SSLV316 – Cracking with propagation imposed with X-FEM

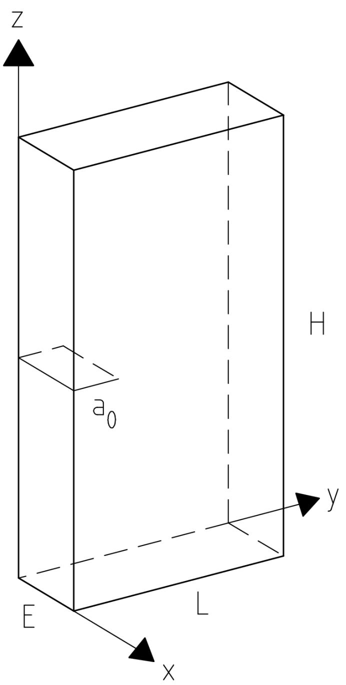
Summarized:

The goal of this test is to check that the methods simplex, upwind and geometrical of operator `PROPA_FISS` correctly calculate the position of the bottom of a crack 3D which propagates in mixed mode.

One simulates several propagations of a crack by imposing a projection and a given direction of propagation. The position of crack after each propagation is thus known and one can check if the position of bottom calculated by the operator `PROPA_FISS` is correct.

1 Problem of reference

1.1 Geometry



Appears 1.1-a: geometry of the fissured plate

geometrical Dimensions of the fissured plate:

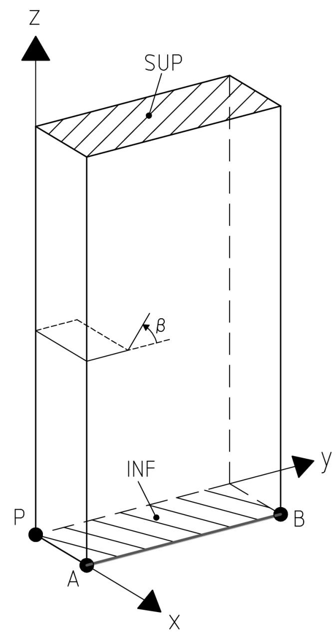
width	$L=8\text{m}$
thickness	$E=1\text{m}$
height	$H=18\text{m}$

initial Length of plane crack: $a_0=2\text{m}$
The crack is positioned in the middle of the height of the plate ($H/2$).

1.2 Properties of the material

Young's modulus $E=205\,000\,MPa$
Poisson's ratio $\nu=0.3$

1.3 Boundary conditions and loadings



Appears 1.3-a: boundary conditions and loadings

Boundary conditions:

Point: P $\Delta X = \Delta Y = \Delta Z = 0$

Points on the segment AB : $\Delta X = \Delta Z = 0$

Points on surface INF : $\Delta Z = 0$

Loading:

Pressure on surface SUP : $P = -1\,MPa$

The loading is constant during the propagation. Three calls with operator `PROPA_FISS` are made to simulate a propagation of initial crack already present in structure. On each call the advance and the direction of propagation of **each point** of the crack tip are imposed:

Point of the bottom advances: $\Delta a = 0.4\,m$

Angle of propagation: $\beta = 30°$

The positive direction of the angle β is visible on Appears 1.3-a.
The bottom of crack remains **always right** during all the propagation. The motivation of this choice will be explained in paragraph 4.

2 Reference solution

2.1 Method of calculating

One wants to check that the position of the bottom after propagation, calculated by the operator `PROPA_FISS`, is correct. One must thus calculate the theoretical position given by the advance and the imposed direction of propagation.
As already noticed, the crack tip remains right during all the propagation. The bottom is always perpendicular to two surfaces of the plate parallel with the plane YZ, like illustrated Appears 1.3-a. The position of the bottom can thus be indicated by means of only the coordinates Y and Z.
The initial position is the following one:

$$y_0 = a_0 = 2.0$$

$z_0 = 9.0$

After the propagation i , the new position of the bottom can be calculated like this:

$y_i = y_{i-1} + \Delta a \cdot \cos(i \cdot \beta)$

$z_i = z_{i-1} + \Delta a \cdot \sin(i \cdot \beta)$

2.2 Quantities and results of reference

For the three propagations calculated in the tests, the position of the bottom is the following one:

Coordinated	Coordinated y_i	propagation z_i
1	2.34641	9.19999
2	2.54642	9.54640
3	2.54644	9.94640

Table 2.1

In the current version of Code Aster, the coordinates of the points of the crack tip are available only in the file .mess and thus one cannot check them directly in the command file.

