SSLV121 - Stretching of a transverse isotropic parallelepiped under its own Summarized

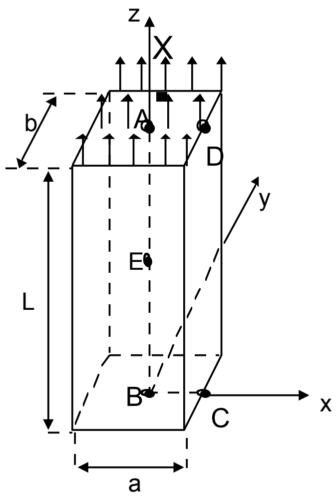
weight:

This test of structural mechanics allows the evaluating of displacements and the stresses of a parallelepiped becoming deformed under its own weight. The material is elastic linear isotropic transverse. The modelization is three-dimensional. The model is similar to test VPCS SSLV07 (but in this case the material is isotropic) and with test SSLV120 (in this case the material is orthotropic.).

The variations of the results got by *Aster* are between $0,00\%$ and $0,4\%$ of the analytically calculated reference.

1 Problem of reference

1.1 Geometry



Hauteur : $L=3m$ Largeur : $a=1m$ Epaisseur : $b=1r$

Coordinated of the points (in meters):

	A	B	C	D	E	X
x	0.	0.	0.5.0.5		0.	0.
y	0.	0.	0.	0.	0.	0.5
z	3.	0.	0.	3.	1.5	3.

1.2 Material properties

Young's moduli in the plane xy and the direction z :

$$E_L = 5.10^{11} Pa \quad E_N = 2.10^{11} Pa .$$

Poisson's ratios relating to the plane xy and the direction z :

$$\nu_{LT} = 0.1 \quad \nu_{LN} = 0.3 .$$

Shear modulus relating to the direction z :

$$G_{LN} = 7.69231\,10^{10} Pa .$$

Density: $\rho = 7800\, kg/m^3$.

1.3 Boundary conditions and loadings

Point: A ($u=v=w=0$, $\theta_x=\theta_y=\theta_z=0$)

Inertia loading following the axis z : $\rho g z$
Uniform stress with the tension for the upper face:

$$\sigma_z = \rho g L = +229554.\,Pa$$

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is resulting from that given in file SSLV07/89 of guide VPCS (while considering in more one transverse isotropic elastic matrix). The analytical statement of the solution is the following one:

Displacements:

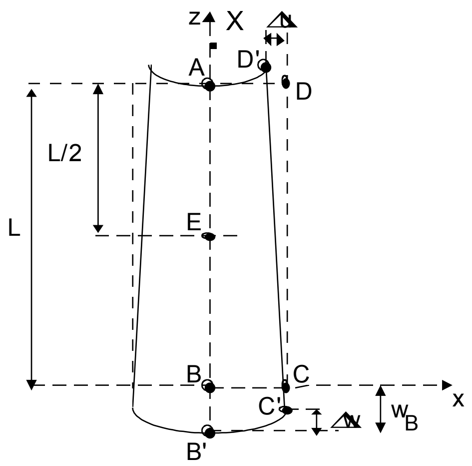
$$u=-\frac{\nu_{\mathrm{NL}}\rho g x z}{E_N}$$

$$v=-\frac{\nu_{\mathrm{NL}}\rho g y z}{E_N}$$

$$w=\frac{\rho g z^2}{2E_N}+\frac{\rho g \nu_{\mathrm{NL}}}{2E_N}(x^2+y^2)-\frac{\rho g L^2}{2E_N}$$

Stresses:

$$\sigma_{zz}=\rho g z \quad \sigma_{zz}=\sigma_{yy}=\sigma_{xy}=\sigma_{yz}=\sigma_{zx}=0$$



2.2 Results of reference

Displacement of the points B C D , E and X.

Stresses σ_{zz} in A and E

2.3 Uncertainty on the solution

exact analytical Results.

