SSLV134 - circular Crack in infinite medium

Summarized

This test allows, after obtaining the field of displacement by MECA_STATIQUE, the computation of the rate of refund of energy room for a circular crack plunged in a presumedly infinite medium.

For the first modelization, only a half space defined by the plane of the crack is represented. The crack tip is then a closed curve (a circle) and is defined as such in DEFI_FOND_FISS. The rate of refund local and total is compared to the analytical solution of reference.

The seven following modelizations make it possible to calculate the stress intensity factors $K1$ and $K3$, in $3\mathrm{D}$ and axisymmetric, calculated by POST_K1_K2_K3 and/or CALC_G.

- The modelization A formula G a mesh tests $3\mathrm{D}$ with closed crack,
- The modelization B tests $K1$ for a mesh $3\mathrm{D}$,
- The modelization C tests $K1$ for an axisymmetric mesh,
- The modelization D tests the combination of $K1$ and $K3$ for a mesh $3\mathrm{D}$.
- The modelization G tests $K1$ for a mesh $3\mathrm{D}$ with a crack tip defined by two coïncidents nodes groups.
- The modelization H tests $K1$ for a crack nonwith a grid (method $\mathrm{X\text{-}FEM}$)
- The modelization I tests $K1$ for an axisymmetric crack nonwith a grid (method $\mathrm{X\text{-}FEM}$)
- The modelization J tests G for a mesh $3\mathrm{D}$ with crack closed for the incompressible elements,
- The modelization K tests $K1$ for an axisymmetric mesh for the incompressible elements,

the modelizations E and F allow to validate the computation of the bilinear form of G on the same problem. Lastly, the modelization F makes it possible to validate options G_MAX and CALC_K_MAX.

1 Problem of reference

1.1 Geometry

the crack is circular (penny shaped ace) of radius a , in the plane Oxy . So that the medium is regarded as infinite, the quantities characteristic of the solid mass are about 5 times higher than the radius a .

1.2 Material properties

Modulus Young: $E=2.10^5 MPa$

Poisson's ratio: $\nu=0.3$

Density: $\rho=7850 kg/m^3$

1.3 Boundary conditions and loadings

lower Face	: uniform stress of tension $\sigma_z=1.MPa$
Upper face	: uniform stress of tension $\sigma_z=1.MPa$

According to the modelization, one also has boundary conditions of symmetry and blocking of rigid body motions.

In the modelization D where only the quarter of the parallelepiped is represented, one uses boundary conditions of antisymetry for the loading of torsion: they amount imposing null tangential displacements on a face. The loading of torsion is introduced in the form of a tangential surface force (shears distributed) applied to the lips of crack.

- Upper lip: $F_X=-\tau\frac{Y}{a}$ and $F_Y=+\tau\frac{X}{a}$
- formulates: $F_X=+\tau\frac{Y}{a}$ and $F_Y=-\tau\frac{X}{a}$

2 Reference solution

2.1 Method of calculating used for the reference solution

For a circular crack of radius a in an infinite medium, subjected to a uniform tension σ according to the norm with the plane of the lips, local rate of energy restitution $G(s)$ is independent of the curvilinear abscisse s and is worth [bib1]:

$$G(s)=\frac{(1-\nu^2)}{\pi E}4\sigma^2 a$$

then the stress intensity coefficient $K1$ is given by the formula of Irwin:

$$G(s)=\frac{(1-\nu^2)}{E}K_1^{\ 2} \text{ maybe } K_I=\frac{2\sigma\sqrt{a}}{\sqrt{\pi}}$$

If this crack is subjected to shears distributed on the lips: $\sigma_{\theta z}=\tau\frac{r}{a}$

(what is equivalent to a torsion ad infinitum), then one is in pure mode 3 and the corresponding stress intensity factor is worth:

$K_3=\frac{4\tau\sqrt{a}}{3\sqrt{\pi}}$ thus by the formula of Irwin $G(s)=\frac{(1+\nu)}{E}K_3^2$

In the presence of the two combined modes, one will have:

$$G(s)=\frac{(1-\nu^2)}{E}K_1^2+\frac{(1+\nu)}{E}K_3^2$$

The theta-method connects total and local rates of energy restitution by the following variational equation:

$$G_{réf}(\boldsymbol{\theta})=\int_{\Gamma}G(s)\boldsymbol{\theta}\cdot\boldsymbol{m}(s)ds$$

where $\boldsymbol{m}(s)$ is the norm with the crack tip Γ and θ is the velocity field of a virtual propagation of crack.

