SSLV131 - Orthotropy in an unspecified reference

Summarized

This case test validates the modelizations relating to linear elasticity which implement orthotropic materials whose properties are known in a reference defined by the user different from the total reference.

1 Problem of reference

1.1 Geometry

the total reference is the reference (A, X, Y, Z). In this reference the coordinates of the nodes are:

$A(0., 0., 0.)$
$B(3., 1., 0.)$
$C(2., 3., 0.)$
$D(3., 1., -1)$

One will study the behavior of the tetrahedron $ABCD$ whose material properties are defined in a local coordinate system (A, x, y, z) obtained by rotation of the total reference according to the nautical angles $(\alpha=30°, \beta=20°, \gamma=10°)$.

1.2 Properties of the material

the materials used are orthotropic and isotropic transverse. In order to validate the orthotropic strains of thermal origin, one does also a thermomechanical calculation.

One adopts the convention of terminology used in Code_Aster. That is to say the suffixes L, T and N mean Longitudinal, Transverse and Normal.

The units are not specified.

$$E_L = 11000,\ E_T = 5000,\ E_N = 8000,$$
$$\nu_{LT} = 0.396,\ \nu_{LN} = 0.15,\ \nu_{TN} = 0.11$$
$$G_{LT} = 10500,\ G_{LN} = 7000,\ G_{TN} = 13000$$
$$\alpha_L = 10^{-3}, \alpha_T = 1.5\,10^{-3}, \alpha_N = 2\,10^{-3}$$

(It is known that $\nu_{LT} = \frac{E_L}{E_T}\nu_{TL}, \nu_{LN} = \frac{E_L}{E_N}\nu_{NL},\ \nu_{TN} = \frac{E_T}{E_N}\nu_{NT}$,

that is to say $\nu_{TL} = 0.18, \nu_{NL} = 0.1091, \nu_{NT} = 0.176$

For the transverse isotropy, one keeps the same values while knowing as:

$$E_T = E_L = 11000, \nu_{TN} = \nu_{LN} = 1.15 \text{ et } G_{LT} = \frac{E_L}{2(1+\nu_{LT})}$$

It is pointed out that these coefficients are defined in a local coordinate system (A, L, T, N) turned with the nautical angles $(30°, 20°, 10°)$ compared to the total reference.

1.3 Boundary conditions and loadings

the boundary conditions are of Dirichlet type. One makes the assumption of a linear field of displacement in x and y so that the strain field is constant.

Conditions of thermal	$dX=2x+3y+4z$ $dY=3x+5y+6z$ $dZ=4x+6y+7z$
Dirichlet Conditions	Temperature imposed on all structure of 100

One will thus impose:

- for the node A $dX=0, dY=0, dZ=0$
- for the node B $dX=9, dY=14, dZ=18$
- for the node C $dX=13, dY=21, dZ=26$
- for the node D $dX=5, dy=8, dZ=11$

2 Reference solution

2.1 Method of calculating

The computation is analytical.
One used the formal calculation programme Mathématica to carry it out.

It is known that the field of displacement is:

$$dX=2x+3y+4z$$
$$dY=3x+5y+6z$$
$$dZ=4x+6y+7z$$

The strain field ε_G in the total reference is thus constant and equal to:

$$\varepsilon_G=\begin{vmatrix}2 & 3 & 4\\ 3 & 5 & 6\\ 4 & 6 & 7\end{vmatrix}$$

That is to say P the transition matrix allowing to make pass a vector of the total reference (A, X, Y, Z) to the local coordinate system (A, L, N, T).

That is to say $\varepsilon_{\mathbf{L}}$ the strain tensor in the local coordinate system. One a: $\varepsilon_L=P\cdot\varepsilon_G\cdot P^T$

the tensor of Hooke H_L is known in the local coordinate system, that is to say σ_L the tensor of the stresses in this reference. One a:

$$\sigma_L=H_L.\varepsilon_L$$

One obtains the tensor σ_G of the stresses in the total reference by: $\sigma_G=P^T.\sigma_L.P$

If a field of temperature is applied, the equations above are modified as follows:

The strain field ε_G in the total reference is always the same one:

$$\varepsilon_G = \begin{vmatrix} 2 & 3 & 4 \\ 3 & 5 & 6 \\ 4 & 6 & 7 \end{vmatrix}$$