2.4 Bibliographical references

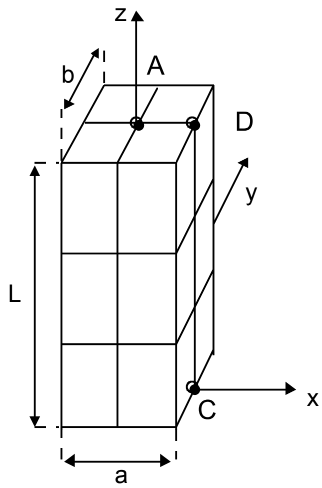
1 TIMOSHENKO (S.P) Theory of elasticity - Paris - Polytechnic Library CH. Béranger, p.279 with 282 (1961)

2 S.W. TSAI, H.T. HAHN - Introduction to composite materials. Technomic Publishing Company (1980).

3 Modelization A

3.1 Characteristic of the modelization

3D



Cutting:

3 elements in height

2 elements in width and thickness

meshes hexa20

limiting Conditions:

on axis AB	DDL_IMPO: (GROUP_NO: ABsansA DX=0., DY=0.)
in A and D	(THE NODE IS OUTSIDE THE FIELD OF DEFINITION WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: A DX=0., DY=0., DZ=0.) (THE NODE IS OUTSIDE THE FIELD OF DEFINITION WITH A RIGHT PROFILE OF THE EXCLU TYPE NODE: D DY=0.)

Names of the nodes: $A=N59$ $B=N53$ $C=N12$
$D=N18$ $E=N56$ $X=N70$

3.2 Characteristics of the mesh

Many nodes: 111

Number of meshes and types: 12 HEXA20

3.3 Quantities tested and results

Identification	Reference	*Aster*	% difference
U_B	0.	10^{-22}	-
V_B	0.	10^{-22}	-

W_B	-1.72165510^{-6}	$-1.721674\ 10^{-6}$	0.001
U_C	0.	$= 10^{-14}$	-
V_C	0.	$= 10^{-19}$	-
W_C	$-1.715916\ 10^{-6}$	$-1.715935\ 10^{-6}$	0.001
U_D	$-6.88662\ 10^{-8}$	$-6.88653\ 10^{-8}$	0.001
V_D	0.	$= 10^{-23}$	-
W_D	$5.73885\ 10^{-9}$	$5.71514\ 10^{-9}$	0.413
U_E	0.	$= 10^{-22}$	
V_E	0.	$= 10^{-23}$	
W_E	$-1.291241\ 10^{-6}$	$-1.291260\ 10^{-6}$	0.002
(Pa)			
$\sigma_{zz}\ (A)$	$2.29554\ 10^{5}$	$2.2956\ 10^{5}$	< 0.01
$\sigma_{zz}\ (E)$	$1.14777\ 10^{5}$	$1.14777\ 10^{5}$	< 0.01
$\sigma_{zz}\ (X)$	$2.29554\ 10^{5}$	$2.29549\ 10^{5}$	< 0.01
U_X	0.	10^{-20}	-
V_X	-6.88662 10-8	-6.886534 10-8	-
W_X	5.73885 10-8	5.71514 10-9	0.413

The modelization in HEXA20 are completely acceptable for this coarse mesh.

4 Summary of the results

the results concerning displacements and the forced are very close to the analytical solution with the adopted modelization ($< 0.2\%$ for displacements, $< 0.5\%$ for the stresses).

The fact that there is only one component of the stresses (σ_{zz}) in the problem makes it possible to test only 2 elastic coefficients (E_N and ν_{LN}).

Although these coefficients are constant, they were introduced in the form of functions to test functionality `ELAS_ISTR_FO`.

The elastic coefficients in the plane XY and the direction Z were selected so as to obtain the same values of displacements at the points B C, D and E that those calculated for an isotropic material (test SSLV07) or orthotropic (test SSLV120). Numerically, these values are very close to those of these tests at the points considered (about 10^{-6}) the difference resulting from the method of construction of the stiffness matrixes in the various cases. At the point X, these values differ but correspond well to the reference solution.