

## RCCM09 - Operator POST\_RCCM : computation of the factor of Summarized

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### starting:

This test of thermomechanical linear static makes it possible to validate the computation of the factor of starting with command `POST_RCCM` (option `AMORCAGE`, results of the type `EVOLUTION`).

The analytical solution is simple, and makes it possible to test postprocessing within the meaning of the `RCC_M`.

More precisely, the modelization A tests only the routines associated with option `PM_PB`, `SN` and `FATIGUE_ZH210` for results of the type `EVOLUTION`.

The modelization B allows to test the group of computation on a fissured plane structure: computation of the stresses in structure, extraction of these stresses on cut lines circular and computation of the factor of starting.

## 1 Problem of reference

### 1.1 Geometry

the problem of reference is plane. One considers a rectangular plate, of width  $W$ , comprising an emerging crack length  $a$ .

One is interested in the factor of starting related to the geometrical singularity in  $P$ , the structure being subjected to several transitory mechanical loadings.

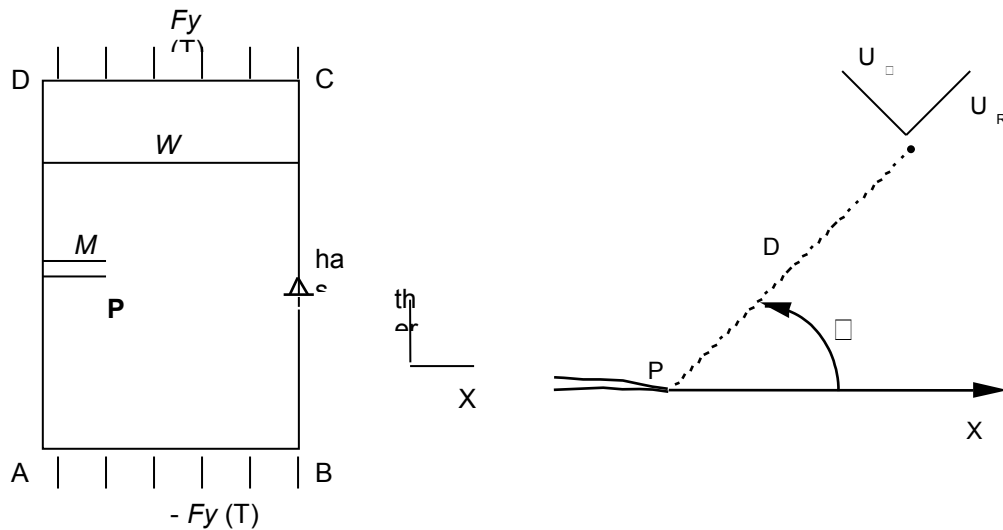


Figure 1.1-a : Definition of the problem of reference and the local coordinate system

### 1.2 Materials properties

the properties material selected are the following ones:

- 1) Young modulus:  $E = 2. E + 05 MPa$  ;
- 2) Poisson's ratio:  $\nu = 0,3$  .

The characteristics suitable for the computation of the factor of starting to meaning RCC-M correspond to the case of an alloy Ni-Cr-Fe (Inconel):

- 1) parameters of the model of starting:  $A\_AMORC = 7091$  and  $B\_AMORC = -0,1823$ ;
- 2) parameter of the effective stress:  $R\_AMORC = 1,63$ ;
- 3) outdistance D with the singularity:  $D\_AMORC = 0,046 mm$  .

The model of starting is expressed in the following way:

$$\Delta\sigma_{eff}(d) = A\_AMORC \cdot (N_a)^{B\_AMORC}$$

with  $N_a$  the number of acceptable cycles and by taking the following relation between amplitude of variation of the stresses real  $\Delta\sigma_{\theta\theta}$  and effective  $\Delta\sigma_{eff}$  :

$$\Delta\sigma_{eff} = \frac{\Delta\sigma_{\theta\theta}}{1 - R/R\_AMORC}, \text{ with } R \text{ ratio of load.}$$

## 1.3 Boundary conditions and loadings

the structure is subjected to two transitory mechanical loadings; all the loadings consist of a linear force  $F_y$  imposed on the segments  $AB$  and  $CD$ .