However, for this case test, one knows the theoretical position and the form (a segment) of the bottom of crack. In fact the bottom is always coincide with the edge which connects the two points $(0, y_i, z_i)$ and $(1, y_i, z_i)$. By means of the commands of postprocessing INTE_MAIL_3D and POST_RELEVE_T, one can of the level sets calculate the values at the points of intersection between this edge and the sides of the elements of the mesh. If the position of the bottom after the propagation is calculated correctly by PROPA_FISS, the value of both level sets must be equal to zero for all the found points of intersection because, by definition, the crack tip is formed by all the points where the level set tangent and norm are equal to zero.

That explains why one decided to give the same advance and direction of propagation to all the points of the crack tip. Even if the crack tip is located by two only coordinates, the propagation is 3D and the algorithms implemented in PROPA_FISS calculate a propagation in 3D.

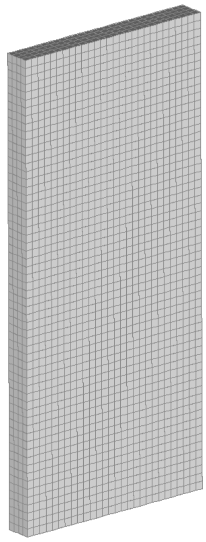
3 Modelization A

3.1 Characteristic of the modelization

the method **upwind** is used by PROPA_FISS to solve the equations of propagation of crack.
No auxiliary grid is used. That is possible because the mesh of structure is very regular. The field of computation is **localised** around the bottom of crack.

3.2 Characteristics of the mesh

the structure is modelled by a mesh made up of 6720 elements HEXA8 (see Appear 3.2-a).



Appear 3.2-a: mesh of structure

The mesh is not very refined to reduce the computing time. The size of the elements is uniform and equal to $0.29 x 0.33 x 0.25 m$.

3.3 Quantities tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the sides of the elements of the mesh by means of operator INTE_MAIL_3D (see Table 2.1). For each one of these points, one calculates the value of the level set norm (LSN) and tangent (LST) by means of operator POST_RELEVE_T and one checks that the values maximum and minimal are almost null:

Propag. i	**Max** LSN_i	**Min** LSN_i	**max LST_ı**	**Min** LST_i
1	-2.91E-16	-8.19E-16	the 0.002	0.002
2	-0.005	-0.005	0.004	0.004
3	-0.006	-0.006	-0.002	-0.002

values obtained are calculated starting from the values with the nodes of the mesh by means of the shape functions of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the mesh. Indeed the accuracy of representation of the level set is it even dependant in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost null. By considering that the mesh is coarse, one affects a tolerance equal to 15% length of the backbone of the mesh in the zone of propagation:

Tolerance used = $0.15 \times 0.33 = 0.05\,m$

3.4 Remarks

All the values tested respect the tolerance used. That means that the position of the crack tip calculated by the method upwind is correct and that the localization of the field goes well for the method upwind without auxiliary grid.

It should be noticed that after the three simulated propagations, the crack deviated of $90°$. On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the method upwind calculated well the position of crack and one thus checked his robustness.

4 Modelization B

4.1 Characteristic of the modelization

the method **upwind** is used by PROPA_FISS to solve the equations of propagation of crack. **Auxiliary grid** is used.
The same model that described for the modelization A is used. The field of computation is **localised** around the bottom of crack.

4.2 Characteristics of the mesh

the same mesh is used as that of modelization A.
auxiliary grid used consists of 1296 regular elements HEXA8 of dimension $0.25\times0.25\times0.25\,m$ (see Figure 4.2-a : 4.2-a).

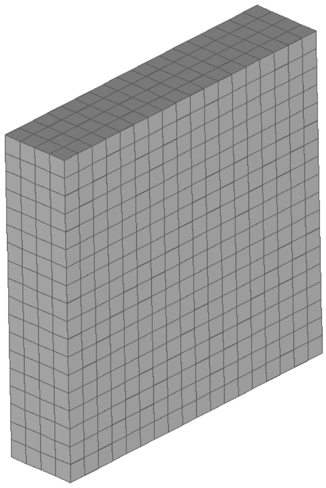


Figure 4.2-a : 4.2-a mesh used to define auxiliary grid

the grid is extended to the only zone of structure interested by the propagation of crack.

