SSLV130 - Hollow roll in incompressible

Abstract:

This test makes it possible to validate the quasi-incompressible elements in static for a three-dimensional, axisymmetric or two-dimensional problem (plane strains). One considers a hollow roll subjected to an internal pressure. The material has a Poisson's ratio equal to 0.4999 and one uses the quasi-incompressible elements (modelization INCO and INCO_UP). In all the cases of modelization, one carries out the test by imposing or not the condition of perfect incompressibility if necessary (DDL_IMPO and GONF=0)

Five modelizations are carried out for this problem.
The modelization INCO is tested in the following configurations:

- modelization A : modelization 3D_INCO with HEXA20;
- modelization B : modelization 3D_INCO with TETRA10;
- modelization E : modelization 3D_INCO with PENTA15;
- modelization C : modelization D_PLAN_INCO with a mixed mesh TRIA6/QUAD8;
- modelization D : modelization AXIS_INCO with a mixed mesh TRIA6/QUAD8.

The modelization INCO_UP is tested in the following configurations:

- modelization A : modelization 3D_INCO_UP with HEXA20;
- modelization B : modelization 3D_INCO_UP with TETRA10 and TETRA4;
- modelization C : modelization D_PLAN_INCO_UP with a mixed mesh TRIA6/QUAD8 and TRIA3;
- modelization D : modelization AXIS_INCO_UP with a mixed mesh TRIA6/QUAD8 and TRIA3.

The modelization INCO_OSGS is tested in the following configurations:

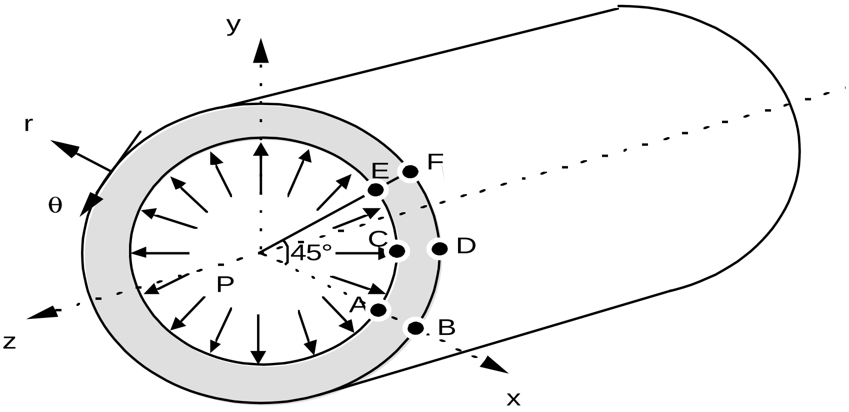
- modelization A : modelization 3D_INCO_OSGS with HEXA8;
- modelization B : modelization 3D_ INCO_OSGS with TETRA4;
- modelization E : modelization 3D_ INCO_OSGS with PENTA6;
- modelization C : modelization D_PLAN_INCO_OSGS with a mixed mesh TRIA6/QUAD8;
- modelization D : modelization AXIS_INCO_OSGS with a mixed mesh TRIA6/QUAD8.

This test is similar to test SSLV100.

The numerical results are satisfactory for all the modelizations. The fact of imposing the condition of incompressibility explicitly influences only very little the results. The linear elements for the modelization at 2 fields are logically a little less precise than the quadratic elements.

1 Problem of reference

1.1 internal



Rayon interne a = 0.1 m
Rayon externe b = 0.2 m

Geometry Radius: $a=0{,}1\ mm$
Radius exerne: $b=0{,}2\ mm$

Coordinated points (mm **)**

	A	B	E	F	C	D
x	0.1	0.2	$0.1\times\cos(45)$	$0.2\times\cos(45)$	$0.1\times\cos(22.5)$	$0.2\times\cos(22.5)$
y	0	0	$0.1\times\sin(45)$	$0.1\times\sin(45)$	$0.1\times\sin(22.5)$	$0.1\times\sin(22.5)$
z	0	0	0	0	0	0

1.2 Properties of the material

the elastic properties of the material considered are the following ones:
Young modulus: $E=2.10^5\ MPa$
Poisson's ratio: $\nu=0.4999$

1.3 Boundary conditions and loadings

the imposed internal pressure is equal to $P=60\ MPa$.

2 Reference solution

2.1 Method of calculating

the general solution in displacement is the following one:

$$\begin{cases} u_r = \dfrac{Pa^2}{E(b^2-a^2)}(1+\nu)\left[(1-2\nu)r+\dfrac{b^2}{r}\right] \\ u_\theta = u_z = 0 \end{cases}$$

In strains:

$$\begin{cases} \varepsilon_{rr} = \dfrac{Pa^2}{E(b^2-a^2)}(1+\nu)\left[(1-2\nu)-\dfrac{b^2}{r}\right] \\ \varepsilon_{\theta\theta} = \dfrac{Pa^2}{E(b^2-a^2)}(1+\nu)\left[(1-2\nu)+\dfrac{b^2}{r}\right] \\ \varepsilon_{r\theta} = \varepsilon_{zz} = 0 \end{cases}$$

In stresses:

$$\begin{cases} \sigma_{rr} = P\dfrac{a^2}{b^2-a^2}\left[1-\dfrac{b^2}{r}\right] \\ \sigma_{\theta\theta} = P\dfrac{a^2}{b^2-a^2}\left[1+\dfrac{b^2}{r}\right] \\ \sigma_{zz} = 2\nu P\dfrac{a^2}{b^2-a^2} \\ \sigma_{r\theta} = 0 \end{cases}$$