If one chooses for the field θ unit normal with the crack tip, one obtains, since $G(s)$ is constant on all the crack tip:

$$G_{réf}(\boldsymbol{\theta})=G(s)\cdot 2\Pi a$$

2.2 Results of Numerical

reference Application (case with loading of tension only):

For the loading considered and $a=2m$, one obtains then:

$$G(s)=11.586\,J/m^2$$
$$G_{réf}=145.60\,J/m$$
$$K1=1.5958\text{E}6\,J/m^2$$

For the modelization G (3 different crack tips) with the same loading, one obtains:

for $a=2m$

$$G(s)=10.586\,J/m^2$$
$$K1=1.5958\mathrm{E}6\,J/m^2$$

$a=1.88m$

$$G(s)=10.891\,J/m^2$$
$$K1=1.5472\mathrm{E}6\,J/m^2$$

for $a=1.76\,m$

$$G(s)=10.196\,J/m^2$$
$$K1=1.4969\mathrm{E}6\,J/m^2$$

Numerical Application (case with loading of torsion only):

$$G(s)=7.3565\,J/m^2$$
$$G_{réf}=92.44\,J/m$$
$$K1=1.0638\mathrm{E}6\,J/m^2$$

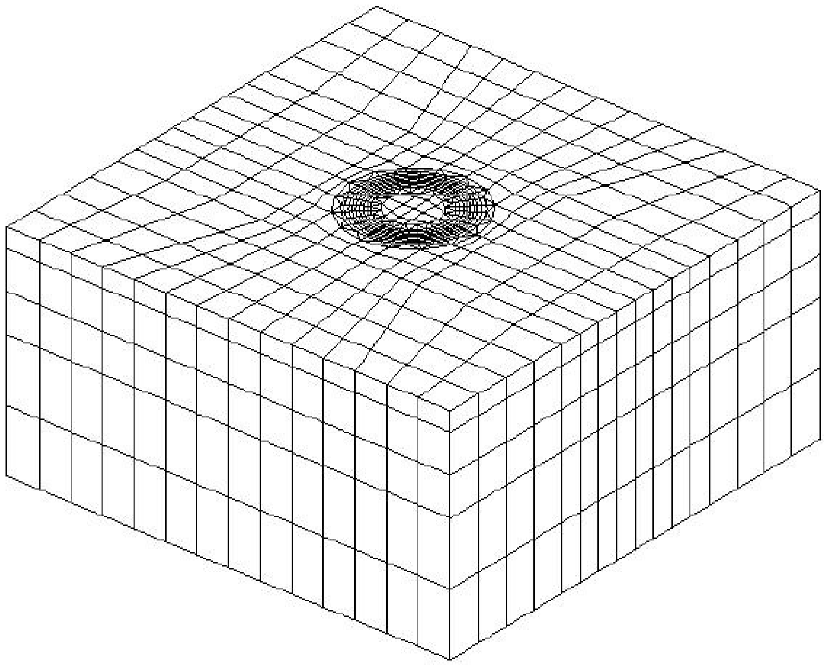
2.3 Bibliographical references

1) Solution of Sneddon (1946) in G.C. SIH: Handbook of stress-intensity factors Institute of Fracture and Solid Mechanics - Lehigh University Bethlehem, Pennsylvania

3 Modelization A

the crack tip is closed. One calculates G .

3.1 Characteristics of the modelization



the interest of this modelization is to represent the entirety of the crack tip which is a closed curve, without benefitting from symmetries of the problem.

Only the loading of tension is taken into account.

3.2 Characteristics of the mesh

Many nodes: 11114
Number of meshes and type: 2432 PENTA15

the nodes mediums of the edges of the elements touching the crack tip are moved with the quarter of these edges.

3.3 Notice

One uses key word SYME in operator CALC_G to multiply automatically by two the rate of energy restitution calculated on only one lip of crack, when operand FOND_FISS is absent. When FOND_FISS is present, information on symmetry is recovered directly in the concept fond_fiss created via DEFI_FOND_FISS.

The principle is the same one for POST_K1_K2_K3. The indication of symmetry induces the computation of the factors of intensity of the stresses and the rate of refund of energy G_IRWIN by interpolation of displacements of a single lip of crack. The displacement of the nodes mediums of the edges of the elements touching the crack tip with the quarter of these edges would make it possible to improve the accuracy of computation.