Motions of rigid bodies are blocked at the point Mr.

Transitoire 1:

$$t=0s \quad |F_y|=0 \quad ; \quad t=1s \quad |F_y|=100 f^0$$

Transient 2:

$$t=0s \quad |F_y|=10 f^0 \quad ; \quad t=1s \quad |F_y|=110 f^0 \quad ; \quad t=2s \quad |F_y|=50 f^0$$

where  $f^0 = 3 N.mm^{-1}$ .

## 2 Reference solution

### 2.1 Results of reference

#### 2.1.1 Computation of the factors of intensity of the stresses

the reference solution for an emerging crack of depth  $a$  in a plate of thickness  $W$  requested in pure  $I$  mode (force  $\sigma$  along the axis  $y$ ) is the following one:

$$K_I = Y \sigma \sqrt{a}, \text{ with } Y = 1.99 - 0.41 \frac{a}{W} + 18.7 \left( \frac{a}{W} \right)^2 - 38.48 \left( \frac{a}{W} \right)^3 + 53.85 \left( \frac{a}{W} \right)^4$$

numerical Application: for a crack 15 mm depth ( $W = 72$  mm) and the loading  $f^0$ , one obtains  $K_I = 2,874$  MPa.  $\sqrt{\text{mm}}$ .

#### 2.1.2 Computation of the stresses around the crack tip

the analytical solution for the stress on a circle of radius  $r$  around the crack tip, obtained for a crack in infinite medium, is the following one:

$$\sigma_{\theta\theta} = \frac{K_I}{4\sqrt{2\pi r}} \left( 3 \cos \frac{\theta}{2} + \cos \frac{3\theta}{2} \right)$$

Numerical application: for the loading  $f^0$  and a radius  $r = R_{AMORC} = 0,046$  mm, one obtains:

$$\theta = 0 : \sigma_{\theta\theta} = 53.5 \text{ MPa} \quad ; \quad \theta = \pi/2 : \sigma_{\theta\theta} = 18.9 \text{ MPa}$$

#### 2.1.3 Computation of the factor of starting

the factor of starting associated with the geometrical singularity  $P$  is calculated by it analytically while following the procedure prescribed in the RCC-M [3]. All provided times are regarded as local extrema and are thus appointed in computation.

One notes  $s^0$  the value of  $\sigma_{\theta\theta}$  raised with a radius  $r$  of the crack tip, with an angle  $\theta$  given for the loading  $f^0$ . One can then build the table of the amplitudes of variation of  $\sigma_{\theta\theta}$  for all the possible combinations:

			Transient 1		Transient 2		
			$Nb_{occur} = i$		$Nb_{occur} = j$		
			$t=0$	$t=1$	$t=0$	$t=1$	$t=2$
Transient 1	$Nb_{occur} = i$	$t=0$		100 s°	10 s°	110 s°	50 s°
		$t=1$			90 s°	10 s°	50 s°
Transient 2	$Nb_{occur} = j$	$t=0$				100 s°	40 s°
		$t=1$					60 s°
		$t=2$					

the taking into account of the ratio of load  $R$  of each combination makes it possible to calculate the amplitude of variation of the effective stresses  $\Delta \sigma_{eff}$  :

			Transient 1		Transient 2		
			$Nb_{occur} = i$		$Nb_{occur} = j$		
			$t=0$	$t=1$	$t=0$	$t=1$	$t=2$
Transient 1	$Nb_{occur} = i$	$t=0$		100 s°	10 s°	110 s°	50 s°
		$t=1$			95,6 s°	22,6 s°	72,1 s°
Transient 2	$Nb_{occur} = j$	$t=0$				105,9 s°	45,6 s°
		$t=1$					83,6 s°
		$t=2$					

The computation of the factor of starting  $FA$  is carried out according to an iterative process:

- 1) identification of the transient of non-zero number of occurrences leading to the maximum of  $\Delta\sigma_{eff}$  ;
- 2) computation of the elementary factor of associated starting, using the fatigue model;
- 3) reactualization amongst occurrences of the treated combination.