One obtains for a perfectly incompressible cylinder $(\nu=0.5)$:

	$r=0{,}1$	$r=0{,}2$
u_r	$6.\,10^{-5}\,mm$	$3.\,10^{-5}\,mm$
ε_{rr}	$-6.\,10^{-4}$	$-1{,}5\,10^{-4}$
$\varepsilon_{\theta\theta}$	$-6.\,10^{-4}$	$1{,}5\,10^{-4}$
σ_{rr}	$-60\,MPa$	$0\,MPa$
$\sigma_{\theta\theta}$	$100\,MPa$	$40\,MPa$
σ_{zz}	$20\,MPa$	$20\,MPa$

The transition in the Cartesian system is done using the following relations:

$$\sigma_{xx} = \sigma_{rr}\cos(\theta)^2 + \sigma_{\theta\theta}\sin(\theta)^2 - 2\sigma_{r\theta}\sin(\theta)\cos(\theta)$$

$$\sigma_{yy} = \sigma_{rr}\sin(\theta)^2 + \sigma_{\theta\theta}\cos(\theta)^2 + 2\sigma_{r\theta}\sin(\theta)\cos(\theta)$$

$$\sigma_{xy} = \sigma_{rr}\sin(\theta)\cos(\theta) - \sigma_{\theta\theta}\cos(\theta)\sin(\theta) - 2\sigma_{r\theta}(\cos(\theta)^2 - \sin(\theta)^2)$$

2.2 Quantities and results of reference

One compared to values of reference:

- displacements (u, v) at the points A and F,
- strains (ε_{xx} ε_{yy}, ε_{xy}) and the forced (σ_{xx} σ_{yy} σ_{zz}, σ_{xy}) at the points A and F,
- strains equivalent and the equivalent stresses to the point A.

Lastly, to test the transition of the quantities of Gauss points to the nodes for the nodes mediums, one also tests the non-zero strains and the forced in a medium node of structure.

2.3 Bibliographical references

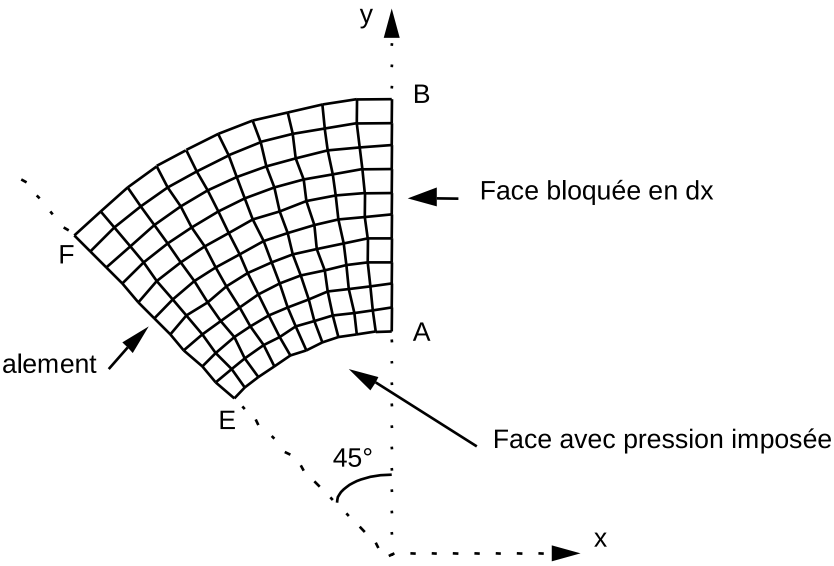
1) Y.C. FUNG: Foundations of solid mechanics. Prenctice-hall, Inc. Englewood Cliffs. NJ. 1965, p. 243-245

2) [V3.04.100] Hollow roll in plane strains

3 Modelization A

3.1 Characteristic of the modelization

Meshes:
1.of type HEXA20 for modelizations 3D_INCO and 3D_INCO_UP
2.of the type HEXA8 for modelization 3D_INCO_OSGS



Along the axis z :

- total thickness $e=0.01\,mm$
- 2 layers of elements

For the needs for examination in a medium node, one defines the node $NOEUMI=A+(0.0.e/4)$ where the strains and the forced are the same ones as in A .

Limiting conditions :

```
DDL_IMPO=GROUP_NO    = ' FACSUP', DZ = 0
   GROUP_NO = ' FACINF', DZ = 0       sides AEFD ( z=0 and z=0.01 )
   GROUP_NO = ' FACEAB', DX = 0       face AB
FACE_IMPO=GROUP_MA   = ' FACEEF', DNOR = 0       face EF
PRES_REP=GROUP_MA    = ' FACEAE', NEAR = 60  face AE
```

3.2 Characteristics of the meshes

Mesh 1 HEXA20:
Many nodes: 1501 nodes
Number of meshes: 240 HEXA20

Mesh 2 HEXA8:
Many nodes: 429 nodes
Number of meshes: 240 HEXA 8

3.3 Quantities tested and Results

results with point: A

- first column `3D_INCO` without imposing $GONF=0$
- second column `3D_INCO` by imposing $GONF=0$
- third column `3D_INCO_UP`
- fourth Standard `column`