4.3 Quantities tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the sides of the elements of the mesh by means of operator INTE_MAIL_3D (see Table 2.1). For each one of these points, one calculates the value of the level set norm (LSN) and tangent (LST) by means of operator POST_RELEVE_T and one checks that the values maximum and minimal are almost null:

Propag. i	**Max** LSN_i	**Min** LSN_i	**max** LST_i	**Min** LST_i
1	1.64E-04	1.64E-04	the 0.004	0.004
2	-0.007	-0.007	0.014	0.014
3	-0.004	-0.004	-0.002	-0.002

values obtained are calculated starting from the values with the nodes of the mesh by means of the shape functions of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the mesh. Indeed the accuracy of representation of the level set is it even dependant in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost null. By considering that the mesh is coarse, one affects a tolerance equal to 15% length of the backbone of the mesh in the zone of propagation:

Tolerance used = $0.15\times 0.33=0.05\,m$

4.4 Remarks

All the values tested respect the tolerance used. That means that the position of the crack tip calculated by the auxiliary method upwind+grill is correct and that the localization of the field goes well for the auxiliary method upwind+grill.

It should finally be noticed that after the three simulated propagations, the crack deviated of $90°$. On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the auxiliary method upwind+grill calculated well the position of crack and one thus checked his robustness.

5 Modelization C

5.1 Characteristic of the modelization

the method **simplex** is used by PROPA_FISS to solve the equations of propagation of crack. **No auxiliary grid** is used. The field of computation is **localised** around the bottom of crack.
The same model that described for the modelization A is used.

5.2 Characteristics of the mesh

One uses the same mesh as that of modelization A.

5.3 Grandeurs tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the sides of the elements of the mesh by means of operator INTE_MAIL_3D (see Table 2.1). For each one of these points, one calculates the value of the level set norm (LSN) and tangent (LST) by means of operator POST_RELEVE_T and one checks that the values maximum and minimal are almost null:

Propag. i	**Max** LSN_i	**Min** LSN_i	**max** LST_i	**Min** LST_i
1	1.94E-16	-3.47E-16	0.002	0.002
2	-0.002	-0.002	0.005	0.005
3	-6.74E-04	-6.74E-04	the 0.003	0.003

values obtained are calculated starting from the values with the nodes of the mesh by means of the shape functions of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the mesh. Indeed, the accuracy of representation of the level set is it even dependant in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost null. By considering that the mesh is coarse, one affects a tolerance equal to 15% length of the backbone of the mesh in the zone of propagation:

Tolerance used = $0.15\times 0.33=0.05\,m$

5.4 Remarks

All the values tested respect the tolerance used. That means that the position of the crack tip calculated by the method simplex is correct and that the localization of the field goes well for the method simplex.

It should finally be noticed that after the three simulated propagations, the crack deviated of $90°$. On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the method simplex calculated well the position of crack and one thus checked his robustness.

6 Modelization D

6.1 Characteristic of the modelization

the method **simplex** is used by PROPA_FISS to solve the equations of propagation of crack. **Auxiliary grid** is used. The field of computation is **localised** around the bottom of crack.
The same model that described for the modelization A is used.

6.2 Characteristics of the mesh

One uses the same mesh as that of modelization A.
the same one auxiliary grid that of the modelization B is used.

6.3 Quantities tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the sides of the elements of the mesh by means of operator INTE_MAIL_3D (see Table 2.1). For each one of these points, one calculates the value of the level set norm (LSN) and tangent (LST) by means of operator POST_RELEVE_T and one checks that the values maximum and minimal are almost null:

Propag. i	**Max** LSN_i	**Min** LSN_i	**max** LST_i	**Min** LST_i
1	1.24E-04	1.24E-04	the 0.003	0.003
2	-0.007	-0.007	0.021	0.021
3	-0.015	-0.015	0.031	0.027

values obtained are calculated starting from the values with the nodes of the mesh by means of the shape functions of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the mesh. Indeed, the accuracy of representation of the level set is it even dependant in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost null. By considering that the mesh is coarse, one affects a tolerance equal to 15% length of the backbone of the mesh in the zone of propagation:

Tolerance used = $0.15\times 0.33=0.05\,m$

6.4 Remarks

All the values tested respect the tolerance used. That means that the position of the crack tip calculated by the auxiliary method simplexe+grill is correct and that the localization of the field goes well for the auxiliary method simplexe+grill.
If one C compares the results got with the results of the modelization, i.e. the results got by means of the same method (simplex) but without the assistance of one auxiliary grid, one can say that the values of the level sets are larger with auxiliary grid. That cannot be explained easily because the use of one auxiliary grid makes it possible to have the best results with the method upwind, as in the modelization B. the got results could show that the method simplex is not very robust. Other tests would be necessary to check it.

