SSLV113 - Estimator of error on a hollow roll bi--materials

Summarized:

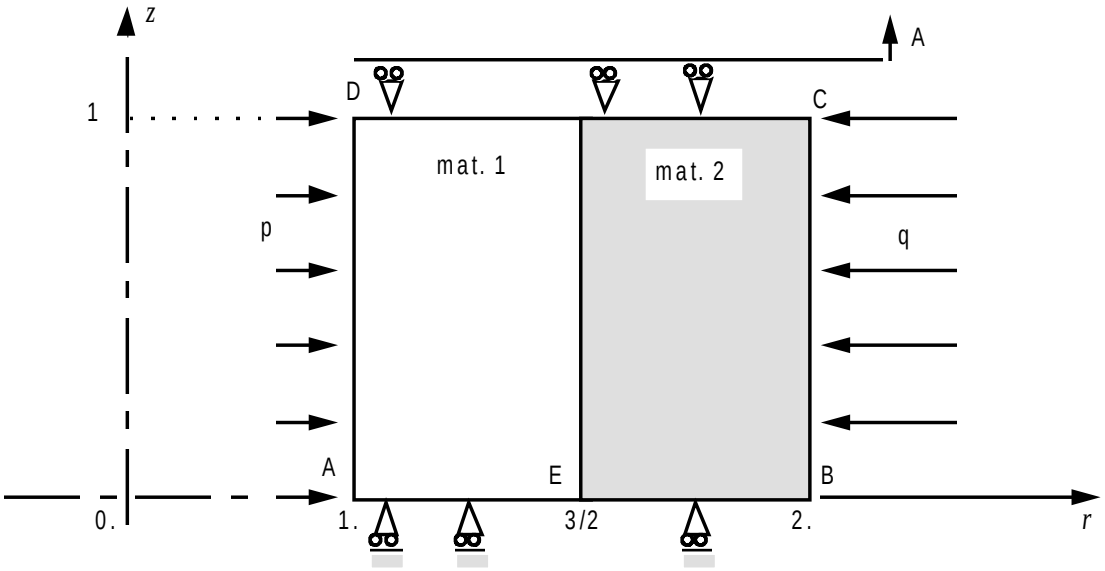
This test 2D validates the estimator of error in pure residue, applied to linear elasticity, in static. One considers a hollow roll made up of two materials and subjected to internal and external pressures.

The 2 modelizations are axisymmetric, on quadrangles with 8 nodes.

The interest of the test lies in the comparison between the exact and calculated stresses, on the one hand, the estimated error and the exact error, on the other hand. This test also makes it possible to show the validity of the estimator in residue on a structure bimatériau, contrary to the estimator of Zhu-Zienkiewicz which is not applicable on structures presenting of discontinuities in the stress field (here with the interface material).

1 Problem of reference

1.1 Geometry



1.2 Material properties

material 1: $E=2.$ $\nu=0.3$
material 2: $E=1.$ $\nu=0.3$

1.3 Boundary conditions and loadings

On AB , $U_Z=0.$

on DC $U_Z=0.91333=A$.

Pressure interns on AD $p=1$.

External pressure on BC $q=2$.

2 Reference solution

2.1 Method of calculating used for the reference solution

$$\mu_i = \frac{E_i}{2(1+\nu)}$$
$$\lambda_i = \frac{\nu E_i}{(1-2\nu)(1+\nu)}$$

$$\left.\begin{array}{llll} a_1 = -0.98097 & b_1 & -1.11741 \\ a_2 = -1.34405 & b_2 & -0.30048 \end{array}\right\}$$ Numerical data calculated starting from the equations of Navier