3D_INCO_OSGS	Identification of reference	Value of reference	Tolerance			
			1	2	3	4
v	"ANALYTIQUE"	6. 10 $^{-5}$	10-5	10-5	10-5	10-5
σ_{xx}	"ANALYTIQUE"	100.	0.10%	0.10%	0.10%	0,15%
σ_{yy}	"ANALYTIQUE"	– 60.	0.50%	0.50%	0.50%	9,0%
σ_{zz}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	5,0%
σ_{xy}	"ANALYTIQUE"	0.	0.50%	0.50%	0.50%	50.00%
ε_{xx}	"ANALYTIQUE"	6. 10 $^{-4}$	10-5	10-5	10-5	10-5
ε_{yy}	"ANALYTIQUE"	– 6. 10 $^{-4}$	0.50%	0.50%	0.50%	7,0%
ε_{zz}	"ANALYTIQUE"	0.	0.50%	0.50%	0.50%	0.50%
ε_{xy}	"ANALYTIQUE"	0.	10-5	10-5	10-5	3,10-5
ε_{eq} - I NVA_2	"ANALYTIQUE"	6.92 10 $^{-4}$	10-5	10-5	10-5	5,10-2
ε_{eq} - PRIN_1	"ANALYTIQUE"	– 6. 10 $^{-4}$	0.50%	0.50%	0.50%	7,0%
ε_{eq} - PRIN_2	"ANALYTIQUE"	0.	0.50%	0.50%	0.50%	10 $^{-5}$
ε_{eq} - PRIN_3	"ANALYTIQUE"	6. 10 $^{-4}$	10 $^{-5}$	10 $^{-5}$	10 $^{-5}$	5,10 $^{-2}$
σ_{eq} - VMIS	"ANALYTIQUE"	138.56	0.50%	0.50%	0.50%	5.00%
σ_{eq} - TRESCA	"ANALYTIQUE"	160.	0.50%	0.50%	0.50%	5.00%
σ_{eq} - PRIN_1	"ANALYTIQUE"	-60.	0.50%	0.50%	0.50%	10,0%
σ_{eq} - PRIN_2	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	5.00%
σ_{eq} - PRIN_3	"ANALYTIQUE"	100.	0.50%	0.50%	0.50%	1,50%
σ_{eq} - VMIS	"ANALYTIQUE"	138.56	0.50%	0.50%	0.50%	5,0%

Results with point: F

- first column 3D_INCO without imposing $GONF=0$
- second column 3D_INCO by imposing $GONF=0$
- third column 3D_INCO_UP
- fourth column 3D_INCO_OSGS

Standard	Identification of reference	Value of reference	Tolerance			
			1	2	3	4
u	"ANALYTIQUE"	– 2.12 10^{-5}	0.10%	0.10%	0.10%	0.10%
v	"ANALYTIQUE"	2.12 10^{-5}	0.10%	0.10%	0.10%	0.10%
σ_{xx}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	3,0%
σ_{yy}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	6,0%
σ_{zz}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	1,0%
σ_{xy}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	2,0%
ε_{xx}	"ANALYTIQUE"	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{yy}	"ANALYTIQUE"	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{xy}	"ANALYTIQUE"	1.5 10^{-4}	0.50%	0.50%	0.50%	2,0%

Checking of the transition to the nodes for the nodes mediums (only for result obtained without imposing GONF = 0) - value with the node is outside the field of definition with a right profile of the EXCLU type node: $NOEUMI$

Standard	identification of reference	Value of reference	Tolerance ($\%$)
σ_{xx}	"ANALYTIQUE"	100.	0.50%
σ_{yy}	"ANALYTIQUE"	– 60.	0.50%
σ_{zz}	"ANALYTIQUE"	20.	0.50%
ε_{xx}	"ANALYTIQUE"	6. 10^{-4}	0.50%
ε_{yy}	"ANALYTIQUE"	– 6. 10^{-4}	0.50%

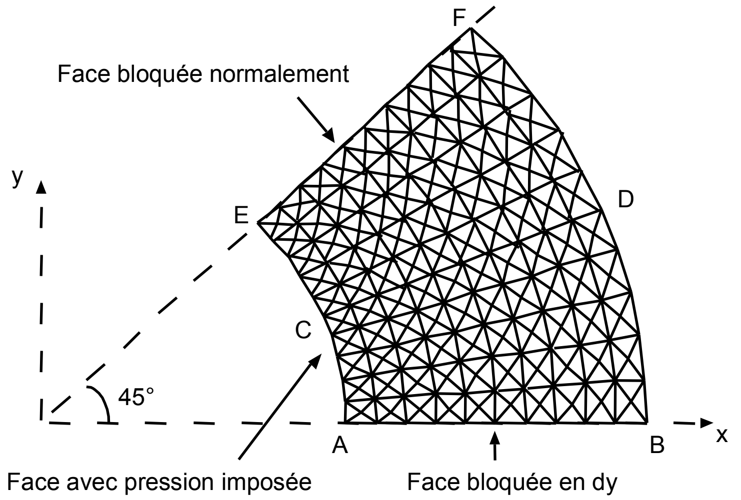
3.4 Remarks

Except for 3D_INCO_OSGS, one gets very good results some is the formulation selected since for all the examined quantities, the difference between the solution obtained with the code and the analytical solution is lower than 0.2%. It is seen that the difference between the solutions obtained by imposing or not the condition $tr(\boldsymbol{\varepsilon})=0$ is unimportant.
The results got with 3D_INCO_OSGS are less precise. That is explained by the fact why the elements used are linear. To find result the more precise, the mesh would have to be refined.

4 Modelization B

4.1 Characteristic of the modelization

Mesh with elements 3D incompressible of type TETRA10 only



AB is on the axis OX (contrary to the modelization A).

For the needs for examination, one defines the node $NOEUMI = A+(0.0.e/4)$ where the strains and the forced are the same ones as in A .