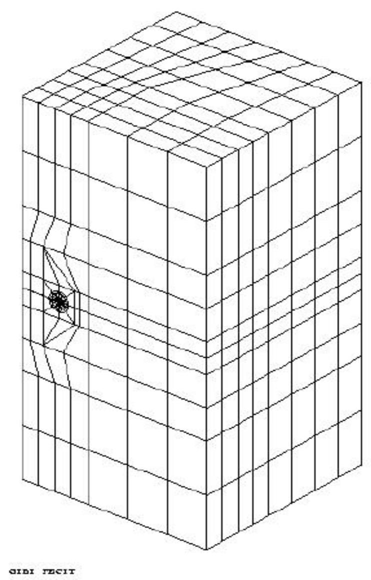
3.4 Quantities tested and results

Identification	Standard	*Reference of reference*	% tolerance
G total	145.6	ANALYTIQUE	1,2
G local Node $N403$ - G Lagrange	11.586	ANALYTIQUE	3.0
G local Node $N2862$ - G Lagrange	11.586	ANALYTIQUE	2.0
G local Node $N375$ - G Lagrange	11.586	ANALYTIQUE	3.0
G local Node $N292$ - G Lagrange	11.586	ANALYTIQUE	2,2
$max(G\,local)$ - G Lagrange	11.59	ANALYTIQUE	2,5
$min(G\,local)$ - G Lagrange	11.59	ANALYTIQUE	2.0
G local Node $N403$ - G Lagrange_no_no	11.586	ANALYTIQUE	2.0
G local Node $N2862$ - G Lagrange_no_no	11.586	ANALYTIQUE	2.0
G local Node $N375$ - G Lagrange_no_no	11.586	ANALYTIQUE	2.0
G local Node $N292$ - G Lagrange_no_no	11.586	ANALYTIQUE	2.0
G local Node $N403$ - G Lagrange_regu	11.586	ANALYTIQUE	1,3
G local Node $N2862$ - G Lagrange_regu	11,726	NON_REGRESSION	0.1
G local Node $N375$ - G Lagrange_regu	11,738	NON_REGRESSION	0.1
G local Node $N292$ - G Lagrange_regu	11,702	NON_REGRESSION	0.1
G (POST_K1_K2_K3) - Node $N403$	11.586	ANALYTIQUE	5.0
G (POST_K1_K2_K3) - Node $N2862$	11.586	ANALYTIQUE	10.0
G (POST_K1_K2_K3) - Node $N375$	11.586	ANALYTIQUE	5.0
G (POST_K1_K2_K3) - Node $N292$	11.586	ANALYTIQUE	5.0

4 Modelization B

Computation with POST_K1_K2_K3 in 3D

4.1 Characteristic of the modelization



This modelization makes it possible to test the computation of $K1$ using POST_K1_K2_K3 (method of extrapolation of displacements on the lips of crack). Parameter ABSC_CURV_MAXI of the operator is calculated in POST_K1_K2_K3 so as to retain 5 nodes on the segment of extrapolation ($dmax=0,35$).

Only the loading of tension is taken into account.

4.2 Characteristics of the mesh

Many nodes: 6536
Number of meshes and type: 432 PENTA15 and 987 HEXA20

the nodes mediums of the edges of the elements touching the crack tip are moved with the quarter of these edges, to obtain a better accuracy.

4.3 Notice

One represents only the quarter of the complete three-dimensional block and thus the quarter of crack. Thus, it is necessary to divide the theoretical value of reference of the total rate of refund by 4: $G_{glob}=145.60/4=36.40\ J/m$

4.4 Quantities tested and Results

4.4.1 results of CALC_G

Identification	Reference (analytical)	% tolerance
G room Node 49	11.59	3,0
G room Node 1710	11.59	2,0
G room total Node	11.59	11,59
G 3,0	36.4	1,2

4.4.2 Results of POST_K1_K2_K3

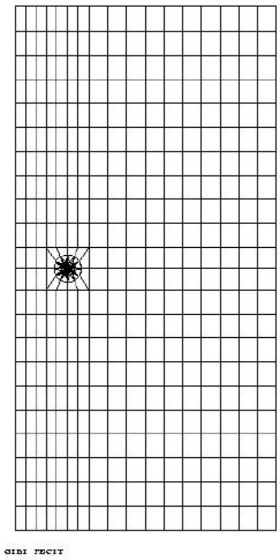
Identification	Reference (analytical)	% tolerance
G Node 49	11.586	2
G Node 77	11.586	2
G Node 1710	11.586	2
$K1$ Node 49	1.60E+006	1
$K1$ Node 77	1.60E+006	1
$K1$ Node 1710	1.60E+006	1

Of other purely data-processing tests of the command POST_K1_K2_K3 are also carried out.

5 Modelization C

Computation with POST_K1_K2_K3 into axisymmetric.

5.1 Characteristics of the modelization



This modelization makes it possible to test the computation of $K1$ using POST_K1_K2_K3 (method of extrapolation of displacements on the lips of crack) into axisymmetric.

Only the loading of tension is retained in this modelization.

Since one is in axisymmetric modelization, the relation between total and local rates of energy restitution is [R7.02.01]:

$G_{réf}(\boldsymbol{\theta})=G(s)\cdot a$ that is to say here $G_{réf}=23.17\,J/m$

5.2 Characteristic of the mesh

Many nodes: 1477
Number of meshes and type: 402 QUAD8 and 60 TRIA6

the nodes mediums of the edges of the elements touching the crack tip are moved with the quarter of these edges, to obtain a better accuracy.

