SSLV155 – Crack lens in tension

Summarized:

The purpose of this test is validating the computation of the stress intensity factors (SIFs) along the bottom of a crack 3D nonplane, in the frame of linear elasticity.

This test formats concerned a cube with a crack of lens, subjected to a hydrostatic tension.

This test contains 3 modelizations:
–Modelization a: the crack is with a grid in 2D-axi (FEM);
–Modelization b: the crack is not with a grid, it is represented by of the level sets (X-FEM) in 2D-axi;
–Modelization C: the crack is not with a grid, it is represented by of the level sets (X-FEM) in 3D;

There is no modelization FEM in 3D because the creation of the mesh is extremely difficult.

For each modelization, SIFs are evaluated by commands `POST_K1_K2_K3` and `CALC_G`.
The numerical values are compared with the analytical values.

1 Problem of reference

1.1 Geometry

One considers a cube of with dimensions $2L$ and a crack in the shape of lens (Lens shaped ace) of radius R such as $\frac{L}{R}=5$ and of angle in the center $\alpha=\frac{\pi}{4}$ (see Figure 1).

The equation characteristic of the form of the surface of crack is:
$x^2+y^2+(z-R)^2=R^2$ with $0\leq z\leq(1-\cos\alpha)R$.

Formula $a=R\sin\alpha$.

The equation characteristic of the crack tip is:
$x^2+y^2=a^2$ with $z=(1-\cos\alpha)R$

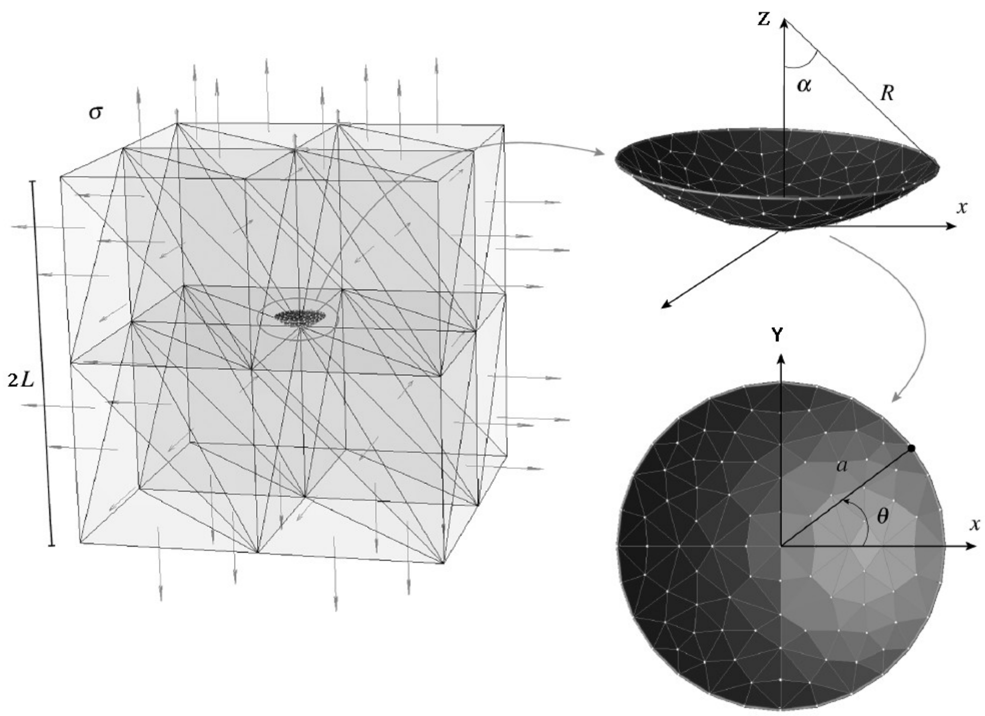

Figure 1: geometry of the cube fissured

1.2 Material properties

the material is elastic isotropic whose properties are:
$E=210\,000\,MPa$
$\nu=0,3$

1.3 Boundary conditions and loadings

the cube is subjected to a hydrostatic tension σ.

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution 3 for a crack in the shape of lens of radius R in an infinite medium, subjected to a uniform pressure σ rather far away from crack shows that the stress intensity factors are constant along the crack tip and are worth:

$$K_I=0{,}877\left(\frac{2}{\pi}\right)\sigma\sqrt{\pi a}$$
$$K_{II}=0{,}235\left(\frac{2}{\pi}\right)\sigma\sqrt{\pi a}$$
$$K_{III}=0$$

with $a=R\sin\alpha$.