7 Modelization E

7.1 Characteristic of the modelization

This modelization is identical to the modelization A safe for the field of computation which **is not localised** around the bottom of crack. The update of the level sets is thus made under all the model.

7.2 Characteristics of the mesh

One uses the same mesh as that of modelization A.

7.3 Grandeurs tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the sides of the elements of the mesh by means of operator INTE_MAIL_3D (see Table 2.1). For each one of these points, one calculates the value of the level set norm (LSN) and tangent (LST) by means of operator POST_RELEVE_T and one checks that the values maximum and minimal are almost null:

Propag. i	**Max** LSN_i	**Min** LSN_i	**max** LST_i	**Min** LST_i
1	-2.91E-16	-8.19E-16	the 0.002	0.002
2	-0.005	-0.005	0.004	0.004
3	-0.006	-0.006	-0.002	-0.002

values obtained are calculated starting from the values with the nodes of the mesh by means of the shape functions of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the mesh. Indeed the accuracy of representation of the level set is it even dependant in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost null. By considering that the mesh is coarse, one affects a tolerance equal to 15% length of the backbone of the mesh in the zone of propagation:

Tolerance used = $0.15\times 0.33=0.05\,m$

7.4 Remarks

All the values tested respect the tolerance used. That means that the position of the crack tip calculated by the method upwind is correct.

It should be noticed that after the three simulated propagations, the crack deviated of $90°$. On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the method upwind calculated well the position of crack and one thus checked his robustness.

8 Modelization F

8.1 Characteristic of the modelization

This modelization is identical to the modelization B safe for the field of computation which **is not localised** around the bottom of crack. The update of the level sets is thus made under all the model.

8.2 Characteristics of the mesh

One uses the same mesh as that of the modelization A and the same one auxiliary grid as that of the modelization B.

8.3 Quantities tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the sides of the elements of the mesh by means of operator `INTE_MAIL_3D` (see Table 2.1). For each one of these points, one calculates the value of the level set norm (LSN) and tangent (LST) by means of operator `POST_RELEVE_T` and one checks that the values maximum and minimal are almost null:

Propag. i	**Max** LSN_i	**Min** LSN_i	**max** LST_i	**Min** LST_i
1	1.64E-04	1.64E-04	0.004	0.004
2	-0.007	-0.007	0.014	0.014
3	-0.003	-0.003	-7.44E-04	-7.44E-04

the values obtained are calculated starting from the values with the nodes of the mesh by means of the shape functions of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the mesh. Indeed the accuracy of representation of the level set is it even dependant in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost null. By considering that the mesh is coarse, one affects a tolerance equal to 15% length of the backbone of the mesh in the zone of propagation:

Tolerance used = $0.15\times 0.33=0.05\,m$

8.4 Remarks

All the values tested respect the tolerance used. That means that the position of the crack tip calculated by the auxiliary method upwind+grill is correct.

It should finally be noticed that after the three simulated propagations, the crack deviated of $90°$. On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the auxiliary method upwind+grill calculated well the position of crack and one thus checked his robustness.

9 Modelization G

9.1 Characteristic of the modelization

This modelization is identical to the modelization C safe for the field of computation which **is not localised** around the bottom of crack. The update of the level sets is thus made under all the model.

9.2 Characteristics of the mesh

One uses the same mesh as that of the modelization C.

9.3 Quantities tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the sides of the elements of the mesh by means of operator `INTE_MAIL_3D` (see Table 2.1). For each one of these points, one calculates the value of the level set norm (LSN) and tangent (LST) by means of operator `POST_RELEVE_T` and one checks that the values maximum and minimal are almost null:

Propag. i	**Max** LSN_i	**Min** LSN_i	**max** LST_i	**Min** LST_i
1	2.08E-16	-3.47E-16	the 0.002	0.002
2	-0.002	-0.002	0.011	0.011
3	-0.005	-0.005	0.001	0.001

values obtained are calculated starting from the values with the nodes of the mesh by means of the shape functions of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the mesh. Indeed, the accuracy of representation of the level set is it even dependant in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost null. By considering that the mesh is coarse, one affects a tolerance equal to 15% length of the backbone of the mesh in the zone of propagation:

Tolerance used = $0.15\times0.33=0.05\,m$

9.4 Remarks

All the values tested respect the tolerance used. That means that the position of the crack tip calculated by the method simplex is correct.