5.3 Quantities tested and results

Identification	Standard	Method	Reference of reference	% tolerance
G	CALC_G	23.2	ANALYTIQUE	1.8
$K1$	POST_K1_K2_K3	1.60E+006	ANALYTIQUE	3
G	POST_K1_K2_K3	11.6	ANALYTIQUE	1.8

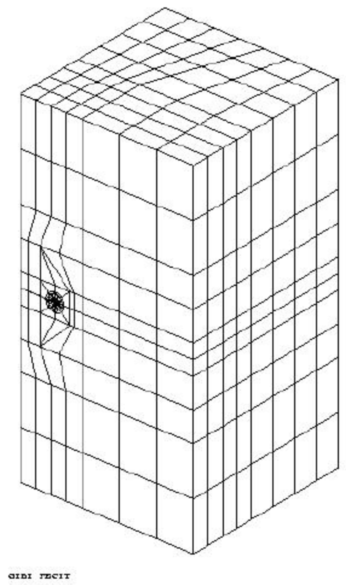
5.4 Notice

an ordering of STAT_NON_LINE is added to validate the modifications brought on xpesro.f. The got results this case test will be compared with those obtained from case test sslv134i for into X-FEM axisymmetric. In addition of the simple tension applied to edges high and low of the plate, a rotation of around $150 trs/min$ the symmetric axis is applied. With
option CALC_G , and $G=21,4$. $K_1=219E+05$ Modelization

6 D Computation

with POST _K1_K2_K3 in 3D for modes 1 and 3. Characteristics

6.1 of the modelization



the following boundary conditions are successively applied: tension

- : as for the modelization B; torsion
- . This

modelization makes it possible to test the computation of and $K1$ combined $K3$ using POST _K1_K2_K3 (method of extrapolation of displacements on the lips of crack).

The nodes mediums of the edges of the elements touching the crack tip are moved with the quarter of these edges, to obtain a better accuracy. Characteristics

6.2 of the mesh Many

nodes: 6536 Number of meshes
and type: 432 PENTA 15 and 987 HEXA20

the nodes mediums of the edges of the elements touching the crack tip are moved with the quarter of these edges, to obtain a better accuracy. Notice

6.3

the two loading cases (tension and torsion) are taken into account. It is thus necessary to cumulate the values of for G the two loadings. Moreover, one represents only the quarter of the complete three-dimensional block and thus the quarter of crack, it is thus necessary to divide the theoretical value of reference of the total rate of refund by 4. Thus

Only

$$G(s)=(11.586+7.356)=18.943\,J/m^2$$

$$G=(145.60+92.44)/4=59{,}511\ J/m$$

the tension contributes to, $K1$ only torsion contributes to. $K3$ Quantities

6.4 tested and results Identification

Method	Standard	Localization	Reference	of reference tolerance	% G
CALC_G	Legendre Node	49 18,94	18.94	3,0	3.0
CALC_G	Legendre Node	1710 18,94	18.94	2,0	2.0
CALC_G	Legendre Node	77 18,94	18.94	3,0	3.0
CALC_G	option CALC_G_GLOB –	59,51	59.51	1,2	1.2
POST	_K1_K1_K3 Node	49 1,596	106 ANALYTIQUE	1,0	1.0
POST	_K1_K1_K3 Node	1710 1,596	106 ANALYTIQUE	1,0	1.0
POST	_K1_K1_K3 Node	77 1,596	106 ANALYTIQUE	2,0	2.0
POST	_K1_K1_K3 Node	49 1,064	106 ANALYTIQUE	2,0	2.0
POST	_K1_K1_K3 Node	1710 1,064	106 ANALYTIQUE	2,0	2.0
POST	_K1_K1_K3 Node	77 1,064	106 ANALYTIQUE	1,0	1.0
POST	_K1_K1_K3 Node	49 18,94	18.94	2,0	2.0
POST	_K1_K1_K3 Node	1710 18,94	18.94	2,0	2.0
POST	_K1_K1_K3 Node	77 18,94	18.94	2,0	2.0

7 E Computation

of the bilinear form of. G Characteristics

7.1 of the modelization The mesh

is identical to that of preceding computations, but only the eighth of the block is retained (quadrant) $Oxyz$ Loading

1: Idem	modelization B. Loading
2: :	$Face\ x=10.$ uniform stress of tension $\sigma_z=1$:
	$Face\ z=10.$ uniform stress of tension (shears $\sigma_x=1$). Loading
3: Loading	1 + Loading 2. Loading
4: Loading	2 – Loading 1. Four

computations are static are carried out producing displacements respectively, $u, v, u+v$ and. $v \quad u$ Characteristics

7.2 of the mesh Many

nodes: 2774	Number of meshes
and type: 392	HEXA20 `and` 216 PENTA `15 Notice`

7.3 The mesh

represents only the eighth of the complete three-dimensional block. On the other hand, one uses operand SYME `in` DEFI `_FOND_FISS, which` amounts representing a quarter of crack. Thus, it is necessary to divide the theoretical value of reference of the total rate of refund by 4.