For the material i , one a:

$$u_r = a_i r + \frac{b_i}{r}$$
$$u_z = A$$

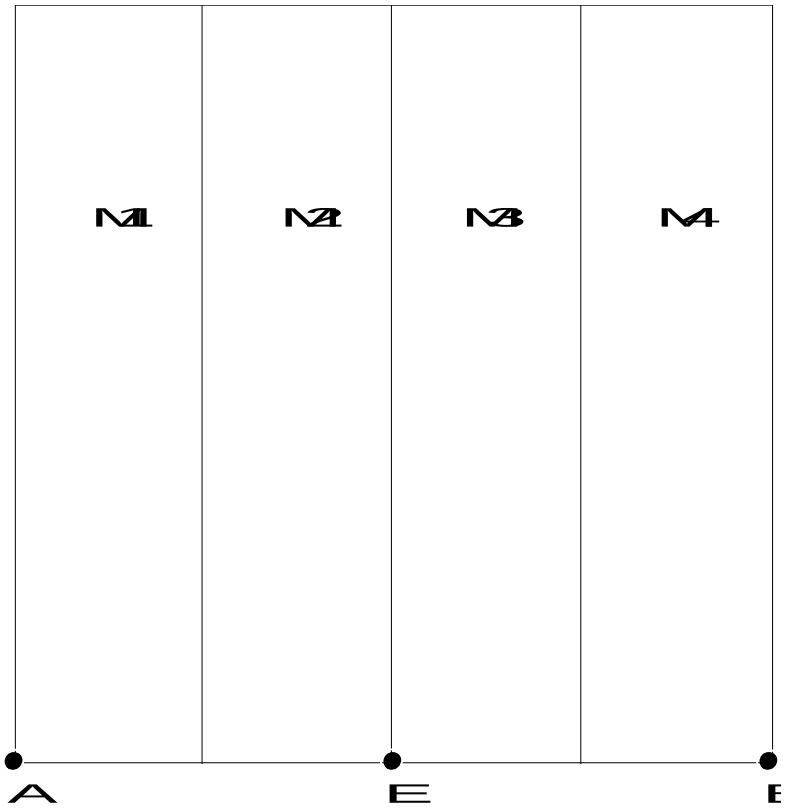
$$\left\{\begin{array}{l} \sigma_{rr} = \lambda_i(2a_i+A)+2\mu_i\left(a_i-\frac{b_i}{r^2}\right) \\ \sigma_{rr} = \lambda_i(2a_i+A)+2\mu_i\left(a_i+\frac{b_i}{r^2}\right) \\ \sigma_{rr} = 2\lambda_i a_i+(\lambda_i+2\mu_i)A \end{array}\right.$$

2.2 Uncertainty on the analytical

solution Solution.

3 Modelization A

3.1 Characteristic of the modelization



3.2 Characteristics of the mesh

Many nodes: 23.

Number of meshes and types: 4 QUAD8.

3.3 Results and quantities tested

Identification		Reference	*Aster*	% difference	tolerance
A	σ_{rr}	– 1.00003	– 1.06833	6.83	7.0
	$\sigma_{\theta\theta}$	– 4.43821	– 4.46731	0.66	2.0
	σ_{zz}	0.19518	0.16596	14.9	15.0
	e_{rel}		2.37%		5.0
E mat. 1	σ_{rr}	– 1.95508	– 1.97893	1.22	2.0
	$\sigma_{\theta\theta}$	– 3.48316	– 3.49330	0.29	2.0
	σ_{zz}	0.19518	0.18498	5.22	6.0
	e_{rel}		1.05%		5.0
E mat. 2	σ_{rr}	– 1.95508	– 1.98398	1.48	2.0
	$\sigma_{\theta\theta}$	– 2.16049	– 2.13394	1.23	2.0

	σ_{zz}	– 0.32135	– 0.32204	0.22	2.0
	e_{rel}		0.152%		5.0
B	σ_{rr}	– 1.99999	– 2.00095	0.048	2.0
	$\sigma_{\theta\theta}$	– 2.11555	– 2.11595	0.012	2.0
	σ_{zz}	– 0.32135	– 0.32174	0.12	2.0
	e_{rel}		0.057%		5.0