That is to say $\varepsilon_{\mathbf{L}}$ the strain tensor in the local coordinate system. One a: $\varepsilon_{\mathbf{L}} = P.\varepsilon_G.P^T$
the tensor of the mechanical strains in the local coordinate system is thus worth:

$$\varepsilon_L^{mec} = \varepsilon_L - \varepsilon_L^{ther} \text{ with } \begin{cases} \varepsilon_{Lxx}^{ther} = \alpha_L\left(T - T_{ref}\right) \\ \varepsilon_{Lyy}^{ther} = \alpha_T\left(T - T_{ref}\right) \\ \varepsilon_{Lzz}^{ther} = \alpha_N\left(T - T_{ref}\right) \end{cases} \text{, the other components being null}$$

the tensor of Hooke H_L is known in the local coordinate system. That is to say σ_L the tensor of the stresses in this reference. One a:

$$\sigma_L = H_L.\varepsilon_L^{mec}$$

One obtains the tensor σ_G of the stresses in the total reference by: $\sigma_G = P^T.\sigma_L.P$

2.2 Results of reference

They are got by carrying out the operations described above with Mathematica.

2.3 Uncertainties on the solution

uncertainty is null because the solution is analytical.

2.4 Bibliographical references

For the description of the matrixes of Hooke for materials isotropic transverse and orthotropic for the modelizations 3D, plane stresses and plane strains, the selected reference was:

“Matrix of Hooke for the orthotropic materials `. Ratio interns applications in Mechanics n° 79 - 018 of Jean-Claude Masson CISI.

3 Modelization A

3.1 Characteristic of the modelization

The modelization 3D is implemented. One tests the materials isotropic transverse and orthotropic (with possibly taken into account of strains D" thermal origins)

Note:

- The transverse isotropy is not tested for the plane stresses because this case corresponds to the isotropy.
- For the axisymmetric case the stress field depends on the point of computation.
- This point is selected at the point of integration of the triangle (i.e it is the center of gravity of the triangle). It
- is pointed out that the orthotropy in an unspecified reference is not available for the modelization as a Fourier because there is then coupling of all the components of the stress tensor:

Implementation the current makes it possible to use only the symmetric components from which one can find the skew-symmetric components but so that it is possible, it is not necessary that the slidings induce tensile stresses. Characteristics

3.2 of the mesh There

is an element tetrahedron with 4 nodes. $ABCD$ Values

3.3 tested Identification

Reference	**Case**
of the transverse isotropy 3D name	
of result: field $Mest1$	
of displacement 21	
$dy(c)$	field
EPSI_ELGA formulates	
$EPXY$	formula
$EPXZ$	formulates
$EPYZ$	field
SIEF_ELGA formulates	
$SIXX$	formula
$SIYY$	formulates
$SIZZ$	formula
$SIXY$	formulates
$SIXZ$	formula
$SIYZ$	field
SIGM_ELNO 3D	
$SIXX$	43310.760
field emel-ELGA Ep	1.19123 E6
Champ emel-elno-ELGA Ep	1.19123 E6

Cas of the orthotropy name	
of result: field $Mest2$	
of displacement 21	
$dy(c)$	field
EPSI_ELGA `formulates`	
$EPXY$	formula
$EPXZ$	formulates
$EPYZ$	field
SIEF_ELGA `formulates`	
$SIXX$	formula
$SIYY$	formulates
$SIZZ$	formula
$SIXY$	formulates
$SIXZ$	formula
$SIYZ$	field
enel-ELGA 1.55286 Ep	.106 field
enel-elno-ELGA 1.55286 Ep	.106 Cases
of the orthotropy with taking into account of the thermal strains (command STAT_NON_LINE) `name`	
of result: field $Mest3$	
of displacement 21	
$dy(c)$	Identification
Reference	**field**
EPSI_ELGA `formulates`	
$EPXY$	formula
$EPXZ$	formulates
$EPYZ$	field
SIEF_ELGA `formulates`	
$SIXX$	formula
$SIYY$	formulates
$SIZZ$	formula
$SIXY$	formulates
$SIXZ$	formula
$SIYZ$	Cases
of the orthotropy with taking into account of the thermal strains (command MECA_STATIQUE) `name`	
of result: field $Mest4$	
of displacement 21	
$dy(c)$	field
EPSI_ELGA `formulates`	
$EPXY$	formula
$EPXZ$	formulates
$EPYZ$	field
SIEF_ELGA `formulates`	

SIXX	formula
SIYY	formulates
SIZZ	formula
SIXY	formulates
SIXZ	formula
SIYZ	Summary

4 of the results

the results provided by Mathématica and Aster *are* identical for all the modelizations usable with materials isotropic transverse and orthotropic.