SSLV110 - elliptic Crack in a Summarized infinite

medium:

It is about a test in static for a three-dimensional problem. This test makes it possible to calculate total and local rate of energy restitution on the crack tip by the method theta (command `CALC_G`).

Radius of integration contours are variable along crack, and local rate of energy restitution is calculated according to 3 different methods (`LEGENDRE, LAGRANGE` and `LAGRANGE_REGU`).

The interest of the test is the validation of the method theta in $3D$ and of the following points:

- comparison between the results and an analytical solution,
- stability of the results according to integration contours,
- comparison between 3 methods different for computation from G room,
- 2 cases of equivalent loadings (distributed pressure and voluminal loading).

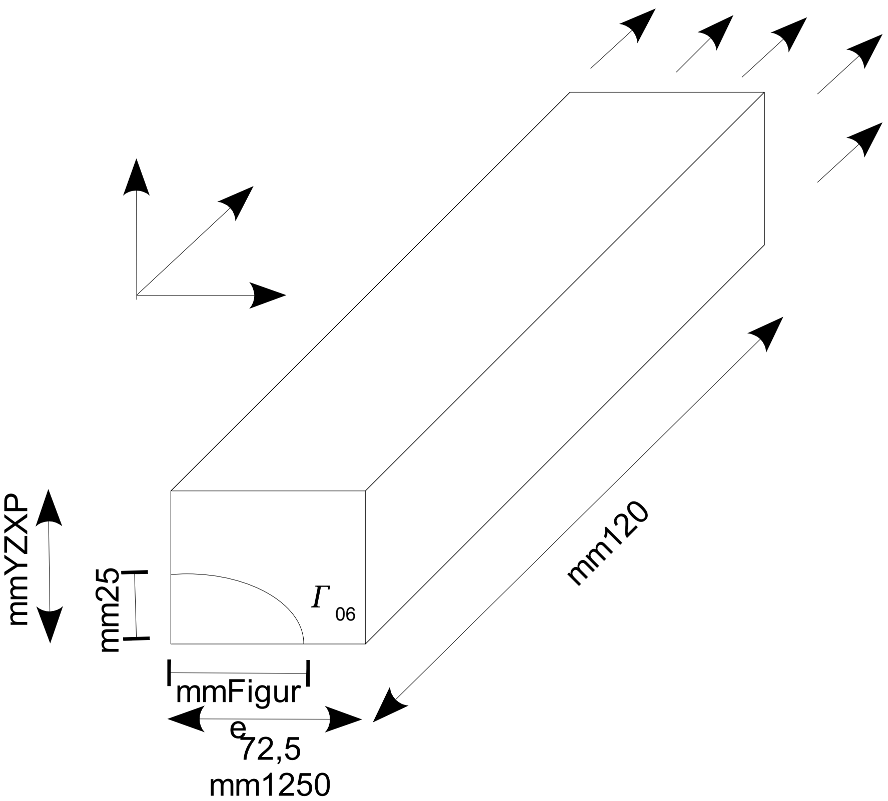
This test contains 5 different modelizations.

The modelization E is a data-processing validation of the taking into account of various loadings applied to the lips of crack in the computation of G.
The modelization F the computation of for $K1$ a crack nonwith a grid (method X-FEM) tests. It also makes it possible to compare the mistakes made on the computation of $K1$ with the operator `POST_K1_K2_K3` or operator `CALC_G`.

1 Problem of reference

1.1 Geometry

It acts of an elliptic crack plunged in a presumedly infinite medium. Only one eighth of a parallelepiped is modelled:



1.1-1: geometry and elliptic crack tip

1.2 Materials properties

$E=210\,000.\text{MPa}$

$\nu=0.3$

1.3 Boundary conditions and loadings

Symmetry compared to the 3 principal planes:

$U_x=0.$ in the plane $X=0$.

$U_Y=0.$ in the plane $Y=0$.

$U_Z=0.$ in the plane $Z=0.$ out of crack

the conditions of loadings are is:

$P=1\,MPa$ in the plane $Z=1250\,mm$ (modelizations A and B)