Two cases are successively studied with the numbers of occurrences of the two following transients:  $i=j=1$  and  $i=1 ; j=2$ . One notes  $N_{kl-mn}$  the number of acceptable cycles for the combination of  $l$ -ième time step of  $k$ - the ième transient and of  $n$ -ième time step of  $m$ - the ième transient (calculated with part of  $\Delta\sigma_{eff}$  this combination and fatigue model).

$$\mathbf{1st\ case:} \quad FA_1 = \frac{1}{N_{11-22}} + \frac{1}{N_{12-21}} + \frac{1}{N_{22-23}}$$

$$\mathbf{2nd\ case:} \quad FA_2 = \frac{1}{N_{11-22}} + \frac{1}{N_{11-12}} + \frac{1}{N_{12-21}} + \frac{1}{N_{22-23}}$$

*Numerical application:* for the modelization A of this case test, one supposes that  $s^\circ = 1$ . One thus has:

$$FA_1 = 2,005 \cdot 10^{-10} ; \quad F A_2 = 2,710 \cdot 10^{-10}$$

For the modelization B, the value of  $s^\circ$  depends on the angle  $\theta$ , and can be calculated using the formula indicated in paragraph 2.1.2.

$$\theta = 0 : \quad \sigma^\circ = 53,5 \text{MPa} \quad FA_1 = 0,607 \quad ; \quad FA_2 = 0,819$$

$$\theta = 90^\circ : \quad \sigma^\circ = 18,9 \text{MPa} \quad FA_1 = 2,01 \cdot 10^{-3} ; \quad FA_2 = 2,72 \cdot 10^{-3}$$

## 2.2 Bibliographical references

- 1) W.F. BROWN, J.E. STRAWLEY: "Planes Strain Ace Toughness Testing of High Strength Material", American Society of Testing and Materials, STP 410
- 2) J.B. LEBLOND: "Brittle and ductile Fracture mechanics", Lavoisier, Paris, 2003.
- 3) RCC-M: "Brittle and ductile Fracture mechanics", Lavoisier, Paris, 2003.

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## 3 Modelization A

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### 3.1 Characteristic of the modelization

The modelization A only aims at validating the functionalities of computation of the factor of starting in operator `POST_RCCM` from tables of evolution of the stresses.

There is thus in this modelization neither mesh, nor mechanical computation.

### 3.2 Values tested and results

the values of the factor of starting calculated are exactly equal to the analytical values of reference.

## 4 Modelization B

### 4.1 Characteristic of the modelization

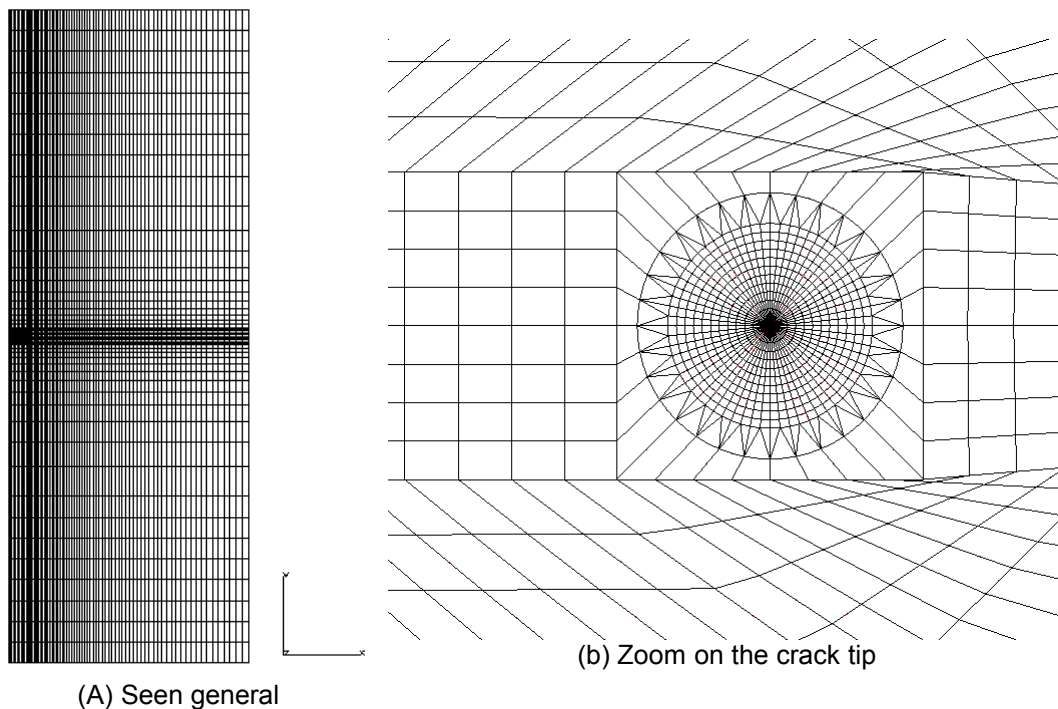
the purpose of the modelization B is to validate the group of the process of the computation of the factor of starting, of the resolution of the mechanical problem to postprocessing with `POST_RCCM`.