3.4 Remarks

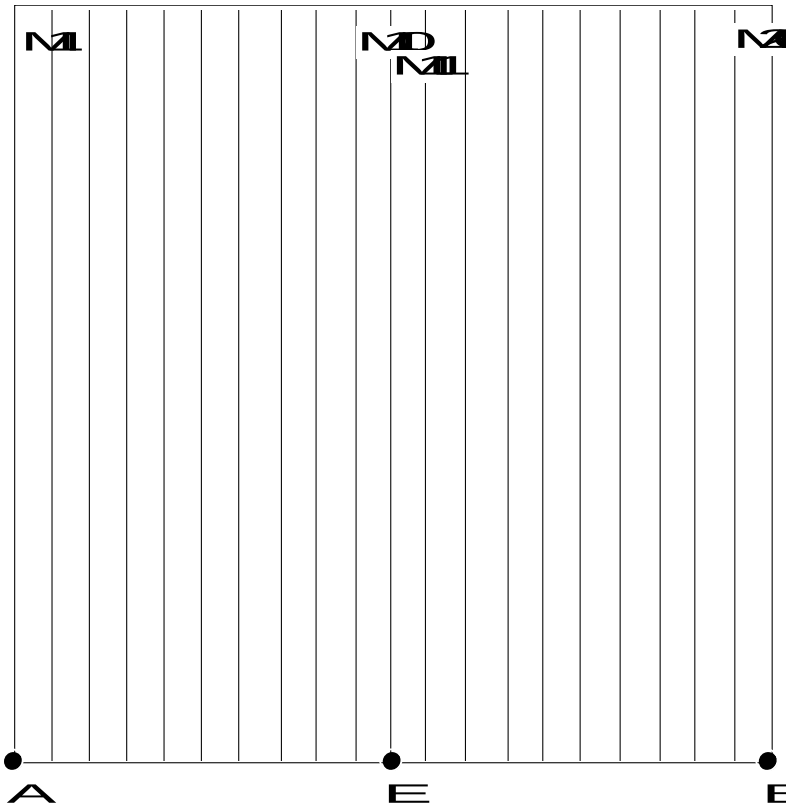
The mesh being coarse (4 elements according to Or), certain stresses close to the axis of axisymetry are badly approximated. The jump of $\sigma_{\theta\theta}$ and σ_{zz} with the interface of the 2 materials is on the other hand well detected.

3.5 Remarks

relative Error considered total = 1.40%.

4 Modelization B

4.1 Characteristic of the modelization



4.2 Characteristics of the mesh

Many nodes:

Number of meshes and types: 20 QUAD8.

4.3 Results and quantities tested

Identification		Reference	*Aster*	% difference	tolerance
A	σ_{rr}	– 1.00003	– 1.00351	0.35	0.5
	$\sigma_{\theta\theta}$	– 4.43821	– 4.43970	0.034	0.05
	σ_{zz}	0.19518	0.19369	0.76	0.8
	e_{rel}		0.57%		0.6
E mat. 1	σ_{rr}	– 1.95508	– 1.95583	0.039	0.05
	$\sigma_{\theta\theta}$	– 3.48316	– 3.48347	0.009	0.01
	σ_{zz}	0.19518	0.19486	0.16	0.2
	e_{rel}		0.14%		0.2
E mat. 2	σ_{rr}	– 1.95508	– 1.96166	0.34	0.5
	$\sigma_{\theta\theta}$	– 2.16049	– 2.15403	0.299	0.5

	σ_{zz}	– 0.32135	– 0.32138	0.009	0.01
	e_{rel}		0.027%		0.03
B	σ_{rr}	– 1.99999	– 2.00003	0.002	0.01
	$\sigma_{\theta\theta}$	– 2.11555	– 2.11558	0.001	0.01
	σ_{zz}	– 0.32135	– 0.32135	0.002	0.01
	e_{rel}		0.0084%		0.01

4.4 Remark

relative Error considered total = 0.24%.

5 Summary of the results

the estimator of error in residue ERRE_ELEM_SIGM gives good performances on the problems out of bi--materials.

Note:

The estimator of error of Zhu-Zienkiewicz does not give correct results. Indeed, with the interface it detects a strong error because it carries out a continuous lissage of the stresses whereas there exists a jump for σ_{zz} *and* $\sigma_{\theta\theta}$ *.*