Limiting conditions :

```
DDL_IMPO=GROUP_NO    = ' FACSUP', DZ = 0
   GROUP_NO = ' FACINF', D Z = 0faces    AEFD ( z=0 and z=0.01 )
   GROUP_NO = ' FACEAB', DY = 0        face AB
FACE_IMPO=GROUP_MA   = ' FACEEF', DNOR = 0      face EF
PRES_REP=GROUP_MA    = ' FACEAE', NEAR = 60     face AE
```

4.2 Characteristics of the mesh

Many nodes: 13907
Number of meshes: 8519 TETRA10

4.3 Quantities tested and results

Result with point: A

- first column `3D_INCO` without imposing $GONF=0$
- second column `3D_INCO` by imposing $GONF=0$
- third column `3D_INCO_UP` with quadratic elements
- fourth column `3D_INCO_UP` with linear elements
- fifth column `3D_INCO_OSGS` with linear elements

Standard	Identification of reference	Value of reference	Tolerance				
			1	2	3	4	5
u	"ANALYTIQUE"	$6.\ 10^{-5}$	0.50%	0.50%	0.50%	0.50%	0.50%
v	"ANALYTIQUE"	0.	10-5	10-5	10-5	10-5	10-5
σ_{xx}	"ANALYTIQUE"	– 60.	1.00%	1.00%	1.00%	4,00%	3,00%
σ_{yy}	"ANALYTIQUE"	100.	1.00%	1.00%	1.00%	1,50%	1.00%
σ_{zz}	"ANALYTIQUE"	20.	2.50%	2.50%	2.50%	10,00%	3.00%
σ_{xy}	"ANALYTIQUE"	0.	2.5	2.5	2.5	2.5	2.5
ε_{xx}	"ANALYTIQUE"	-6. 10-4	0.50%	0.50%	0.50%	1.50%	2,0%
ε_{yy}	"ANALYTIQUE"	6. 10-4	0.50%	0.50%	0.50%	1.00%	1.00%
ε_{xy}	"ANALYTIQUE"	0.	$3.\ 10^{-5}$	$3.\ 10^{-5}$	$3.\ 10^{-5}$	$3.\ 10^{-5}$	$3.\ 10^{-5}$
ε_{eq} - I NVA_2	"ANALYTIQUE"	6.92 10-4	0.50%	0.50%	0.50%	1.00%	1,50%
ε_{eq} - PRIN_1	"ANALYTIQUE"	– 6. 10-4	0.50%	0.50%	0.50%	1.50%	2,0%
ε_{eq} - PRIN_2	"ANALYTIQUE"	0.	10-5	10-5	10-5	10-5	10-5
ε_{eq} - PRIN_3	"ANALYTIQUE"	6. 10-4	0.50%	0.50%	0.50%	1.00%	1,0%
σ_{eq} - VMIS	"ANALYTIQUE"	138.56	1.00%	1.00%	1.00%	1.00%	1,5%
σ_{eq} - TRESCA	"ANALYTIQUE"	160.	1.00%	1.00%	1.00%	1.00%	1,5%
σ_{eq} - PRIN_1	"ANALYTIQUE"	-60.	3.00%	3.00%	3.00%	4,00%	3.00%
σ_{eq} - PRIN_2	"ANALYTIQUE"	20.	3.00%	3.00%	3.00%	10,00%	3.00%
σ_{eq} - PRIN_3	"ANALYTIQUE"	100.	1.00%	1.00%	1.00%	1,50%	1.00%
σ_{eq} - VMIS	"ANALYTIQUE"	138.56	1.00%	1.00%	1.00%	1.00%	1,5%

Result with point: F

* first column `3D_INCO` without imposing $GONF=0$
* second column `3D_INCO` by imposing $GONF=0$
* third column `3D_INCO_UP` with quadratic elements
* fourth column `3D_INCO_UP` with linear elements
* fifth column `3D_INCO_OSGS` with linear elements

Standard	Identification of reference	Value of reference	Tolerance				
			1	2	3	4	5
u	"ANALYTIQUE"	2.12 10-5	0.50%	0.50%	0.50%	0.50%	0.50%
v	"ANALYTIQUE"	2.12 10-5	0.50%	0.50%	0.50%	0.50%	0.50%
σ_{xx}	"ANALYTIQUE"	20.	1.00%	1.00%	1.00%	1.50%	1,50%
σ_{yy}	"ANALYTIQUE"	20.	1.00%	1.00%	1.00%	1.00%	1.00%
σ_{zz}	"ANALYTIQUE"	20.	1.00%	1.00%	1.00%	1,50%	1.00%
σ_{xy}	"ANALYTIQUE"	-20.	1.00%	1.00%	1.00%	1.00%	1.00%
ε_{xx}	"ANALYTIQUE"	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{yy}	"ANALYTIQUE"	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{xy}	"ANALYTIQUE"	-1.5 10-4	0.50%	0.50%	0.50%	1.00%	1.00%

Checking of the transition to the nodes for the nodes mediums (only for result obtained without imposing `GONF` = 0) - value with the node is outside the field of definition with a right profile of the EXCLU type node: $NOEUMI$

Standard	identification of reference	Value of reference	Tolerance ($\%$)
σ_{xx}	"ANALYTIQUE"	-60.	1.70%
σ_{yy}	"ANALYTIQUE"	100.	0.60%
σ_{zz}	"ANALYTIQUE"	20.	3.50%
ε_{xx}	"ANALYTIQUE"	-6. 10-4	0.50%
ε_{yy}	"ANALYTIQUE"	6. 10-4	0.50%