2.2 Results of reference

For the loading considered $\sigma=1\,MPa$ and the characteristic geometrical following:
$L=10\,m$ $R=2\,m$ $a=\frac{\sqrt{2}}{2}$, one finds

$$K_I=1{,}177\,MPa.\sqrt{m}$$
$$K_{II}=0{,}3153\,MPa.\sqrt{m}$$
$$K_{III}=0$$

2.3 bibliographical References

(1) J.P. Pereira, A.C. Duarte, D. Guoy and X. Jiao: HP-Generalized FEM and ace surfaces representation for NON-double diffusion 3D aces, Int. J. Numer. Engng, vol. 77,2009

3 Modelization a: Modelization FEM 2D-axi

3.1 Characteristic of the modelization

In this modelization, the crack is with a grid (case FEM) and the structure is modelled in 2D-axisymetry.

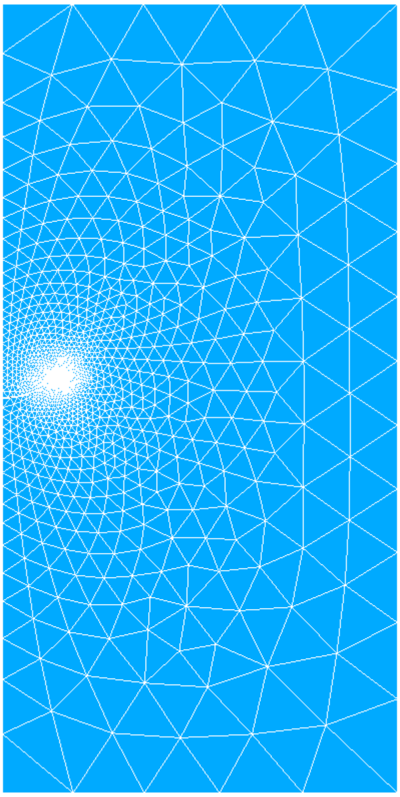

Figure 3.1-1: mesh 2d-axi (FEM)

3.2 Characteristic of the mesh

Many nodes: 5211
Number of meshes and type: 2550 `TRIA6`
the length characteristic of an element close to the crack tip are of H = 0,025m.

The nodes mediums of the edges of the elements touching the crack tip are moved with the quarter of these edges (elements of Barsoum).

3.3 Boundary conditions and loadings

–a surface force of tension is applied to the sides higher, lower and that of right;
–Displacements following Ox of the nodes of the rotational axis are blocked, as that is advised for the axisymmetric modelizations;
–The rigid mode of displacement following the axis Oy is blocked via the blocking of a node following this axis.

3.4 Quantities tested and results

One tests the values of K_I and K_{II} in crack tip exits of commands CALC_G and POST_K1_K2_K3. These values are compared with the analytical solution.

Integration contours of the field theta for command CALC_G are:
$R_{\text{inf}}=2h$ and $R_{\text{sup}}=5h$.

Parameter ABS_CURV_MAXI of operator POST_K1_K2_K3 is selected by default.

3.4.1 Values resulting from **CALC_G**

the values are in $Pa.\sqrt{m}$.

Standard	identification of reference	Value of reference	% Tolerance
K_I	"ANALYTIQUE"	1,177 106	2%
K_{II}	"ANALYTIQUE"	0,3153 106	2%

3.4.2 Values resulting from **POST_K1_K2_K3**

the values are in $Pa.\sqrt{m}$.

Standard	identification of reference	Value of reference	% Tolerance
K_I	"ANALYTIQUE"	1,177 106	2%
K_{II}	"ANALYTIQUE"	0,3153 106.12%	12%

4 Modelization b: Modelization X-FEMs 2D-axi

4.1 Characteristic of the modelization

In this modelization, the crack is not with a grid (case X-FEM) and the structure is modelled in 2D-axisymetry.