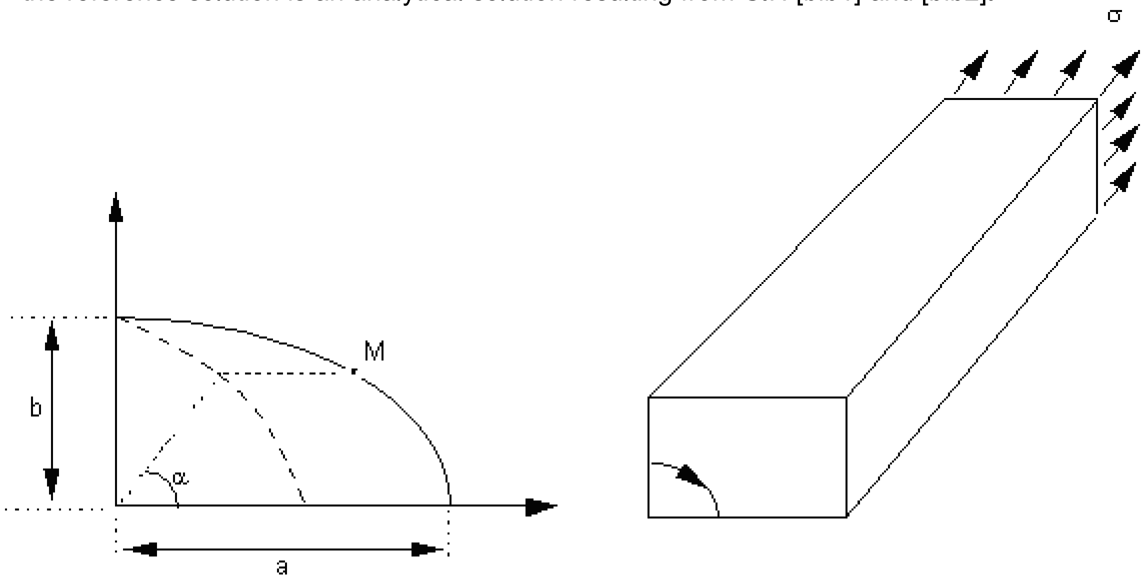
that is to say:

$FZ=8.10^{-4} N/mm^3$ on all the volume elements (loading are equivalent to the precedent) (modelizations C and D).

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is an analytical solution resulting from SIH [bib1] and [bib2].



It is noted that the angle α indicates here the parametric angle of the point M (angle compared to the axis Ox of the project of M on the circle of radius b) and not the polar coordinate of this point.

Here: $a=25\,mm$ and $b=6\,mm$, therefore $k=0{,}9707728$

the values of the elliptic integral $E(k)$ are tabulated in [bib3], according to $asin(k)$ which is worth here $76{,}11°$. One finds then: $E(k)=1{,}0672$.

From where the factor of intensity of the stresses in $MPa.\sqrt{mm}$:

$$K_I(\alpha)=4.0680\left[\sin^2\alpha+\frac{b^2}{a^2}\cos^2\alpha\right]^{1/4}$$

Then, from the formula of Irwin (plane strain):

The total rate of refund of energy G_{ref} is calculated by integration of $G(\alpha)$:

$G_{ref}=5{,}76.10^{-3}\,J/mm$.

2.2 Bibliography

1) G.C. SIH: Mathematical Theories of Brittle Fractures - FRACTURE, flight II - Academic Close - 1968

2) M.K. KASSIN and G.C. SIH: Three-dimensional stress distribution around year elliptical ace under arbitrary loadings J. Appl. Mech., 88,601-611, 1966.

3) H. TADA, P.PARIS, G. IRWIN: The Stress Analysis of Cracks Handbook - Third Edition - International ASM - 2000

3 Modelization A

3.1 Characteristic of the modelization

$A=N01099$ ($s=0.$)
$B=N01259$ ($s=26.68$)
$C=N01179$ ($s=17.8$; $\alpha=\pi/4$)

Loading: Unit pressure distributed on the face of the block opposed to the plane of the lip:

$P=1.\mathrm{MPa}$ in the plane $Z=1250.\mathrm{mm}$.

3.2 Characteristics of the mesh

Many nodes: 1716
Number of meshes and types: 304 `PENTA15` and 123 `HEXA20`

3.3 Quantities tested and results

the values tested are:

- total rate of energy restitution G ,
- local rate of energy restitution g in all the nodes of the crack tip.

  The mesh only one of the lips of crack understands, it is thus necessary to use key word "`SYME`" automatically to multiply by 2 in Aster computation the rate of refund of energy calculated by virtual extension of the single lip.

  In the same way, G the total one calculated here corresponds to the quarter of G reference defined previously, only a eighth of parallelepiped being represented.

Identification	**Reference**	$\%$ **tolerance**
G Crowns C_1	1.44 10-3	-2.1
G Contour C_2	1.44 10-3	0.8
G Contour C_3	1.44 10-3	-1.1
$g(A)$ contour C_1 **("LEGENDRE")**	7.171 10-5	-4.8
$g(A)$ contour C_2 **("LEGENDRE")**	7.171 10-5	0.95
$g(A)$ contour C_3 **("LEGENDRE")**	7.171 10-5	-4.3
$g(B)$ contour C_1 **("LEGENDRE")**	1.721 10-5	-13.8
$g(B)$ contour C_2 **("LEGENDRE")**	1.721 10-5	-8.7
$g(B)$ contour C_3 **("LEGENDRE")**	1.721 10-5	-6.9
$g(C)$ contour C_1 **("LEGENDRE")**	5.215 10-5	-4.3
$g(C)$ contour C_2 **("LEGENDRE")**	5.215 10-5	-1.7
$g(C)$ contour C_3 **("LEGENDRE")**	5.215 10-5	-3.9
$g(A)$ contour C_1 **("LAGRANGE_REGU")**	7.171 10-5	-7.7