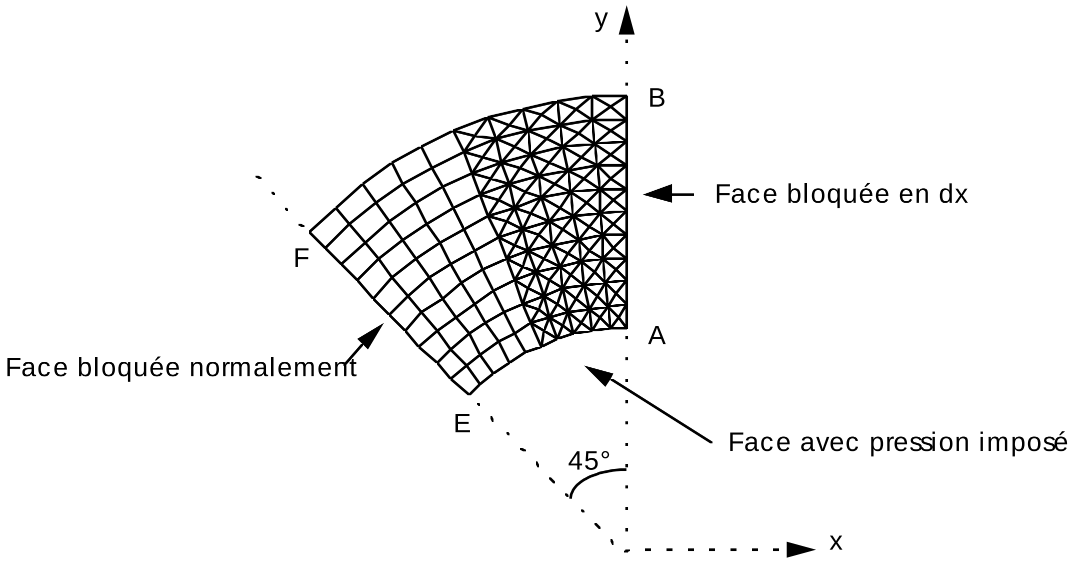
4.4 Remarks

the results got here are a little less good than in the case of the modelization A , but the discretization is coarser since there are approximately 2 times less nodes in this benchmark. The results are satisfactory all the same since the variations are lower than 0.2% for displacements, lower than 0.5% for the strains and lower than 1% for the stresses. It is noted again that there is no significant improvement of result when one imposes explicitly $tr\,\varepsilon=0$.

5 Modelization C

5.1 Characteristic of the modelization

Mesh with elements 2D incompressible of type QUAD8 and TRIA6.



Limiting conditions:

```
DDL_IMPO=GROUP_NO    = ' GRNM11', DX = 0côté          AB
FACE_IMPO=GROUP_MA   = ' GRMA12', DNOR = 0coté     EF
PRES_REP=GROUP_MA    = ' GRMA13', NEAR =   60face  AE
```

Name of the nodes:

$A=N2 \quad B=N361 \quad C=N121 \quad D=N584 \quad E=N155 , \quad F=N503$

5.2 Characteristic of the mesh

Many nodes: 591
Number of meshes: 200 TRIA6, 50 QUAD8.

5.3 Quantities tested and Results

results with point: A

- first column D_PLAN_INCO without imposing $GONF=0$
- second column D_PLAN_INCO by imposing $GONF=0$
- third column D_PLAN_INCO_UP with quadratic mesh
- fourth column D_PLAN_INCO_UP with linear mesh
- fifth column 3D_INCO_OSGS with linear elements

Standard	Identification of reference	Value of reference	Tolerance				
			1	2	3	4	5
u	"ANALYTIQUE"	0.	10-5	10-5	10-5	10-5	10-5
v	"ANALYTIQUE"	6. 10^{-5}	0.50%	0.50%	0.50%	0.50%	0.50%
σ_{xx}	"ANALYTIQUE"	100.	0.50%	0.50%	0.50%	0,60%	0.50%
σ_{yy}	"ANALYTIQUE"	– 60.	0.50%	0.50%	0.50%	2,50%	2,10%
σ_{zz}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	5,00%	1,60%
σ_{xy}	"ANALYTIQUE"	0.	0.3	0.3	0.3	1,5.0,9	0.9
ε_{xx}	"ANALYTIQUE"	6. 10^{-4}	0.50%	0.50%	0.50%	0.50%	0.50%
ε_{yy}	"ANALYTIQUE"	– 6. 10^{-4}	0.50%	0.50%	0.50%	1.50%	1.50%
ε_{xy}	"ANALYTIQUE"	0.	10-5	10-5	10-5	10-5	10-5
ε_{eq} - I NVA_2	"ANALYTIQUE"	6.92 10^{-4}	0.50%	0.50%	0.50%	1.00%	1.00%
ε_{eq} - PRIN_1	"ANALYTIQUE"	– 6. 10^{-4}	0.50%	0.50%	0.50%	1.50%	1.50%
ε_{eq} - PRIN_2	"ANALYTIQUE"	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{eq} - PRIN_3	"ANALYTIQUE"	6. 10^{-4}	0.50%	0.50%	0.50%	0.50%	0.50%
σ_{eq} - VMIS	"ANALYTIQUE"	138.56	1.00%	1.00%	1.00%	1.00%	1.00%
σ_{eq} - TRESCA	"ANALYTIQUE"	160.	1.00%	1.00%	1.00%	1.00%	1.00%
σ_{eq} - PRIN_1	"ANALYTIQUE"	-60.	1.00%	1.00%	1.00%	2,50%	2,10%
σ_{eq} - PRIN_2	"ANALYTIQUE"	20.	1.00%	1.00%	1.00%	5,00%	1,60%
σ_{eq} - PRIN_3	"ANALYTIQUE"	100.	1.00%	1.00%	1.00%	1.00%	1.00%
σ_{eq} - VMIS	"ANALYTIQUE"	138.56	1.00%	1.00%	1.00%	1.00%	1.00%