The crack is represented by of the level sets:

$$lsn=\sqrt{x^2+(y-R)^2}-R$$

$$lst=\sqrt{x^2+(y-y_h)^2}-R'$$

$$y_h=R-\frac{R}{\cos(\alpha)} \text{ and } R'=R\tan(\alpha)$$

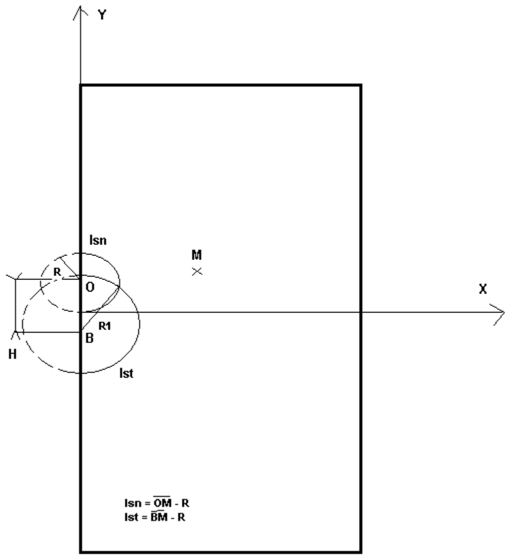

Figure 4.1-1: level setsavec

4.2 Caractéristiques of the initial

mesh The mesh healthy are relatively coarse: 252 nodes and 442 TRIA3. The size of meshes is $h_0=1\,m$. One uses a procedure of successive refinement to lead to a target size corresponding to half of the size of meshes of the modelization A, that is to say $h_c=0{,}0125\,m$. Indeed, the modelization A uses quadratic elements, one thus needs finer linear elements to obtain an equivalent accuracy. For that, one calls Homard in an iterative way. After refinement, the size of meshes close to the crack tip is worth $h=0{,}0078125\,m$. One meshes refines all in a disc of radius $5\,h$ around the crack tip.

