SSLV311 - Murakami 9.39. Fissure in quarter of ellipse to the corner of a thick disc in rotation

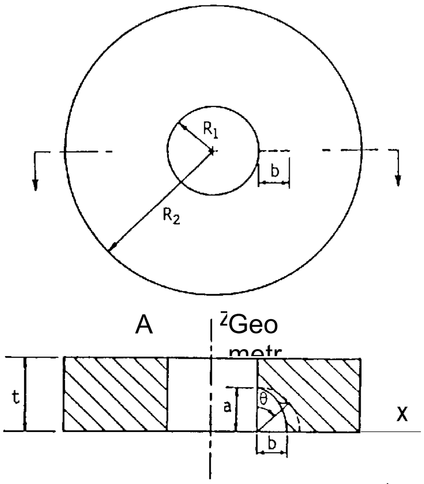
Summarized:

This test is resulting from the validation independent of the version in fracture mechanics.

Scope of application:	Standard linear
fracture mechanics of analysis:	Standard
static of behavior:	Standard isotropic linear
elastic of model:	Three-dimensional
Number of modelizations:	1
Purpose:	Basic test into three-dimensional for the isotropic elastic materials, in field limited in three directions, in the presence of a voluminal loading.
Explored parameters:	-
Fixed Parameters:	Ratios a/t b/a R_2/R_1 , t/R_1
Accuracy of the results:	Average standard deviation of 3% with the analytical reference solution

1 Problem of reference

1.1 internal



B Radius: $R_1=0{,}1\,m$
External radius: $R_2=0{,}6\,m$
Thickness: $t=0{,}2\,m$
Half main roads: $a=0{,}05\,m$
Small half centers: $b=0{,}0125\,m$

1.2 Properties of the material

Modulus Young $E=2\,10^5\,MPa$
Poisson's ratio $\nu=0.3$
Density $\rho=7800\,kg/m^3$

1.3 Boundary conditions and loading

The model will be restricted with the part of the thick disc located in the half space $Y\geq 0$, the plane of the crack vertical being a symmetry plane.

In the absence of nodes on the axis of revolution, a rigid mode will be blocked by a linear relation between degrees of freedom.

That is to say points: $A(R_1, 0, t)$ $B(-R_1, 0, t)$

Blocking of the translation in X : $UX(A)+UX(B)=0$
Blocking of the translation in Y : $UY=0$ in the plane XOZ , except for the lips of crack.
Blocking of the translation in Z : $UZ(A)=0$

Blocking of rotation around OX : ensured by the boundary condition of symmetry in the Blocking XOZ
plane of rotation around OY : $UZ(B)=0$

Blocking of rotation around OZ : ensured by the boundary condition of symmetry in the Loading XOZ

