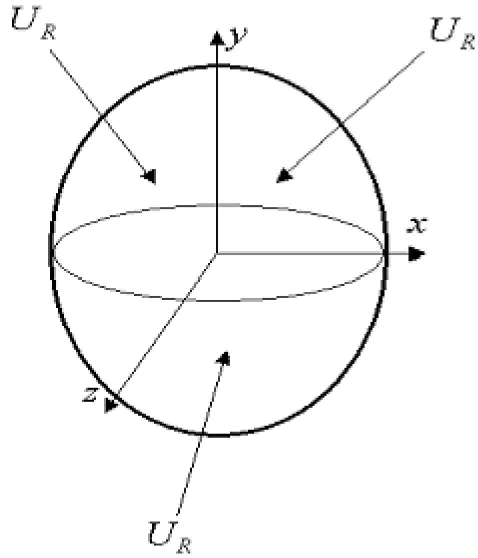
SSLV117 – Validation of the modelization second gradient of thermal expansion in 3D

Summarized:

This test makes it possible to validate the modelization second gradient [R5.04.03] in 3D from analytical solutions. It is about an elastic ball of unit radius subjected to an imposed radial displacement and an internal pressure.

1 Problem of reference

1.1 Geometry



Appears 1.1-the

geometry of structure is a unit ball of radius for the analytical solution.

1.2 Properties of the materials

In the context of the mediums of the second gradient, it is necessary to define materials properties for the parts attached to the first and second gradients of the field of displacement. The properties of the material do not have any physical relevance but were established to simplify the analytical applications.

- Young modulus: $\mathrm{E}=1\,\mathrm{Pa}$;
- Poisson's ratio: $\nu=0$;
- Modulate microscopic stiffness: $a_1=0.25\,\mathrm{Pa.m}^2$.
- Material parameters of the mediums of the second gradient: $a_2=a_3=a_4=a_5=0\,\mathrm{Pa.m}^2$

1.3 Boundary conditions and loadings

For numerical reasons of simplifications, it is not considered that a eighth of the ball by symmetry:

- $DX=0$ on GROUP_MA "SYME_X"
- $DY=0$ on GROUP_MA "SYME_Y"
- $DZ=0$ on GROUP_MA "SYME_Z"

For numerical reasons of simplifications, one does not model the ball but the cube registers this ball for the application of the boundary conditions. One applies thus in the reference $X=(x,y,z)$:

- $U_R(X)=\left(\frac{r^4}{7}-\frac{1}{3}\right)X$ for the components DX , DY and DZ of GROUP_MA " BORD_EXT " where r is the distance between the origin of the ball and the point of coordinate X .

- $G_R(X)=\left(r^4-1\right)$ for components GONF of GROUP_MA " BORD_EXT ".

One applies finally the following internal forces in the reference $X=(x, y, z)$:
$F_I(X)=4(10-r^2)X$ for the components FX , FY and FZ of GROUP_MA " BORD_EXT "

2 Reference solution

2.1 Quantities and results of reference

the analytical solution is the following one:
$\nabla\chi(X)=4r^2X$. where $\nabla\chi(X)$ are components DGONFX1, DGONFX2, DGONFX3 of field SIEF_ELGA.

$\varepsilon_{xx}(X)=\left(\frac{r^4}{7}-\frac{1}{3}\right)+\frac{4r^2}{7}x*x$. where r indicates the distance between the origin of the ball and the point of coordinate $X=(x, y, z)$ (component EPXX of field EPSI_ELGA).
$\varepsilon_{yy}(X)=\left(\frac{r^4}{7}-\frac{1}{3}\right)+\frac{4r^2}{7}y*y$. (component EPYY of field EPSI_ELGA).
$\varepsilon_{zz}(X)=\left(\frac{r^4}{7}-\frac{1}{3}\right)+\frac{4r^2}{7}z*z$. (component EPZZ of field EPSI_ELGA).
$\varepsilon_{xy}(X)=\frac{4r^2}{7}x*y$. (component EPXY of field EPSI_ELGA).
$\varepsilon_{xz}(X)=\frac{4r^2}{7}x*z$. (component EPXZ of field EPSI_ELGA).
$\varepsilon_{yz}(X)=\frac{4r^2}{7}y*z$. (component EPYZ of field EPSI_ELGA).

2.2 Uncertainties on the analytical

solution Solution.

3 Modelization A

3.1 Characteristic of the modelization

the characteristics are identical to the reference solution. The modelization bearing on the first gradient of the field of displacement is 3D and that bearing on the second gradient of thermal expansion is 3D_DIL with the choice of interpolation P2-P1-P1.

3.2 Characteristics of the mesh

the mesh group CUBE_REG is obtained by duplication of the named mesh group CUBE , whose purpose is to accommodate the modelization second gradient of thermal expansion for the regularization.

Many nodes 9009
Many TRIA6 768
Many TETRA1012288
Number of group of mailles6

3.3 Quantities tested and results

the quantities tested relate to the norms of the errors $\theta(\nabla X)$ and $\theta(\varepsilon)$ according to the relations:

$$\theta(\nabla \chi)=\sqrt{\int_{boule}\left(\nabla \chi^{analytique}(X)-\nabla \chi^{numérique}(X)\right)}$$

$$\theta(\varepsilon)=\sqrt{\int_{boule}\left(\varepsilon^{analytique}(X)-\varepsilon^{numérique}(X)\right)}$$

the reference is regarded here as " NON_REGRESSION " as from the moment when the test relates to the norms of the errors $\theta(\nabla X)$ and $\theta(\varepsilon)$ and not on the components ∇X and ε .

Value tested	Urgent	Reference	Criterion	Aster	Tolerance
$\theta(\varepsilon)$	1.0	"NON_REGRESSION"	RELATIF	0.0038977	0.1%
$\theta(\nabla X)$	1.0	"NON_REGRESSION"	RELATIF	0.06	0.1%

4 Summary of the results

This test makes it possible to validate the correct operation of the modelization second gradient of thermal expansion in 3D per comparison with an analytical solution.