SSLV146 – Cubic full reinforced by reinforcements under triaxial loading

Abstract

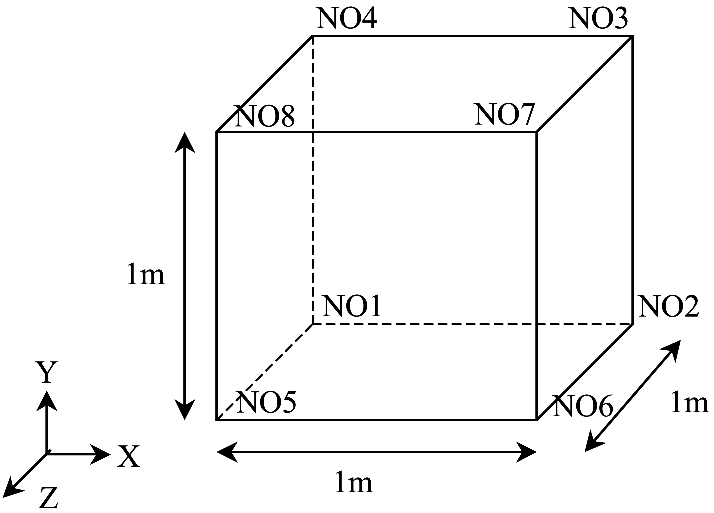
This test $3D$ enters the frame of the validation of formulation GRILLE_MEMBRANE. It is about a cube full of unit size. One places on each face a three-dimensions function of steel reinforcements so as to sweep all the possible directions. The loading consists into cubes displacements imposed on all the nodes of structure.

The principal interest of this test is to test modelization GRILLE_MEMBRANE for various directional senses and various elements (linear and quadratic). The results are compared with an analytical solution.

The units of all the numerical values are in IF.

1 Problem of reference

1.1 Geometry



One defines six three-dimensions functions of reinforcements (one by side of the cube):

- $GEOX$ (2 three-dimensions functions): sides $NO1NO4NO8NO5$ and $NO2NO6NO7NO3$
- $GEOY$ (2 three-dimensions functions): sides $NO1NO2NO6NO5$ and $NO4NO3NO7NO8$
- $GEOZ$ (2 three-dimensions functions): sides $NO1NO2NO3NO4$ and $NO5NO6NO7NO8$

1.2 Properties of the materials

For the full cube:

Modelization	A	B	C	D
$E(Pa)$	2	2	2E14	2E14
ν	0	0	0	0

For the three-dimensions functions of reinforcements (any confused modelization)

$E=2\text{E}11\,Pa$, $\nu=0$

- Three-dimensions function $GEOX$: section per linear meter $0.01\,m^2/ml$, eccentring 0, directional sense (ANGL_REP) $(30\,;0)$
- Three-dimensions function $GEOY$: section per linear meter $0.02\,m^2/ml$, eccentring 0, directional sense (ANGL_REP) $(0\,;40)$
- Three-dimensions function $GEOZ$: section per linear meter $0.03\,m^2/ml$, eccentring 0, directional sense (ANGL_REP) $(15\,;70)$

1.3 Boundary conditions and loadings

the boundary conditions are the following ones:

$DX=0$ to the face $NO2NO3NO7NO6$
$DY=0$ on the face $NO1NO2NO6NO5$
$DZ=0$ on the face $NO1NO2NO3NO4$

the loading is applied in an increment in the following way (imposed displacements):

$DX=1$ on the face $NO1NO4NO8NO5$
$DY=2$ on the face $NO4NO3NO7NO8$
$DZ=3$ on the face $NO5NO6NO7NO8$

2 Reference solution

2.1 formal Solution

One seeks to define the strain ε according to the principal direction of a three-dimensions function of reinforcement located in the plane $(x_1 ; y_1)$.

Taking into account the boundary conditions chosen, one can write:

$$\varepsilon = u_{x1}\cos^2(\theta)+u_{y1}\sin^2(\theta)$$

with (u_{x1}, u_{y1}) the components of the vector displacement in the plane $(x_1 ; y_1)$ and θ the angle the principal direction of the three-dimensions function of reinforcement enters and x_1.