plane: steady angular velocity $\omega = 500\, rad/s$

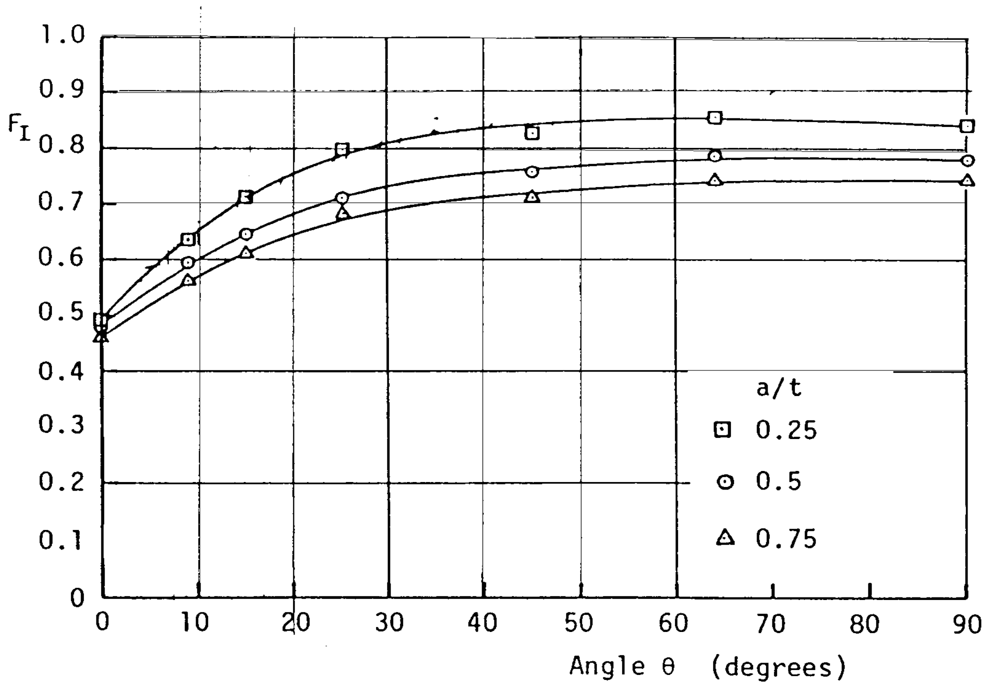
2 Reference solution

2.1 Method of calculating used for the reference solution

Method of integral equation of border.

2.2 Results of reference

$K_I = \frac{3+\nu}{4} \cdot \rho\,\omega^2 \left(R_2^2 + \frac{1-\nu}{3+\nu} R_1^2 \right) \cdot \sqrt{\pi\, b} \cdot F_I$ where the geometrical factor of correction is given, according to the parametric angle of the ellipse θ, with the figure below.



The selected a/t ratio corresponds to the higher curve (squares).

2.3 Uncertainty on the solution

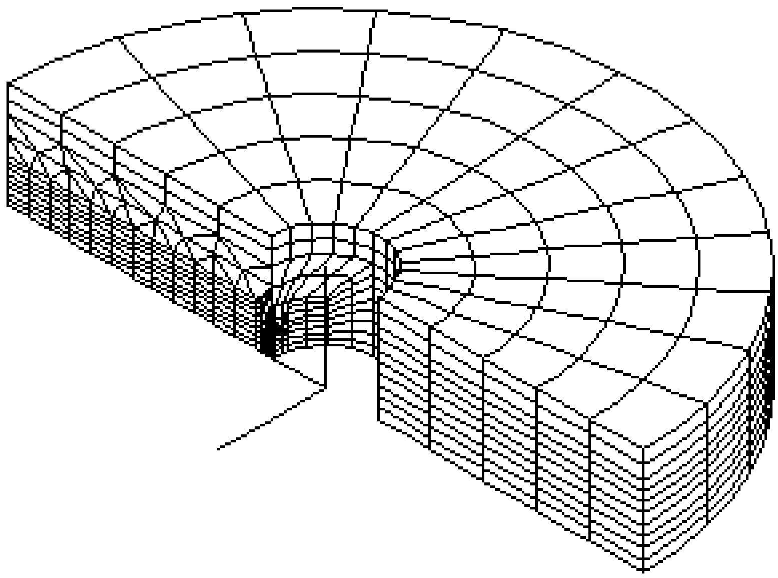
the maximum change between the points marked and the curve being of 2%, the misreading on the curve is lower than the announced maximum error (5%).