Results with point: F

- first column D_PLAN_INCO without imposing $GONF=0$
- second column D_PLAN_INCO by imposing $GONF=0$
- third column D_PLAN_INCO_UP with quadratic mesh
- fourth column D_PLAN_INCO_UP with linear mesh
- fifth column 3D_INCO_OSGS with linear elements

Standard	Identification of reference	Value of reference	Tolerance				
			1	2	3	4	5
u	"ANALYTIQUE"	– 2.12 10-5	0.50%	0.50%	0.50%	0.50%	0.50%
v	"ANALYTIQUE"	2.12 10-5	0.50%	0.50%	0.50%	0.50%	0.50%
σ_{xx}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	1.00%	1.00%
σ_{yy}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	2,00%	1.50%
σ_{zz}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	0.50%	0.50%
σ_{xy}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	0.50%	0.50%
ε_{xx}	"ANALYTIQUE"	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{yy}	"ANALYTIQUE"	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{xy}	"ANALYTIQUE"	1.5 10-4	0.50%	0.50%	0.50%	0.50%	0.50%

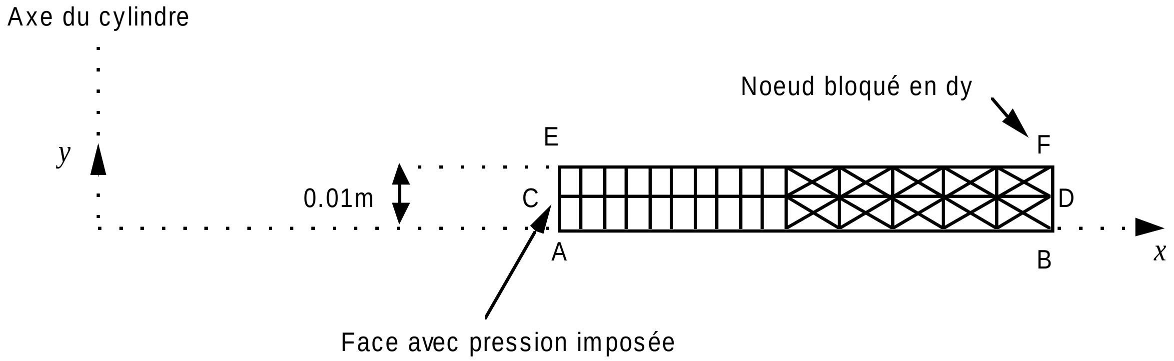
5.4 Remarks

As for the modelization 3D, the got results are completely satisfactory.

6 Modelization D

6.1 Characteristic of the modelization

incompressible `Elements` AXI (`TRIA6` + `QUAD8`)



For the needs for examination, one defines the nodes:

- $NOEUMIA = A + (0.0.01/4)$ where the strains and stresses are the same one as in A
- $NOEUMIB = B + (0.0.01/4)$ where the strains and stresses are the same one as in B

limiting Conditions:

```
DDL_IMPO=GROUP_NO    = ' FACSUP', DY = 0          y=0.1
   GROUP_NO = ' FACINF', DY = 0        y=0
PRES_REP=GROUP_MA    = ' FACEAE', NEAR = 60face     AE
```

6.2 Characteristic of the mesh

Many nodes: 175.
Number of meshes and types: 20 `QUAD8`, 40 `TRIA6`.

6.3 Quantities tested and Results

results with point: A

- first column AXIS_INCO without imposing $GONF=0$
- second column AXIS_INCO by imposing $GONF=0$
- third column AXIS_INCO_UP with quadratic mesh
- fourth column AXIS_INCO_UP with linear mesh
- fifth column 3D_INCO_OSGS with linear elements

Standard	Identification of reference	Value of reference	Tolerance				
			1	2	3	4	5
u	"ANALYTIQUE"	6. 10 -5	0.10%	0.10%	0.10%	0.10%	0.10%
v	"ANALYTIQUE"	0.	10-5	10-5	10-5	10-5	10-5
σ_{xx}	"ANALYTIQUE"	– 60.	0.50%	0.50%	0.50%	0,70%	1,50%
σ_{yy}	"ANALYTIQUE"	3/20/12	0.50%	0.50%	0.50%	1.00%	9,00%
σ_{zz}	"ANALYTIQUE"	100.	0.50%	0.50%	0.50%	0.50%	6,00%
σ_{xy}	"ANALYTIQUE"	0.	10-5	0.01	10-5	0,08	0,2
ε_{xx}	"ANALYTIQUE"	-6. 10-4	0.50%	0.50%	0.50%	0.60%	0.60%
ε_{yy}	"ANALYTIQUE"	0.	10-5	10-5	10-5	10-5	10-5
ε_{xy}	"ANALYTIQUE"	0.	10-5	10-5	10-5	10-5	10-5
ε_{eq} - I NVA_2	"ANALYTIQUE"	6.92 10-4	0.50%	0.50%	0.50%	0.50%	3,0%
ε_{eq} - PRIN_1	"ANALYTIQUE"	– 6. 10-4	0.50%	0.50%	0.50%	1.00%	6,0%
ε_{eq} - PRIN_2	"ANALYTIQUE"	0.	10-5	10-5	10-5	10-5	10-5
ε_{eq} - PRIN_3	"ANALYTIQUE"	6. 10-4	0.50%	0.50%	0.50%	0.50%	0,60%
σ_{eq} - VMIS	"ANALYTIQUE"	138.56	1.00%	1.00%	1.00%	1.00%	3,00%
σ_{eq} - TRESCA	"ANALYTIQUE"	160.	1.00%	1.00%	1.00%	1.00%	3,00%
σ_{eq} - PRIN_1	"ANALYTIQUE"	-60.	1.00%	1.00%	1.00%	1.00%	2,00%
σ_{eq} - PRIN_2	"ANALYTIQUE"	20.	1.00%	1.00%	1.00%	1.00%	23, %
σ_{eq} - PRIN_3	"ANALYTIQUE"	100.	1.00%	1.00%	1.00%	1.00%	5,5%
σ_{eq} - VMIS	"ANALYTIQUE"	138.56	1.00%	1.00%	1.00%	1.00%	3,00%