It should finally be noticed that after the three simulated propagations, the crack deviated of $90°$. On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the method simplex calculated well the position of crack and one thus checked his robustness.

10 Modelization H

10.1 Characteristic of the modelization

This modelization is identical to the modelization D safe for the field of computation which **is not localised** around the bottom of crack. The update of the level sets is thus made under all the model.

10.2 Characteristics of the mesh

One uses the same mesh as that of the modelization C and the same one auxiliary grid as that of modelization D.

10.3 Grandeurs tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the sides of the elements of the mesh by means of operator `INTE_MAIL_3D` (see Table 2.1). For each one of these points, one calculates the value of the level set norm (LSN) and tangent (LST) by means of operator `POST_RELEVE_T` and one checks that the values maximum and minimal are almost null:

Propag. i	**Max** LSN_i	**Min** LSN_i	**max** LST_i	**Min** LST_i
1	1.24E-04	1.24E-04	the 0.003	0.003
2	-0.006	-0.006	0.020	0.020
3	-0.017	-0.017	0.030	0.030

values obtained are calculated starting from the values with the nodes of the mesh by means of the shape functions of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the mesh. Indeed, the accuracy of representation of the level set is it even dependant in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost null. By considering that the mesh is coarse, one affects a tolerance equal to 15% length of the backbone of the mesh in the zone of propagation:

Tolerance used = $0.15\times0.33=0.05\,m$

10.4 Remarks

All the values tested respect the tolerance used. That means that the position of the crack tip calculated by the auxiliary method simplexe+grill is correct.

It should finally be noticed that after the three simulated propagations, the crack deviated of $90°$. On the other hand the total advance is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the method simplex calculated well the position of crack and one thus checked his robustness.

11 Modelization I

11.1 Characteristic of the modelization

the geometrical **method** is used by PROPA_FISS for the computation of the new position of crack. The field of computation is **localised** around the bottom of crack. **No auxiliary grid** is used.

11.2 Characteristics of the mesh

One uses the same mesh as that of modelization A.

11.3 Grandeurs tested and results

One calculates the points of intersection between the edge which gives the theoretical position of the bottom and the sides of the elements of the mesh by means of operator INTE_MAIL_3D (see Table 2.1). For each one of these points, one calculates the value of the level set norm (LSN) and tangent (LST) by means of operator POST_RELEVE_T and one checks that the values maximum and minimal are almost null:

Propag. i	**Max** LSN_i	**Min** LSN_i	**max** LST_i	**Min** LST_i
1	5.55E-16	2.22E-16	-1.94E-16	-3.747E-16
2	-0.0123	-0.0123	-1.97E-4	-1.97E-4
3	-0.027	-0.027	-6.72E-3	-6.72E-3

the values obtained are calculated starting from the values with the nodes of the mesh by means of the shape functions of the elements. One thus expects that these values are affected by an error which depends on the size of the elements of the mesh. Indeed, the accuracy of representation of the level set is it even dependant in keeping with elements. Consequently one uses a tolerance to check if the level sets calculated are almost null. By considering that the mesh is coarse, one affects a tolerance equal to 15% length of the backbone of the mesh in the zone of propagation:

Tolerance used = $0.15\times 0.33=0.05\,m$

11.4 Remarks

All the values tested respect the tolerance used. That means that the position of the crack tip calculated by the geometrical method is correct.

It should finally be noticed that after the three simulated propagations, the crack deviated of $90°$. On the other hand the total projection is small. Thus a propagation in very severe mixed mode was simulated. The conditions used are more severe than the conditions than one finds normally for real structures. However the geometrical method calculated well the position of crack and one thus checked his robustness.

12 Summary of the results

All the methods used (upwind, upwind+grill auxiliary, simplex, simplexe+grill auxiliary, geometrical) made it possible to calculate the position of a crack well propagating in mixed mode in severe conditions. That also made it possible to validate the implementation of these methods in the operator PROPA_FISS and in particular the possibility of locating update zone level-sets.

If it is considered that the mesh used in the cases test is coarse, one can say that the methods calculate in a sufficiently precise way the position of the crack tip.