To define the principal direction of the three-dimensions function, one uses the nautical angles $(\alpha ; \beta)$ given by key word `ANGL_REP`. They define a vector v whose projection x_p on the tangent level of the three-dimensions function fixes the principal direction.

$$v=\cos(\alpha)\cos(\beta)x+\sin(\alpha)\cos(\beta)y+\sin(\beta)z$$

with (x, y, z) the initial reference. For our application, the vector displacement U is written:

$$v=1.x+2.y+3.z$$

For the three-dimensions function $GEOX$ (plane $(y ; z)$):

$$x_p=\sin(30)y$$

The principal direction of the three-dimensions function is y ($\theta=90°$). The strain is written then:

$$\varepsilon = u_y = 2$$

For the three-dimensions function $GEOY$ (plane $(x ; z)$):

$$x_p=\cos(40)x+\sin(40)z$$

The principal direction of the three-dimensions function thus forms an angle of $40°$ with the plane of the three-dimensions function. The strain is written then:

$$\varepsilon = u_x\cos^2(40)+u_z\sin^2(40)=1.82635$$

For the three-dimensions function $GEOZ$ (plane $(x\,;y)$):

$$x_p = \cos(15).\cos(70)x + \sin(15)\cos(70)y$$

The principal direction of the three-dimensions function thus forms an angle of $15°$ with the plane of the three-dimensions function. The strain S "writes then:

$$\varepsilon = u_x \cos^2(15) + u_y \sin^2(15) = 1.067$$

These three values will be the analytical values of reference for the validation of computations.

3 Modelizations

According to the modelizations, the objects are with a grid with different elements:

Modelization A: cubic: 1 element HEXA8 with 8 nodes
sides: 1 element QUAD4 with 4 nodes

Modelization B: cubic: 6 elements TETRA4 with 4 nodes
sides: 2 elements TRIA3 with 3 nodes

Modelization C: even modelization qu" in A with quadratic elements

Modelization D: even modelization that in B with quadratic elements

3.1 Quantities tested and results of the modelization To

the modelization A consists of an element CUB8 for the cube and of an element QUA4 for each face. The behavior is elastic (commands MECA_STATIQUE then STAT_NON_LINE (checking)).
One tests the values given by EPSI_ELGA; EPSI_ELNO, SIEF_ELGA, SIEF_ELNO, SIEF_ELGA, SIGM_ELNO in various points in the principal directions of the three-dimensions functions of reinforcement, obtained respectively with commands MECA_STATIQUE and STAT_NON_LINE.

For **MECA_STATIQUE** :

Points of integration	EPSI_ELGA (in the principal direction of the three-dimensions function of reinforcement)			SIEF_ELGA (in the principal direction of the three-dimensions function of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 – Point 1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA2 – Point 1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA3 – Point 1 (three-dimensions function GEOY)	1.8264	1.8263	0	3.6527E11	3.6527E11	0
MA4 – Point 1 (three-dimensions function GEOY)	1.8264	1.8263	0	3.6527E11	3.6527E11	0
MA5 – Point 1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0
MA6 – Point 1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0

Node	EPSI_ELNO (in the principal direction of the three-dimensions function of reinforcement)			SIGM_ELNO (in the principal direction of the three-dimensions function of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation

NO1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
NO1 (three-dimensions function GEOY)	1.8264	1.8263	0	3.6527E11	3.6527E11	0
NO1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0

For **STAT_NON_LINE**

Points of integration	EPSI_ELGA (in the principal direction of the three-dimensions function of reinforcement)			SIEF_ELGA (in the principal direction of the three-dimensions function of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 – Point 1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA2 – Point 1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA3 – Point 1 (three-dimensions function GEOY)	1.8264	1.8263	0.003	3.6527E11	3.6527E11	0
MA4 – Point 1 (three-dimensions function GEOY)	1.8264	1.8263	0.003	3.6527E11	3.6527E11	0
MA5 – Point 1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0
MA6 – Point 1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0

Node	EPSI_ELNO (in the principal direction of the three-dimensions function of reinforcement)			SIEF_ELNO (in the principal direction of the three-dimensions function of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 - NO1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA2 - NO1 (three-dimensions function GEOY)	1.8264	1.8263	0.003	3.6527E11	3.6527E11	0
MA3 - NO1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	the 0

value of total potential energy is also tested starting from STAT_NON_LINE computation . The analytical solution is calculated from all the uniaxial strains in the grids, from the equation:

$$E_{pot}=\frac{1}{2}\int_{element}\varepsilon.A.\varepsilon dv$$

where A the elasticity tensor indicates.

