1.0e-2
$u_y(E)$ (node $N133$)	ANALYTIQUE	-0.54	1.0e-2
$u_y(F)$ (node $N178$)	ANALYTIQUE	-0.54	1.0e-2
$u_y(G)$ (node $N125$)	ANALYTIQUE	-0.54	1.0e-2
$u_y(H)$ (node $N179$)	ANALYTIQUE	-0.54	1.0e-2

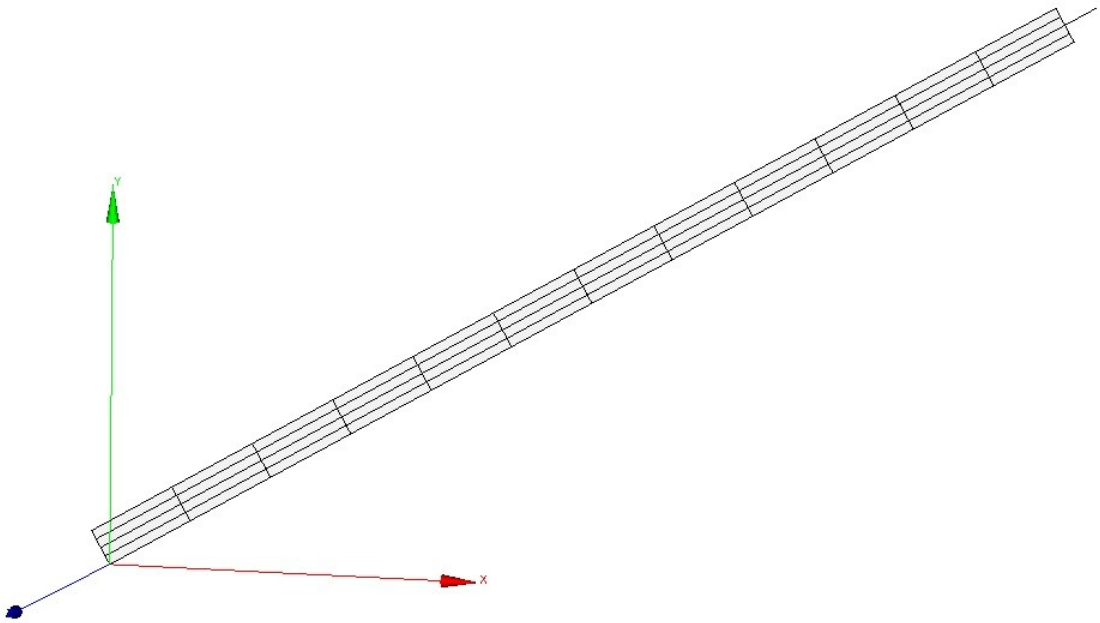
7 Modelization G

7.1 Characteristic of the modelization

Modelization G = Modelization F (C_PLAN , 2D_DIS_TR) + rotation of 30° in the beginning.

7.2 Characteristics of the mesh

Many nodes: 179
Number of meshes and types : 8 SEG3, 1 SEG2, 48 QUAD8 (=12 X 4)



7.3 Quantities tested and Standard

Identification		results of Reference	Reference	Tolerance (%)
Charge 1: *force* F_x				
$u_x(A)$	(node $N13$)	ANALYTIQUE	2.165064	1.0e-6
$u_x(B)$	(node $N75$)	ANALYTIQUE	6.495191	1.0e-6
$u_x(C)$	(node $N131$)	ANALYTIQUE	10.825318	1.0e-6
$u_x(D)$	(node $N141$)	ANALYTIQUE	12.990381	1.0e-6
$u_x(E)$	(node $N133$)	ANALYTIQUE	12.990381	1.0e-6
$u_x(F)$	(node $N178$)	ANALYTIQUE	12.990381	1.0e-6

$u_x(G)$	(node $N125$)	ANALYTIQUE	12.990381	1.0e-6
$u_x(H)$	(node $N179$)	ANALYTIQUE	13.208874	1.0e-6
Load 2: *force* F_y				
$u_y(A)$	(node $N13$)	ANALYTIQUE	-1.8403e-2	1.0e-2
$u_y(B)$	(node $N75$)	ANALYTIQUE	-0.146142	1.0e-2
$u_y(C)$	(node $N131$)	ANALYTIQUE	-0.351823	1.0e-2
$u_y(D)$	(node $N141$)	ANALYTIQUE	-0.467654	1.5
$u_y(E)$	(node $N133$)	ANALYTIQUE	-0.467654	1.0e-2
$u_y(F)$	(node $N178$)	ANALYTIQUE	-0.467654	1.0e-2
$u_y(G)$	(node $N125$)	ANALYTIQUE	-0.467654	1.5
$u_y(H)$	(node $N179$)	ANALYTIQUE	-0.467654	1.0e-2

