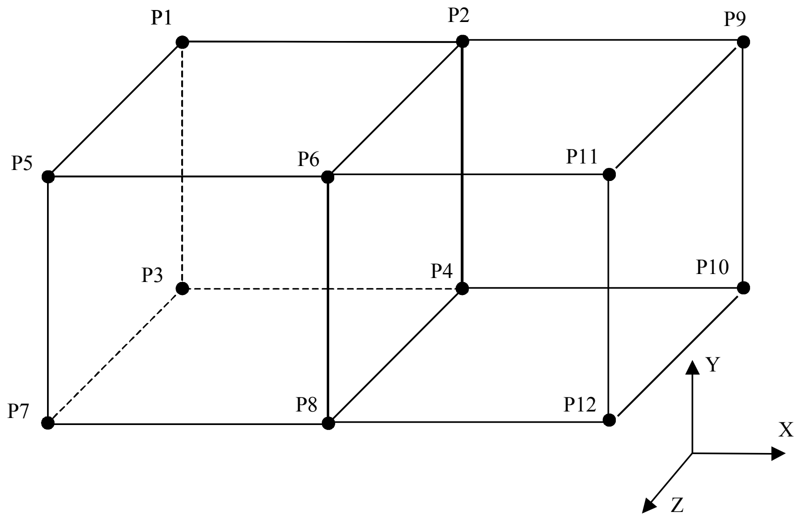
SSLV140 - Computation of effective moduli by a Python method

Summarized:

One presents a test here having an analytical reference. The treated geometry is a set of two cubes having different elastic properties. The goal is to find the Young modulus of the mixture made up of these two cubes along two directions.

1 Problem of reference

1.1 Geometry



One defines following surfaces:

- Face $YZ1$: containing the nodes $P1$, $P3$, $P5$ and $P7$.
- Face $YZ2$: containing the nodes $P9$, $P10$, $P11$ and $P12$.
- Face $XY1$: containing the nodes $P1$, $P2$, $P9$, $P3$, $P4$ and $P10$.
- Face $XY2$: containing the nodes $P5$, $P6$, $P11$, $P7$, $P8$ and $P12$.
- Face $XZ1$: containing the nodes $P3$, $P4$, $P10$, $P7$, $P8$ and $P12$.
- Face $XZ2$: containing the nodes $P1$, $P2$, $P9$, $P5$, $P6$ and $P11$.

and the following elements:

- Element $M1$: containing the nodes $P1$, $P2$, $P3$, $P4$, $P5$, $P6$, $P7$ and $P8$.
- Element $M2$: containing the nodes $P2$, $P9$, $P4$, $P10$, $P6$, $P11$, $P8$ and $P12$.

1.2 Material properties

Two materials are used:

- Material $MAT1$ allotted to the element $M1$:

  Young modulus: $E1=200000\,MPa$

  Poisson's ratio: $\nu_1=0.3$

- Material $MAT2$ allotted to the element $M2$:

  Young modulus: $E2=100000\,MPa$

  Poisson's ratio: $\nu_2=0.3$

1.3 Boundary conditions and loadings

the First computation:
It is a simple computation of tension according to the direction X :

- •One imposes a linear elastic strain $\varepsilon_{xx}=1$ on surface $YZ2$.
- •Surface $YZ1$ does not move according to the direction X .

The second computation:
It is a simple computation of tension according to the direction Y :

- •One imposes a linear elastic strain $\varepsilon_{yy}=1$ on surface $XZ2$.
- •Surface $XZ1$ does not move according to the direction Y .

2 Reference solution

2.1 Method of calculating

According to the general theory of the homogenization of the composites [bib1], the moduli Young effective E_{xx}^{eff} and E_{yy}^{eff} according to the directions X and Y a mixture having the form given above, are given by the following formulas:

$$\frac{1}{E_{xx}^{eff}}=\frac{f_1}{E_1}+\frac{f_2}{E_2}$$
$$E_{yy}^{eff}=f_1 E_1+f_2 E_2$$

f_1 and f_2 are the voluminal fractions of each material, in our case:

$$f_1=f_2=0.5$$

2.2 Bibliographical references

1) Mr. BORNET, T. BRETHEAU and P. GILORMINI: Homogenization in mechanics of materials (T1). Hermes Science Publications - 2001.

3 Modelization A

3.1 Characteristic of the mesh

Many nodes: 12.
Modelization 3D : 2 quadratic volume elements: HEXA8.

3.2 Functionalities tested

Of the commands Python are inserted directly in the file of Aster commands. These commands are used of results to write functions of postprocessing on the fields, like the averages, the trace of a strain tensor or stresses,… etc the fields of results are recovered by the command EXTR_COMP.

3.3 Values tested

the First computation:
The Young modulus following the direction X in this case is the average of the stresses σ_{xx} :

$$E_{xx}^{eff}=\langle\sigma_{xx}\rangle$$

The second computation:
The Young modulus following the direction Y in this case is the average of the stresses σ_{yy} :

$$E_{yy}^{eff}=\langle\sigma_{yy}\rangle$$

Identification	Reference	*Aster*	$\%$ difference
$\langle\sigma_{xx}\rangle$	133333	134134	1.00
$\langle\sigma_{yy}\rangle$	150000	150000	0.00

4 Summary of the results

the got results are in perfect agreement with the reference solution.