The got results are the following:

	Analytical	Code_Aster Reference	Variation
total Potential energy (J)	2.8173E10	2.8173E10	0.3.2

Quantities tested and results of the modelization B

The modelization B consists of six elements TETRA4 for the cube and of two elements TRIA3 for each face.
The behavior is elastic by means of command MECA_STATIQUE then command STAT_NON_LINE (checking).
One tests the values given by EPSI_ELGA; EPSI_ELNO, SIEF_ELGA, SIEF_ELNO ; SIEF_ELGA; SIGM_ELNO in various points in the principal directions of the three-dimensions functions of reinforcement, obtained respectively with commands MECA_STATIQUE and STAT_NON_LINE.

For **MECA_STATIQUE** :

Points of integration	EPSI_ELGA (in the principal direction of the three-dimensions function of reinforcement)			SIEF_ELGA (in the principal direction of the three-dimensions function of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 – Point 1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA2 – Point 1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA3 – Point 1 (three-dimensions function GEOZ)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA4 – Point 1 (three-dimensions function GEOZ)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA5 – Point 1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0

MA6 – Point 1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0
MA11 – Not 1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA21 – Point 1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA31 – Point 1 (three-dimensions function GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA41 – Point 1 (three-dimensions function GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA51 – Point 1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0
MA61 – Point 1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0

Node	EPSI_ELNO (in the principal direction of the three-dimensions function of reinforcement)			SIGM_ELNO (in the principal direction of the three-dimensions function of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 - NO1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA2 - NO1 (three-dimensions function GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA3 - NO1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0
MA11 - NO1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA21 - NO1 (three-dimensions function GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA31 - NO1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0

For **STAT_NON_LINE**

Points of integration	EPSI_ELGA (in the principal direction of the three-dimensions function of reinforcement)			SIEF_ELGA (in the principal direction of the three-dimensions function of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 – Point 1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0

MA2 – Point 1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA3 – Point 1 (three-dimensions function GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA4 – Point 1 (three-dimensions function GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA5 – Point 1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0
MA6 – Point 1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0
MA11 – Point 1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA21 – Point 1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA31 – Point 1 (three-dimensions function GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA41 – Point 1 (three-dimensions function GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA51 – Point 1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0
MA61 – Point 1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0

Node	EPSI_ELNO (in the principal direction of the three-dimensions function of reinforcement)			SIEF_ELNO (in the principal direction of the three-dimensions function of reinforcement)		
	Code_Aster	Reference	Variation (%)	*Code_Aster*	Reference	Variation
MA1 - NO1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA2 - NO1 (three-dimensions function GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA3 - NO1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0
MA11 - NO1 (three-dimensions function GEOZ)	1.067	1.067	0.001	2.13397E11	2.13397E11	0
MA21 - NO1 (three-dimensions function GEOY)	1.8263	1.8263	0	3.6527E11	3.6527E11	0
MA31 - NO1 (three-dimensions function GEOX)	2	2	0	4E11	4E11	0

4 Results of the modelization C

The modelization C is identical to the modelization A by means of quadratic elements (command CREA_MAILLAGE, option LINE_QUAD).

One finds the same results as for the modelization A (error compared to the reference solution lower than $0.002\ \%$).

5 Results of the modelization D

The modelization D is identical to the modelization B by means of quadratic elements.

One finds the same results as for the modelization D (error compared to the reference solution lower than $0.002\ \%$).

6 Summary of the results and general remarks

the results got for these modelizations are identical to the reference solutions. They validate modelization GRILLE_MEMBRANE for four element types different in the case from a linear mechanical computation (MECA_STATIQUE and STAT_NON_LINE).