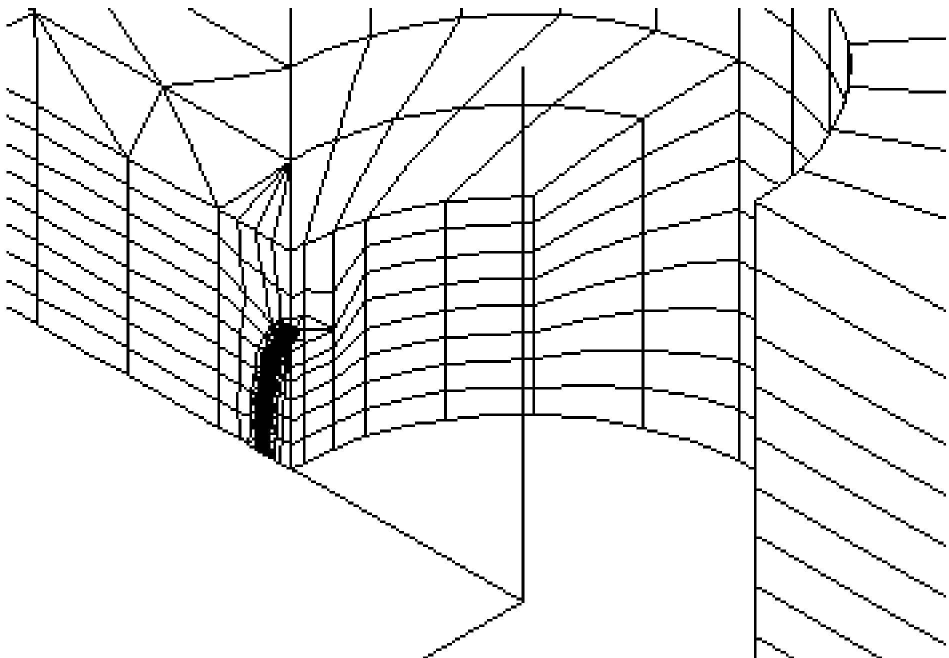
2.4 Bibliographical references

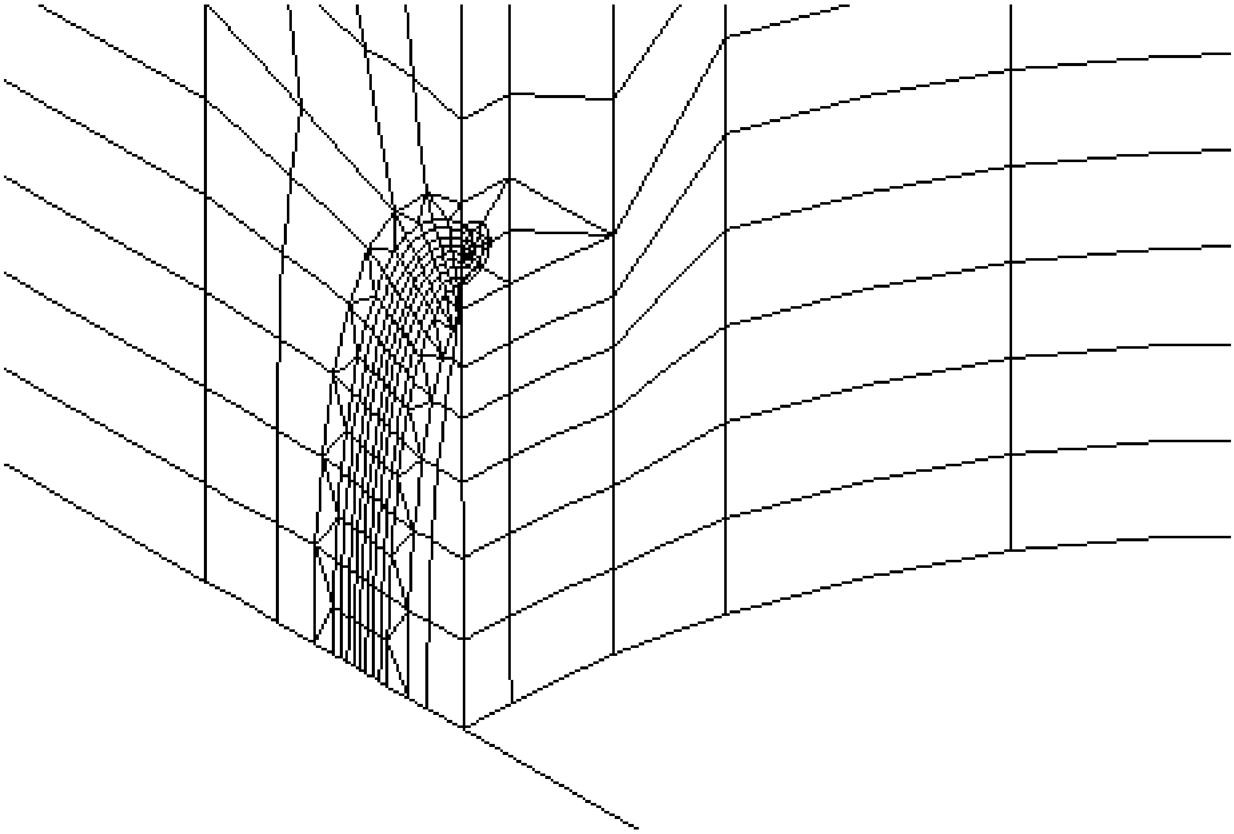
1) Y. MURAKAMI: Stress Intensity Factors Handbook, box 9.39, pages 786-791. The Society of Materials Science, Japan, Pergamon Near, 1987.

3 Modelization A

3.1 Characteristic of the modelization







3.2 Characteristics of the mesh

The mesh initial consists of 8890 nodes and 2203 elements, including 1264 elements $CU20$ and 939 elements $PR15$.