The computation is carried out on a modelization 2D plane strains, with a material with the linear elastic behavior.

### 4.2 Characteristics of the mesh

Many nodes: 30000

Number of meshes: 10000 meshes quadratic (SEG3, TRIA6 and QUAD8)



4.2-a 4.2-a : Mesh – cases test rccm09b

### 4.3 Values tested and results

the various values tested are gathered in the table below.

The good accuracy on the factor of intensity of the stresses shows that the mesh used is sufficiently fine to collect the singularity of displacements in crack tip.

The comparison of the stresses  $\sigma_{\theta\theta}$  according to the angle  $\theta$  is also satisfactory: the variation on the maximum value of  $\sigma_{\theta\theta}$ , obtained for  $\theta=0$ , is of 2%. The difference between analytical solution and numerical solution can be explained by the effects of edge of structure at the stress field: one indeed compares an analytical solution obtained in an infinite medium with a computation on a finished structure (crack 15 mm depth, plates of 72 mm width).



The maximum factor of starting, calculated for  $\theta=0$ , is close to the analytical solution for the two studied cases. For  $\theta=90^\circ$ , the variation is important, like consequence of the variation already noted on the stresses.

Value tested	Reference	Aster	Variation
$K_I$ ( MPa. $\sqrt{mm}$ ) (method 3)	2,874	2,855	-0,6%
$\sigma_{\theta\theta}$ ( $\theta=0^\circ$ ) ( MPa )	53,5	54,5	2%
$\sigma_{\theta\theta}$ ( $\theta=90^\circ$ ) ( MPa )	18,9	17,5	-7%
$FA_1$ ( $\theta=0^\circ$ )	0,607	0,673	11%
$FA_1$ ( $\theta=90^\circ$ )	2,01.10 <sup>-3</sup>	1,32.10 <sup>-3</sup>	-34%
$FA_2$ ( $\theta=0^\circ$ )	0,819	0,909	11%
$FA_2$ ( $\theta=90^\circ$ )	2,72.10 <sup>-3</sup>	1,786.10 <sup>-3</sup>	-34%

**Note::** the stress  $\sigma_{\theta\theta}$  corresponds, in the table produced by MACR\_LIGN\_COUPE, with the component *SIZZ*.

## 5 Summary of the results

This document presents the two cases test of validation of the development of the computation of the factor of starting in operator POST\_RCCM for results of the type EVOLUTION.

The modelization A validates the developments carried out in operator POST\_RCCM.

The modelization B allows to test the group of computation on a fissured plane structure: computation of the stresses in structure, extraction of these stresses on cut lines circular with MACR\_LIGN\_COUPE and computation of the factor of starting with POST\_RCCM. The results are satisfactory, but underline the strong influence of the value of the stresses (expressed in the local coordinate system) on the factor of calculated use.

One thus points out the need for using sufficiently fine meshes (preferably radiant) in crack tip to collect correctly the singularity of stresses related to the presence of the geometrical singularity.