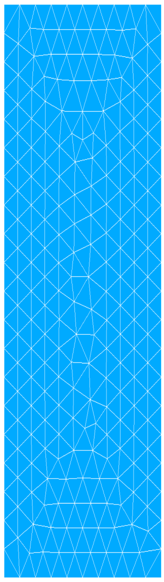

Figure 4.2-1: initial sane mesh

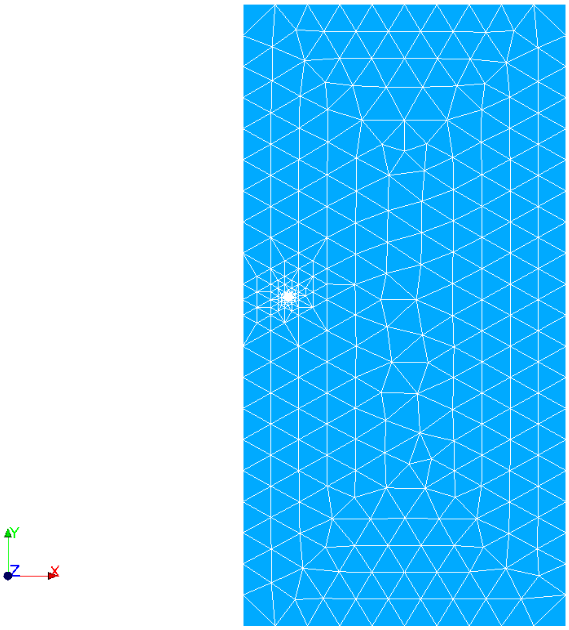

Figure 4.2-2: sane mesh refined

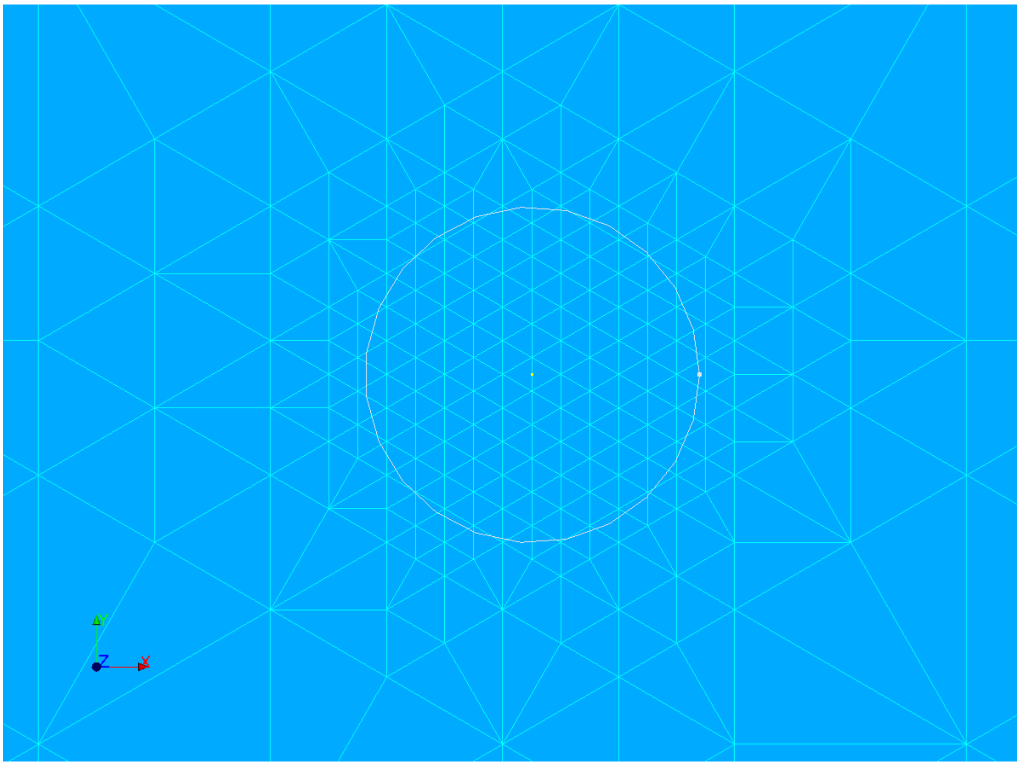

Figure 4.2-3: zoom on the refined part (the disc of the zone of refinement is also indicated)

Many nodes: 722
Number of meshes and type: 1442 TRIA3
the length characteristic of an element close to the crack tip are of 0,0078 Mr.
This size is lower than the target size (for reasons of whole division by 2 during refinement).

4.3 Boundary conditions and loadings

–a surface force of tension is applied to the sides higher, lower and that of right;
–Displacements according to Ox of the nodes of the rotational axis are blocked, as that is advised for the axisymmetric modelizations;
–The rigid mode of displacement following the axis Oy is blocked via the blocking of a node following this axis.

4.4 Quantities tested and results

the choice of the numerical parameters for the postprocessing of SIFs is identical to that done for modelization a: $R_{\text{inf}}=2\text{h}$ and $R_{\text{sup}}=5\text{h}$, but h is worth here less than half of h modelization A.

4.4.1 Valeurs resulting from `CALC_G`

the values are in $Pa.\sqrt{m}$.

Standard	identification of reference	Value of reference	% Tolerance
K_I	"ANALYTIQUE"	1,177 106	2.0%
K_{II}	"ANALYTIQUE"	0,3153 106	7.0%

4.4.2 Values resulting from `POST_K1_K2_K3`

the values are in $Pa.\sqrt{m}$.

Standard	**identification of reference**	**Value of reference**	**% Tolerance**
K_I	`“ANALYTIQUE C`	” 1,177	6.0%
K_{II}	`formula “ANALYTIQUE`	” 0,3153	8.0%

5 8,0% Modelization: Modelization X-FEMs 3D

5.1 Characteristic of the modelization

In this modelization, the crack is not with a grid (case X-FEM) and the structure is modelled in 3D. Only a quarter of structure is modelled, for reasons of symmetry.

The crack is represented by of the level sets:

$$lsn=\sqrt{x^2+(y-R)^2+z^2}-R$$

$$lst=\sqrt{x^2+(y-y_h)^2+z^2}-R'$$

with $y_h=R-\frac{R}{\cos(\alpha)}$ and $R'=R\tan(\alpha)$

Note:
In 3D, there exists an arbitrary choice of directional sense of the local base in crack tip. According to the directional sense of this base, the sign of K_{II} and K_{III} will be different (because related to the base). But that does not affect the result physical one (possible angle of bifurcation of crack expressed in the total reference). Here, to find $K_{II}>0$ as in the reference solution, it is necessary to define the level set norm like the opposite one of the formula above. In this modelization, one will retain finally $lsn=-\sqrt{x^2+(y-R)^2+z^2}+R$. That remains a convention and does not change the result physical one!
In 2d, the sign of the level set norm does not influence the sign of K_{II} because in 2d, there is not fall to define the local base.

5.2 Characteristics of the initial

mesh The mesh healthy is relatively coarse: 2508 nodes and 11945 `TETRA4` . The size of meshes is $h_0=1\,m$. One uses a procedure of successive refinement to lead to a size targets virtually identical to that of the modelization A, that is to say $h_c=0{,}025\,m$. For that, one calls Homard in an iterative way. After refinement, the size of meshes close to the crack tip is $h=0{,}015625\,m$. One meshes refines all in a disc of radius $5\,h$ around the crack tip.