After the conversion of the mesh of quadratic to linear, the number of nodes is reduced to 2230. This conversion is made essential by the use of the operator `DEFI_FISS_XFEM`, who functions for the moment only with linear elements.

3.3 Functionalities tested

Computation of the factors of intensity of the stresses buildings, in all the nodes of the crack tip, by the THETA method .

The factors of intensity of the stresses buildings are calculated on a contour of lower radius $Rinf=0{,}00075\,m$ and higher radius $\mathrm{Rsup}=0{,}0025\,m$.

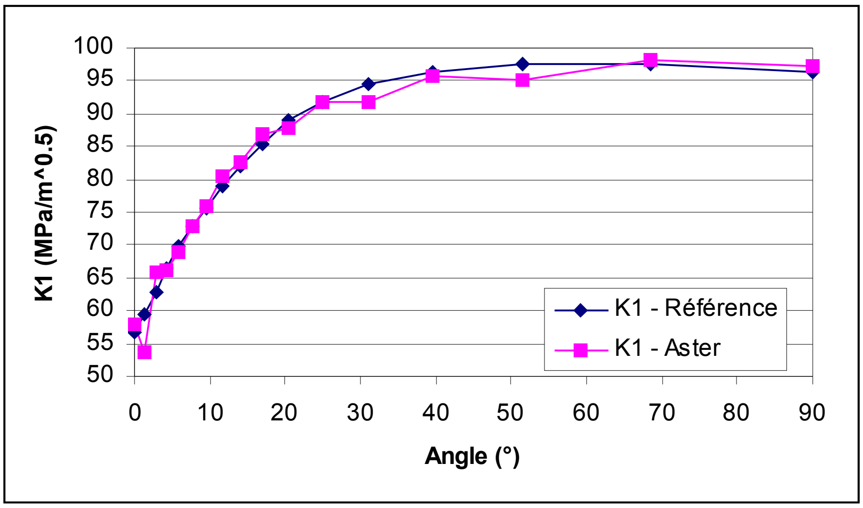
3.4 Values tested and results of the modelization A

Identification	Reference ($Pa.\sqrt{m}$)	*Aster* ($Pa.\sqrt{m}$)	% difference
K_I , θ = 0 degrees	5,657E+07	5,789E+07	-2,33
K_I , θ = 1,4 degrees	5,945E+07	5,360E+07	9,84
K_I , θ = 2,8 degrees	6,292E+07	6,596E+07	-4,84
K_I , θ = 4,3 degrees	6,638E+07	6,606E+07	0,48
K_I , θ = 5,9 degrees	6,984E+07	6,902E+07	1,18

K_I, θ = 7,6 degrees	7,273E+07	7,289E+07	-0,22
K_I, θ = 9,5 degrees	7,562E+07	7,597E+07	-0,47
K_I, θ = 11,6 degrees	7,908E+07	8,053E+07	-1,83
K_I, θ = 14,4 degrees	8,197E+07	8,261E+07	-0,78
K_I, θ = 16,9 degrees	8,543E+07	8,695E+07	-1,78
K_I, θ = 20,5 degrees	8,889E+07	8,785E+07	1,17
K_I, θ = 25,1 degrees	9,178E+07	9,190E+07	-0,13
K_I, θ = 31,1 degrees	9,466E+07	9,173E+07	3,09
K_I, θ = 39,5 degrees	9,640E+07	9,562E+07	0,81
K_I, θ = 51,5 degrees	9,755E+07	9,510E+07	2,51
K_I, θ = 68,5 degrees	9,755E+07	9,824E+07	-0,71
K_I, θ = 90 degrees	9,640E+07	9,720E+07	-0,83

the parametric angles of the values tested corresponds to the position of the 17 points of the crack tip. The figure below makes it possible to compare result computation with the reference solution. The average standard deviation is very satisfactory:

Average standard deviation = $$\varepsilon=\sqrt{\frac{\int_{\Gamma}(K_I^{ref}-K_I^{aster})^2 ds}{\int_{\Gamma}(K_I^{ref})^2 ds}}=3.11\%$$



Note::

The voluminal loading is introduced here using key word `FORCE_INTERNE` *(command* `AFFE_CHAR_MECA` *) and of* `FORMULA` *. The results are equivalent if key word ROTATION* `is used.`

4 Summary of the results

Within sight of the accuracy announced on the results of reference (5%) and of the average standard deviation obtained ($3{,}11\%$), the results provided by *Code_Aster* are satisfactory.