The bilinear form checks $g(u, v)$ the following properties: formulate

$$g(u, u)=G(u)$$

$$g(u, v)=\frac{G(u+v)-G(u-v)}{4}$$

formula

$$g(u-v, u+v)=\frac{G(2\mathrm{u})-G(-2\mathrm{v})}{4}=G(u)-G(v)$$

7.4 tested and results Identification

Standard	Reference	*of reference difference*	% total
G : 36,4 $G(u)$	AUTRE_ASTER	`0,41%`	0.41%
G : 36,4 $g(u, u)$	AUTRE_ASTER	`0,41%`	0.41%
G : 13,72 $G(v)$	13.72	`0,10%`	0.10%
G : 13,72 $g(v, v)-form.1$	13.72	`0,10%`	0.10%
G : 95,04 $G(u+v)$	95.04	`0,10%`	0.10%

G : 95,04 $g(u+v, u+v)-form.1$	95.04	0,10%	0.10%
G : 5,48 $G(u-v)$	5.48	0,10%	0.10%
G : 5,48 $g(u-v, u-v)-form.1$	5.48	0,10%	0.10%
G : 22,4 $g(v, u)-form.2$	22.4	0,10%	0.10%
G : 22,82 $g(u-v, u+v)-form.3$	22.82	0,10%	0.10%

indicate by u displacement corresponding to loading 1, and v displacement corresponding to loading 2. Loadings 3 and 4 corresponding with displacements and $(u+v)$. $(v-u)$ Modelization

8 F Computation

of the bilinear shape of room G . Characteristics

8.1 of the modelization The mesh

is identical to that of the modelization E. Loading

1: Idem	modelization B. Loading
2: :	$Face\ x=10.$ uniform stress of tension $\sigma_z=1$:
	$Face\ z=10.$ uniform stress of tension (shears $\sigma_x=1$). Loading
3: Loading	1 + Loading 2. Loading
4: Loading	2 – Loading 1. Four

static computations are carried out producing displacements respectively, $u, v, u+v$ and. $v \quad u$ Characteristics

8.2 of the mesh Many

nodes: 2774	Number of meshes
and type: 392	HEXA20 and 216 PENTA the 15

nodes mediums of the edges of the elements touching the crack tip are moved with the quarter of these edges. Notice

8.3 The mesh

represents only the eighth of the complete three-dimensional block. On the other hand, one uses operand SYME in DEFI _FOND_FISS, which amounts representing a quarter of crack. Thus, it is necessary to divide the theoretical value of reference of the total rate of refund by 4.

The bilinear form checks $g(u,v)$ the following properties: formulate

$$g(u,u)=G(u)$$

$$g(u,v)=\frac{G(u+v)-G(u-v)}{4}$$

formula

$$g(u-v,u+v)=\frac{G(2\mathrm{u})-G(-2\mathrm{v})}{4}=G(u)-G(v)$$

with options “ G_MAX” and “ CALC_K_MAX” illustrates the method to maximize or G in the presence of $K1$ signed and not signed loads. Loadings 1 and 3 definite like are signed here, loadings 2 and 4 being not signed.

The two options are characterized by the taking into account (CALC_K_MAX) or not (G_MAX) from the possible interpenetration from the lips from crack; in these case test, the maximum approach obtained corresponds to the same combination of loads in both cases. Quantities

8.4 tested and results

the first two values tested are compared with the analytical reference solution. The other tests are of standard NON-regression. Node

Lissage	R_	inf Standard	R_ sup	Identification	of Reference Reference	*Tolerance*	() % N2667
Lag	- Lag 0.1.1.0		room	G : AUTRE_ASTER $G(u)$	11,58	11.58	0.80%
Lag	- Lag 0.1.1.0		bilinear	G : AUTRE_ASTER $g(u,u)$	11,58	11.58	0.80%
Lag	- Lag 0.1.1.0		G_MAX	: NON_REGRESSION $g(u,u)$	30,26	0,60%	0.60%
Leg	- Leg 0.5.1.5		room	G : NON_REGRESSION $G(v)$	4,38	0,50%	0.50%
Leg	- Leg 0.5.1.5		bilinear	G : NON_REGRESSION $g(v,v)$	4,38	0,50%	0.50%
Lag	- Lag 0.5.1.5		total	G : NON_REGRESSION $G(u+v)$	30,45	0,50%	0.50%
Lag	- Lag 0.5.1.5		CALC_K_MAX	: NON_REGRESSION $max(G)$	153,57	0,50%	0.50%

9 G Computation

with POST _K1_K2_K3 in 3D. Characteristics

9.1 of the modelization This

modelization makes it possible to test computations of and $K1$ using G POST _K1_K2_K3 when the crack tip was defined either by only one nodes list, but by two lists of nodes in operator DEFI _FOND_FISS. Note:

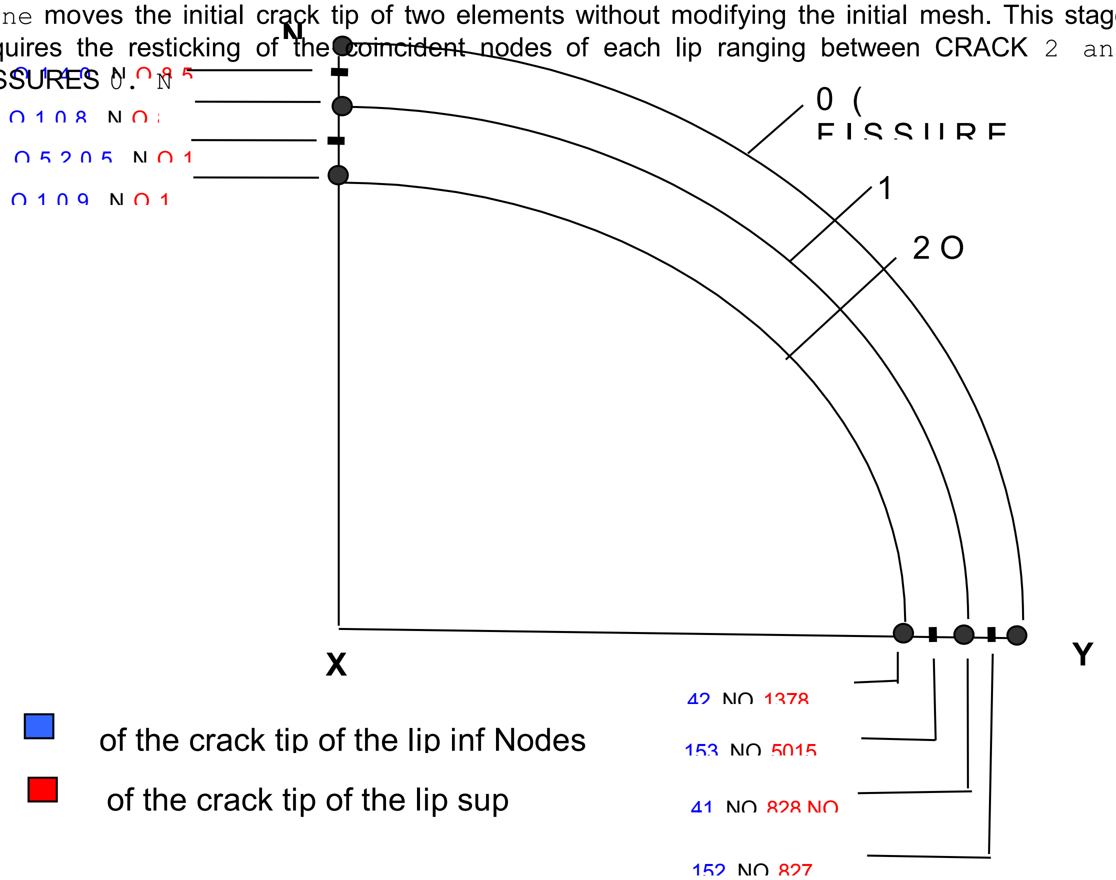
This kind of

> *crack tip is obtained by the key words MELTS _INF and BOTTOM _SUP of operator DEFI _FOND_FISS [U4.82.01]. The two crack tips must be geometrically confused. The mesh*

to that of modelization b: it is identical contains a circular crack noted CRACK 0 on the diagram below. One

carries out the computation of the coefficients of intensity of stresses for the 3 following crack tips: FISSURE

- 0: fissure modelization B (FISSURE0) defined by only one entity; FISSURE
- 1: one moves the initial crack tip of an element without modifying the initial mesh. This stage requires the resticking of the coincident nodes of each lip ranging between CRACK 1 and FISSURES 0; FISSURE
- 2: one moves the initial crack tip of two elements without modifying the initial mesh. This stage requires the resticking of the coincident nodes of each lip ranging between CRACK 2 and FISSURES 0.



: When

one applies conditions of resticking (LIAISON_GROUP) and symmetry to the nodes of the lips, one generates superabundant conditions of blockings. To cure this problem, the nodes of only one of the lips have simultaneously conditions of symmetry and of resticking and for the other lip, the nodes have only conditions of resticking. Only

the loading of tension is taken into account. Characteristics

9.2 of the mesh Many

nodes: 5227 Number of meshes
and type: 432 PENTA `15 and` 784 HEXA20 `Note`

:

The nodes mediums of the edges of the elements touching the crack tip are moved with the quarter of these edges, to obtain a better accuracy (elements of Barsoum). However , the presence of nodes to the quarter in the restuck part of crack disturbs computation significantly. It is thus recommended to duplicate the data structure mesh as many times as crack tips thanks to command CREA_MAILLAGE , and successively to move the nodes with the quarter on each mesh. It is what is made in this case test. Quantities