Results with point: F

- first column AXIS_INCO without imposing $GONF=0$
- second column AXIS_INCO 0.5 by $GONF=0$
- imposing third column AXIS_INCO_UP with quadratic
- mesh fourth column AXIS_INCO_UP with linear
- mesh fifth column 3D_INCO_OSGS with linear

elements	Standard Identification of	reference Value of reference					
			1	2	3	4	5
u	5 "ANALYTIQUE	" 3. 10	0.10%	0.10%	0.10%	0.10%	0,10%
v	2,0% "ANALYTIQUE	" 0	. 10	-5 10	-5 10	-5 10	-5 10
σ_{xx}	-5 "ANALYTIQUE	" 0	.	0.03	0.03	0.03	0.10
σ_{yy}	0.70 "ANALYTIQUE	" 20	0.50%	0.50%	0.50%	0.50%	0,50%
σ_{zz}	1,1% "ANALYTIQUE	" 40	0.50%	0.50%	0.50%	0.50%	0,50%
σ_{xy}	0,60% "ANALYTIQUE	"	. 10-3	5. 10 -	5. 10-3	0.01	0,2
ε_{xx}	"ANALYTIQUE"	-1.5 10-4	0,50%	0,50%	0,50%	0,50%	1,0%
ε_{yy}	"ANALYTIQUE"	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{xy}	"ANALYTIQUE"	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}

Checking of the transition to the nodes for the nodes mediums (only for result obtained without imposing $GONF=0$)

NOEUMIA

Standard	Identification of reference	Value of reference	Tolerance (%)
σ_{xx}	"ANALYTIQUE"	-60.	0.50%
σ_{yy}	"ANALYTIQUE"	20.	0.50%
σ_{zz}	"ANALYTIQUE"	100.	0.50%
ε_{xx}	"ANALYTIQUE"	$-6.\ 10^{-4}$	0.50%
ε_{yy}	"ANALYTIQUE"	$6.\ 10^{-4}$	0.50%

NOEUMIB

Standard	Identification of reference	Value of reference	Tolerance (%)
σ_{xx}	"ANALYTIQUE"	0.	0.50%
σ_{yy}	"ANALYTIQUE"	20.	0.50%
σ_{zz}	"ANALYTIQUE"	40.	0.50%

ε_{xx}	"ANALYTIQUE"	$-1.5\ 10^{-4}$	0.50%
ε_{yy}	"ANALYTIQUE"	$1.5\ 10^{-4}$	0.50%

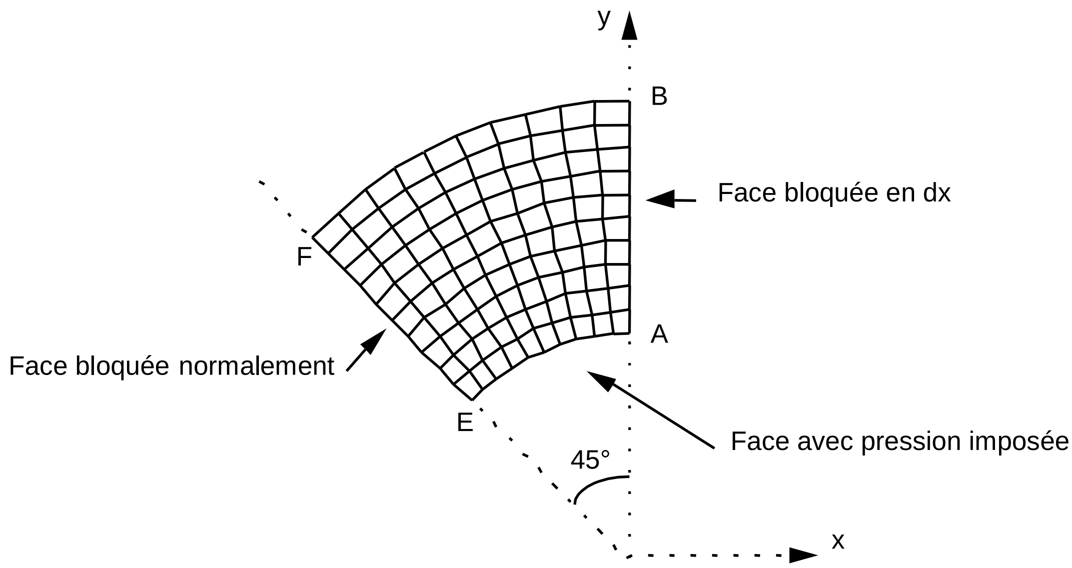
6.4 Remarks

the accuracy obtained are very good.

7 Modelization E

7.1 Characteristic of the modelization

Mesh with elements 3D incompressible of type PENTA15 only



Along the axis z :

- total thickness $e=0.01\,mm$
- 2 layers of elements

For the needs for examination in a medium node, one defines the node $NOEUMI=A+(0.0.e/4)$ where the strains and the forced are the same ones as in A .