Many nodes: 18166
Number of meshes and type: 103079 `TETRA4`
the length characteristic of an element close to the crack tip are of 0,0156 Mr.

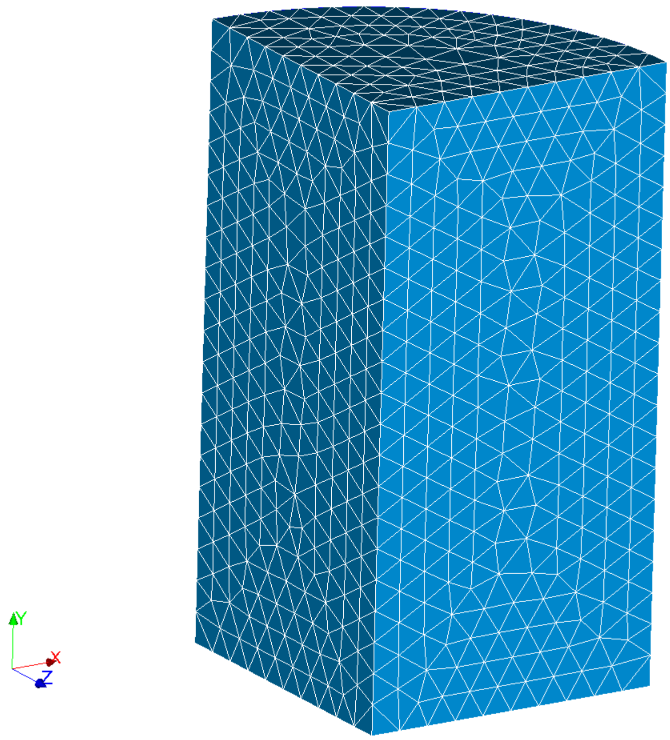

Figure 5.2-1: initial mesh

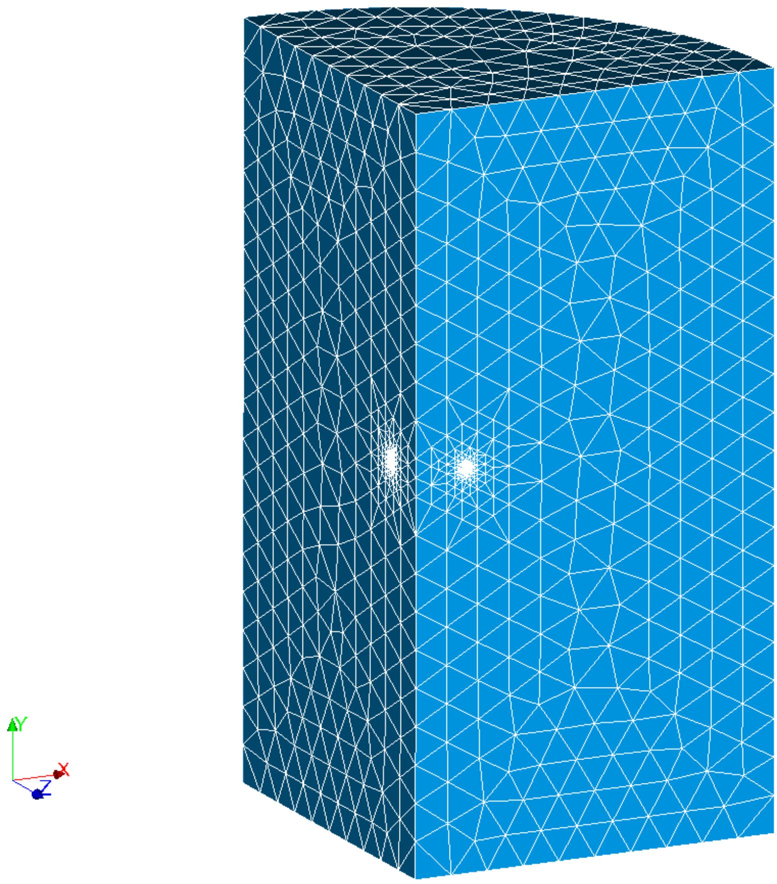

Figure 5.2-2: mesh refined

5.3 Boundary conditions and loadings

- a surface force of tension is applied to the sides higher, lower and external;
- The conditions of symmetry on the side sides are applied;
- The rigid mode of displacement following the axis Oy is blocked via the blocking of a node following this axis.

5.4 Quantities tested and results

the choice of the numerical parameters for the postprocessing of SIFs is identical to that done for modelization a: $R_{\mathrm{inf}}=2h$ and $R_{\mathrm{sup}}=5h$.