9.3 tested and Results results

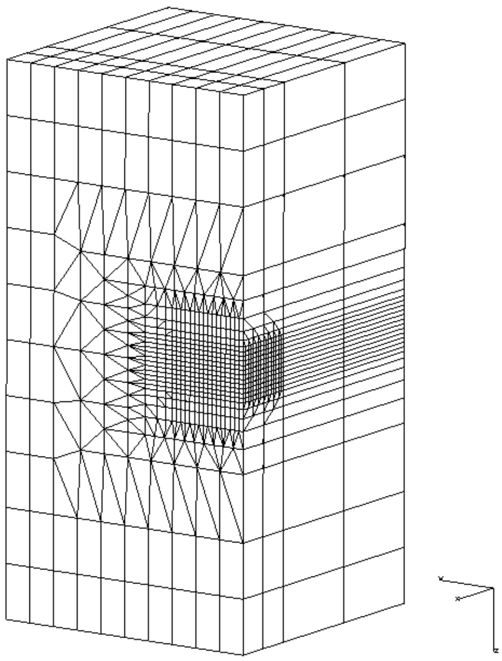
of POST `_K1_K2_K3 Fissures`

Identification	**Standard**	**Node**	**Reference**	***of reference tolerance***	**% Fissures**
1 Node	$K1$	69 1,547	E+06 ANALYTIQUE	0,50	0.50
	G	69 1,089	E+01 ANALYTIQUE	1,00	1.00
Fissures 2	$K1$	Node 70	1,497 E+06	ANALYTIQUE	0.50
	G	Node 70	1,020 E+01	ANALYTIQUE	1.00

10 Modelization Method

X-FEM. Characteristics

10.1 of the modelization This



modelization makes it possible to test the computation of using $K1$ POST `_K1_K2_K3 and` CALC_G `(option` CALC_K_G `) on` a crack nonwith a grid (method). X-FEM Only

the loading of tension is taken into account. Conditions of symmetry are imposed on the two side sides. Characteristics

10.2 of the mesh Many

nodes: 6100 Number of meshes
and type: 1500 PENTA `6 and` 4600 HEXA `8 (linear` mesh) Quantities

10.3 tested and results

the values tested are the factors of intensity of the stresses along $K1$ the crack tip, calculated either by POST `_K1_K2_K3 (method` 3), or by CALC_G . The test is carried out in 3 points of the crack tip: items 1 (first point), 10 and 24 (last point).

The average quadratic error corresponds to the following quantity: Standard

$$\varepsilon=\sqrt{\frac{\int_{\Gamma}(K_I^{ref}-K_I^{Aster})^2\,ds}{\int_{\Gamma}(K_I^{ref})^2\,ds}}$$

identification	Reference	*of reference tolerance*	% POST

_K1_K2_K3 -			
$K1$ point 1 1.595	e6 ANALYTIQUE	12,00	12.00
$K1$ point 10 1.595	e6 ANALYTIQUE	3,00	3.00
$K1$ point 24 1.595	e6 ANALYTIQUE	18,00	18.00
quadratic Error 9,90			9.90

-			
$K1$ point 1 1.595	e6 ANALYTIQUE	6,00	6.00
$K1$ point 10 1.595	e6 ANALYTIQUE	7,00	7.00
$K1$ point 24 1.595	e6 ANALYTIQUE	10,00	10.00
quadratic Error 5,3			5.3

the accuracy of the results got on a crack nonwith a grid (method) X-FEM is slightly less less good than for a crack with a grid. The results got with operator CALC_G remain however satisfactory. It

is pointed out that the mesh used is linear; the use of a finer mesh makes it possible to improve the accuracy of result, but to the detriment of the computing times.

The radius of enrichment around the crack tip (parameter RAYON_ENRI of DEFI_FISS_XFEM) has a negligible influence on the results of CALC_G and a weak influence on the results of POST _K1_K2_K3. For operator CALC_G , the lissages of the type LAGRANGE do not allow to have easily useable results; a lissage of the type LEGENDRE is thus to privilege. Modelization

11 I Method

X-FEM into axisymmetric. Characteristics

11.1 of the modelization This

modelization makes it possible to test the computation of using $K1$ POST `_K1_K2_K3 and` CALC_G `(option` CALC_K_G `) on` an axisymmetric crack nonwith a grid (method X-FEM). Two

types of loadings are considered. First is of simple tension applied to edges high and low of the plate. Second is of simple tension applied to edges high and low of the plate and a rotation of around $150\,trs/min$ the symmetric axis. Characteristics

11.2 of the mesh Many

nodes: 20301 Number of meshes
and type: 20000 QUA `4 and` 600 SE `2 (linear` mesh) Quantities

11.3 tested and results

the values tested are the factors of intensity of the stresses to $K1$ the crack tip, calculated S by CALC_G `(option` CALC_K_G `).` Loading

1: simple tension applied to edges high and low of the Standard plate

Identification	of reference Value of reference	Tolerance	() % “ANALYTIQUE
G	″ 11,59	2,1%	“ANALYTIQUE
$K1$	″ 1,60	1.60E+006	“AUTRE_ASTER
G	″ 11,78	0,4%	“AUTRE_ASTER
$K1$	″ 1,643	E+06 0,8%	Loading