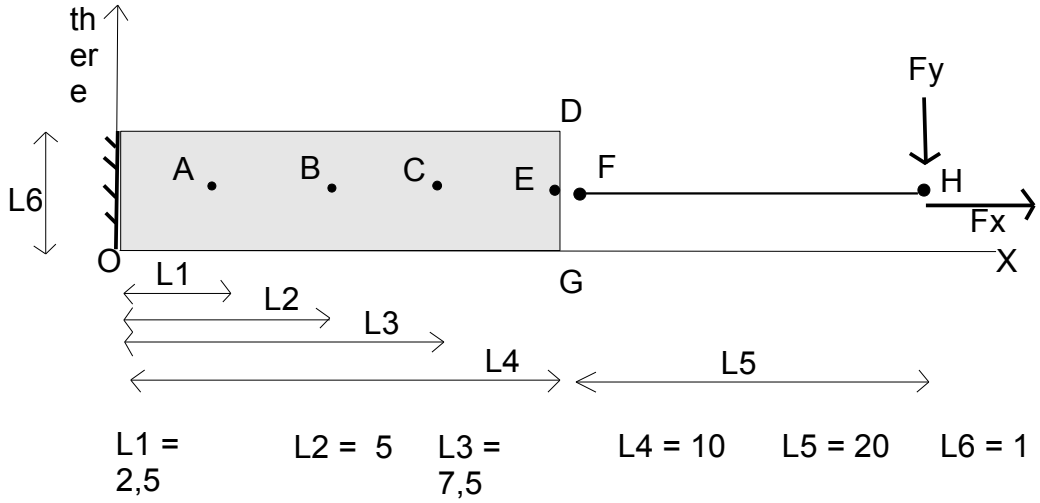
7.4 Remarks

the analytical values of displacement are those of the modelization G multiplied by the cosine of 30 degrees.

8 Modelization H

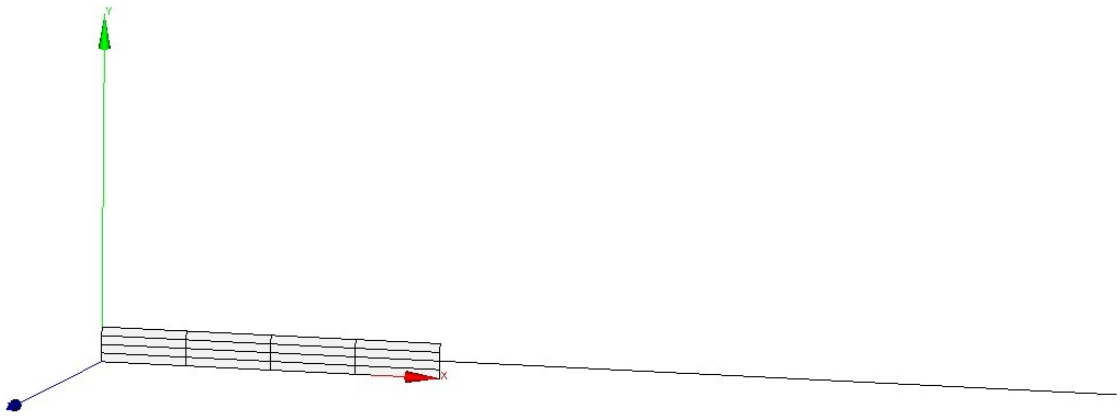
8.1 Characteristic of the mixed

modelization Modelization: C_PLAN , POU_D_E



8.2 Characteristic of the mesh

Many nodes: 67
Number of meshes and types : 8 SEG3, 1 SEG2, 16 QUAD8 (=4 X 4)



8.3 Quantities tested and Standard

Identification		results of Reference	Reference	Tolerance (%)
Charge 1: *force* F_x				
$u_x(A)$	(node $N12$)	ANALYTIQUE	1.25	1.0e-6
$u_x(B)$	(node $N13$)	ANALYTIQUE	2.50	1.0e-6
$u_x(C)$	(node $N14$)	ANALYTIQUE	3.75	1.0e-6
$u_x(D)$	(node $N25$)	ANALYTIQUE	5.00	1.0e-6
$u_x(E)$	(node $N15$)	ANALYTIQUE	5.00	1.0e-6
$u_x(F)$	(node $N66$)	ANALYTIQUE	5.00	1.0e-6
$u_x(G)$	(node $N5$)	ANALYTIQUE	5.00	1.0e-6
$u_x(H)$	(node $N67$)	ANALYTIQUE	15.00	1.0e-6
Load 2: *force* F_y				
$u_y(A)$	(node $N12$)	ANALYTIQUE	-5.46875e-3	1.0e-2
$u_y(B)$	(node $N13$)	The mesh	ANALYTIQUE	-2.125e-2
$u_y(C)$	1.0e-2 ($N14$) node		ANALYTIQUE	-4.640625e-2
$u_y(D)$	1.0e-2 ($N25$) node		-0.08	-0,08
$u_y(E)$	1.0e-2 ($N15$) node		-0.08	-0,08
$u_y(F)$	1.0e-2 ($N66$) node		-0.08	-0,08
$u_y(G)$	1.0e-2 ($N5$) node		-0.08	-0,08
$u_y(H)$	1.0e-2 ($N67$) node		-0.54	-0,54

9 1.0e-3 Summary of

- the results is very coarse in elements $3D$ and plate. The test deserves a finer modelization, since the results are influenced by the way of describing the conditions of fixed support in O . The modelization A led to an error of 14% to the maximum,
- however with a good taking into account of these conditions, the solution is definitely better (the modelization B leads to an error of 3% maximum).
- The comparisons of the stresses and forces give good performances (modelization B). For the shell element $1D$, the results are very good.