5.4.1 Values resulting from `CALC_G`

the values are in $Pa.\sqrt{m}$.

Standard	identification of reference	Value of reference	Tolerance
$max(K_I)$	`"ANALYTIQUE"`	1,177 106	5%
$min(K_I)$	`"ANALYTIQUE"`	1,177 106	2%
$max(K_{II})$	`"ANALYTIQUE"`	0,3153 106.15%	15%
$min(K_{II})$	`"ANALYTIQUE"`	0,3153 106	5%

5.4.2 Values resulting from `POST_K1_K2_K3`

the values are in $Pa.\sqrt{m}$.

Standard	identification of reference	Value of reference	Tolerance
$max(K_I)$	`"ANALYTIQUE"`	1,177 106	2%
$min(K_I)$	`"ANALYTIQUE"`	1,177 106	9%
$max(K_{II})$	`"ANALYTIQUE"`	0,3153 106.14%	14%
$min(K_{II})$	`"ANALYTIQUE"`	0,3153 106	2%

5.4.3 Comments

By more refining the mesh, one can decrease the error, but the computing time becomes incompatible with that of a benchmark.

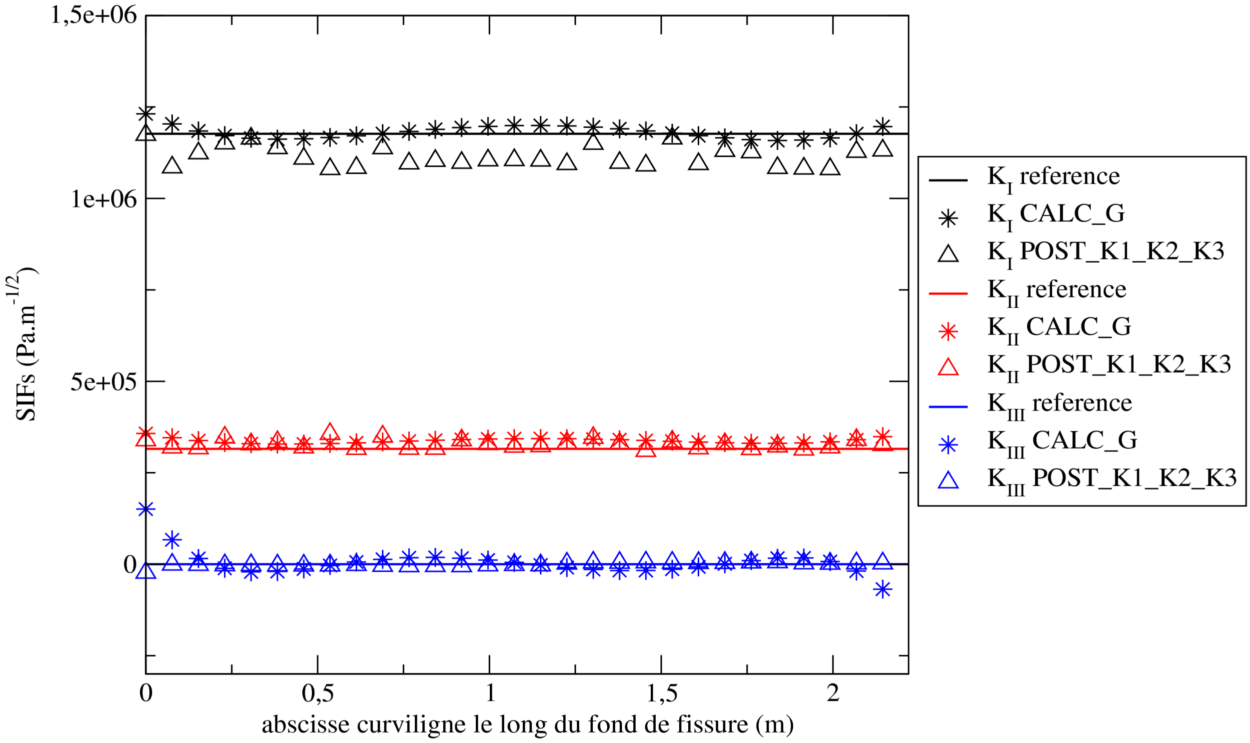

Figure 5.4.3-1: comparison of K enter the various methods

6 Summary of the results

This benchmark validates the computation of the stress intensity factors of a nonplane crack in 2D and 3D.