2: simple tension applied to edges high and low of the plate and a rotation of 150 around $trs/\ min$ the symmetric axis Standard_

Identification	of reference Value of reference	Tolerance	() % “AUTRE_ASTER
G	″ 2136,52	0,3%	“AUTRE_ASTER
$K1$	″ 2,191	E+07 1,0%	Remarks

11.4 In

this case test, the ratio between a/W the size of crack and a the width is W . $0{,}2$ The edge effects thus contribute to the difference between the numerical solution for a finished edge and the reference solution for a continuum. Modelization

12 J closed

Crack tip, computation of for G the incompressible elements. Characteristics

12.1 of the modelization Identical

to the modelization A except the elements used are 3D_INCO and 3D_INCO_UP . Characteristics

12.2 of the mesh Identical

to the modelization A Remark

12.3 One

uses key word SYME in operator CALC_G to multiply automatically by two the rate of energy restitution calculated on only one lip of crack, when operand FOND_FISS is absent. When FOND_FISS is present, information on symmetry is recovered directly in the concept melts _fiss created via DEFI _FOND_FISS. Quantities

12.4 tested and results Standard

Identification	Reference	*of reference formulates*	% G
total 145,600	145.600	2,0	G
local Node G $N403$ - G Lagrange	11,586	ANALYTIQUE	4,0
G local Node $N2862$ - Lagrange 11,586	11.586	1,0	G
local Node G $N375$ - G Lagrange	11.586	ANALYTIQUE	3,5
G local Node $N292$ - Lagrange 11,586	11.586	4,0	G
$max(G\,local)$ - G Lagrange	11,590	ANALYTIQUE	5,0
$min(G\,local)$ - Lagrange 11,59	11.586	1,0	G
local Node G $N403$ - G Lagrange_no_no	11.586	ANALYTIQUE	2,5
G local Node $N2862$ - Lagrange_no_no 11,586	11.586	2,0	2.0
local Node G $N375$ - G Lagrange_no_no	11.586	ANALYTIQUE	2,0
G local Node $N292$ - Lagrange_no_no 11,586	11.586	3,0	formulates
$max(G\,local)$ G Lagrange_no_no 11,715	11.715	2,0	formulates
$min(G\,local)$ G Lagrange_no_no 11,575	11.575	2,0	G
local Node G $N403$ - G Lagrange_regu	11.586	ANALYTIQUE	2,0
G local Node $N2862$ - Lagrange_regu 11,76000	NON_REGRESSION	1,00	E-004 G
local Node G $N375$ - G Lagrange_regu	11,76472	NON_REGRESSION	1,00 E-004
G local Node $N292$ - Lagrange_regu 11,74686	NON_REGRESSION	1,00	E-004 -
$max(G\,local)$ G Lagrange_regu 11,78463	NON_REGRESSION	1,00	E-004 -

$min(G\,local)$	G Lagrange_regu 11,73482	NON_REGRESSION	1,00	E-004 Modelization

13 K One

tests into K axisymmetric for the incompressible elements Characteristic

13.1 of the modelization Identical

to the modelization C except the elements used are AXIS_INCO and AXIS_INCO_UP . Characteristics

13.2 of the mesh Identical

to the modelization C Quantities

13.3 tested and results Identification

Standard	Method	Reference	of reference formulates	% formulates
G	23,2	23.2	2	2
G	23,2	23.2	2	2
$K1$	1.595769	E6 ANALYTIQUE	3,5	formulates
$K1$	1.643	E+06 ANALYTIQUE	3	3

14 of the results

the conclusions of this case test are the following ones:

- The definition and the computation of room G on closed crack tips are validated. One checks in particular the independence of room G with respect to the angle for an axisymmetric crack and a loading. One notes a variation of less on 2% the group of the crack tip by methods " LAGRANCE", " LAGRANGE_NO_NO" and " LAGRANGE_REGU".
- The command POST _K1_K2_K3, which makes it possible to calculate the stress intensity factors by exploiting the jump of displacements on the lips of crack, is also validated. This method, less precise than CALC_G , makes it possible to obtain here (with a suitable mesh: nodes mediums of the edges touching the crack tip moved with the quarter of these edges) of the values of and $K1$ to $K3$ less of $2\,\%$ the reference. Three

  methods of interpolation are used and give close results. Method 3 is interesting because it provides a single value of the stress intensity factors and not a maximum value and a minimal value.

  The use of POST _K1_K2_K3 to study a crack by relaxation of nodes is tested and gives satisfactory results. One
- and the validates the computation of the bilinear form of G options G_MAX and CALC_K_MAX .
- The method makes it possible X-FEM to evaluate the factors of intensity of the stresses on K a mesh not fissured with an error lower than. 10% One
- validates computations for elements 3D_INCO , 3D_INCO_UP, AXIS_INCO and AXIS_INCO_UP