Limiting conditions:

```
DDL_IMPO=GROUP_NO    = ' FACSUP', DZ = 0
   GROUP_NO = ' FACINF', DZ = 0faces     AEFD ( z=0 and z=0.01 )
   GROUP_NO = ' FACEAB', DX = 0face     AB
FACE_IMPO=GROUP_MA   = ' FACEEF', DNOR = 0face     EF
PRES_REP=GROUP_MA    = ' FACEAE', NEAR =   60.face     AE
```

7.2 Characteristic of the mesh

Many nodes: 1501 nodes
Number of meshes: 480 PENTA15

7.3 Quantities tested and Results

results with point: A

- first column `3D_INCO` without imposing $GONF=0$
- second column `3D_INCO` by imposing $GONF=0$
- third column `3D_INCO_UP`
- fourth column `3D_INCO_ OSGS` with linear elements

Standard	Identification of reference	Value of reference	Tolerance			
			1	2	3	4
u	"ANALYTIQUE"	0.	10-5	10-5	10-5	10-5
v	"ANALYTIQUE"	6. 10 -5	0.10%	0.10%	0.10%	0,50%
σ_{xx}	"ANALYTIQUE"	100.	0.50%	0.50%	0.50%	6,0%
σ_{yy}	"ANALYTIQUE"	– 60.	0.50%	0.50%	0.50%	15,0%
σ_{zz}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	5.00%
σ_{xy}	"ANALYTIQUE"	0.	0.6	0.6	0.6	03/06/12
ε_{xx}	"ANALYTIQUE"	6. 10 -4	0.50%	0.50%	0.50%	1,0%
ε_{yy}	"ANALYTIQUE"	– 6. 10 -4	0.50%	0.50%	0.50%	1,0%
ε_{zz}	"ANALYTIQUE"	0.	10-5	10-5	10-5	10-5
ε_{xy}	"ANALYTIQUE"	0.	10-5	10-5	10-5	5,10-5
ε_{eq} - I NVA_2	"ANALYTIQUE"	6.92 10 -4	0.50%	0.50%	0.50%	8,0%
ε_{eq} - PRIN_1	"ANALYTIQUE"	– 6. 10 -4	0.50%	0.50%	0.50%	8,0%
ε_{eq} - PRIN_2	"ANALYTIQUE"	0.	10 -5	10 -5	10 -5	10 -5
ε_{eq} - PRIN_3	"ANALYTIQUE"	6. 10 -4	0.50%	0.50%	0.50%	8,0%
σ_{eq} - VMIS	"ANALYTIQUE"	138.56	0.50%	0.50%	0.50%	10, %
σ_{eq} - TRESCA	"ANALYTIQUE"	160.	0.50%	0.50%	0.50%	10, %
σ_{eq} - PRIN_1	"ANALYTIQUE"	-60.	0.50%	0.50%	0.50%	12, %
σ_{eq} - PRIN_2	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	5.00%
σ_{eq} - PRIN_3	"ANALYTIQUE"	100.	0.50%	0.50%	0.50%	6,0%
σ_{eq} -VMIS	"ANALYTIQUE"	138.56	0.50%	0.50%	0.50%	10, %

Results with point: F

- first column 3D_INCO without imposing $GONF=0$
- second column 3D_INCO by imposing $GONF=0$
- third column 3D_INCO_UP
- fourth column 3D_INCO_ OSGS with linear elements

Standard	Identification of reference	Value of reference	Tolerance			
			1	2	3	4
u	"ANALYTIQUE"	– 2.12 10^{-5}	0.10%	0.10%	0.10%	0,30%
v	"ANALYTIQUE"	2.12 10^{-5}	0.10%	0.10%	0.10%	0,30%
σ_{xx}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	50.00%
σ_{yy}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	6,0%
σ_{zz}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	0.50%
σ_{xy}	"ANALYTIQUE"	20.	0.50%	0.50%	0.50%	1,50%
ε_{xx}	"ANALYTIQUE"	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{yy}	"ANALYTIQUE"	0.	10^{-5}	10^{-5}	10^{-5}	10^{-5}
ε_{xy}	"ANALYTIQUE"	1.5 10^{-4}	0.50%	0.50%	0.50%	1,50%

Checking of the transition to the nodes for the nodes mediums (only for result obtained without imposing $GONF=0$) - value with the node is outside the field of definition with a right profile of the EXCLU type node: $NOEUMI$

Standard	identification of reference	Value of reference	Tolerance ($\%$)
σ_{xx}	"ANALYTIQUE"	100.	0.50%
σ_{yy}	"ANALYTIQUE"	– 60.	0.50%
σ_{zz}	"ANALYTIQUE"	20.	0.50%
ε_{xx}	"ANALYTIQUE"	6. 10^{-4}	0.50%
ε_{yy}	"ANALYTIQUE"	– 6. 10^{-4}	0.50%

7.4 Remarks

Except for modelization 3D_INCO_OSGS, one gets very good results some is the formulation selected since for all the examined quantities, the difference between the solution obtained with the code and the analytical solution is lower than 0.5%. It is seen that the difference between the solutions obtained by imposing or not the condition $tr(\boldsymbol{\varepsilon})=0$ is unimportant.
The results got with 3D_INCO_OSGS are less precise. That is explained by the fact why the elements used are linear. To find result the more precise, the mesh would have to be refined.

8 Summary of the results

With a Poisson's ratio ν very close to 0.5, one finds the results of the incompressible analytical solution with a weak difference. It is noticed that it is not necessary to explicitly impose the condition of incompressibility $tr\,\varepsilon=0$ to get good results since the results are quasi-identical that one activates or not, the condition $GONF=0$ with `DDL_IMPO`.