$g(A)$ contour C_2 **("LAGRANGE_REGU")**	7.171 10-5	-3.3
$g(A)$ contour C_3 **("LAGRANGE_REGU")**	7.171 10-5	-4.3
G (C) contour C1 **("LAGRANGE_REGU")**	5.215 10-5	-3.5
G (C) contour C2 **("LAGRANGE_REGU")**	5.215 10-5	-1
G (C) contour C3 **("LAGRANGE_REGU")**	5.215 10-5	-2.3

3.4 Remark

the results are rather stable between contours safe at the point B where the variation of $g(s)$ is larger and the results far away from the reference solution. One can explain this variation by the poor mesh of quality. Lissages "LEGENDRE" and "LAGRANGE_REGU" provide relatively close results.

4 Modelization B

4.1 Characteristic of the modelization

$A = N01099$ ($s = 0.$)
$B = N01259$ ($s = 26.68$)
$C = N01179$ ($s = 17.8$)

Loading: Unit pressure distributed on the face of the block opposed to the plane of the lip:

$P = 1\,MPa$ in the plane $Z = 1250\,mm$.

4.2 Characteristics of the mesh

Many nodes: 1716
Number of meshes and types: 304 PENTA15 and 123 HEXA20

4.3 Quantities tested and results

the values tested are:

- total rate of energy restitution G ,
- local rate of energy restitution g in all the nodes of the crack tip.

The mesh only one of the lips of crack understands, it is thus necessary to use key word "SYME" automatically to multiply by 2 in Aster computation the rate of refund of energy calculated by virtual extension of the single lip.

In the same way, G the total one calculated here corresponds to the quarter of G reference defined previously, only a eighth of parallelepiped being represented.

Identification	**Reference**	**% tolerance**
G Crowns C_1	1.44 10-3	-2.1
G Contour C_2	1.44 10-3	0.8
G Contour C_3	1.44 10-3	-1.1
$g(A)$ contour C_1	7.171 10-5	-0.7
$g(A)$ contour C_2	7.171 10-5	3.9
$g(A)$ contour C_3	7.171 10-5	3.6
$g(B)$ contour C_1	1.721 10-5	-6.6
$g(B)$ contour C_2	1.721 10-5	-3.4
$g(B)$ contour C_3	1.721 10-5	-0.9
$g(C)$ contour C_1	5.215 10-5	-4.5
$g(C)$ contour C_2	5.215 10-5	-2.3
$g(C)$ contour C_3	5.215 10-5	-3.9

Remark

the results are better than in the modelization A at the point B, but the disparity between contours remains strong.

5 Modelization D

5.1 Characteristic of the modelization

$A=N01099$ ($s=0.$)
$B=N01259$ ($s=26.68$)
$C=N01179$ ($s=17.8$)

Loading: Volume force Fz equivalent to a unit pressure distributed on the face of the block opposed to the plane of the lip:

`FORCE_INTERNE: FZ` = $8.10^{-4} N/mm^3$ on all the volume elements.

5.2 Characteristics of the mesh

Many nodes: 1716
Number of meshes and types: 304 `PENTA15` and 123 `HEXA20`

5.3 Quantities tested and results

the values tested are:

- total rate of energy restitution G,
- local rate of energy restitution g in all the nodes of the crack tip.

The mesh only one of the lips of crack understands, it is thus necessary to use key word "`SYME`" automatically to multiply by 2 in Aster computation the rate of refund of energy calculated by virtual extension of the single lip.

In the same way, G the total one calculated here corresponds to the quarter of G reference defined previously, only a eighth of parallelepiped being represented.

Identification	Reference	% tolerance
G Crowns C_1	1.44 10-3	-0.2
G Contour C_2	1.44 10-3	2.7
G Contour C_3	1.44 10-3	0.7
$g(A)$ contour C_1	7.171 10-5	1.2
$g(A)$ contour C_2	7.171 10-5	5.9
$g(A)$ contour C_3	7.171 10-5	5.7
$g(B)$ contour C_1	1.721 10-5	– 4.9
$g(B)$ contour C_2	1.721 10-5	– 1.7
$g(B)$ contour C_3	1.721 10-5	0.7
$g(C)$ contour C_1	5.215 10-5	– 2.7
$g(C)$ contour C_2	5.215 10-5	0.4
$g(C)$ contour C_3	5.215 10-5	– 2.1

Remark

the results are better than in the modelization C at the point B .

6 Modelization E

The mesh is the same one as that of modelization D.

the goal of this modelization is only data-processing: to test that command CALC_G functions well for loads of pressure on the lips of crack. The pressure is modelled in 3 different ways:

- a pressure function (AFFE_CHAR_MECA_F/PRES_REP),
- a constant distributed force (AFFE_CHAR_MECA/FORCE_FACE)
- and a distributed force function (AFFE_CHAR_MECA_F/FORCE_FACE).

It should be noted that one only mechanical resolution is carried out with a load of constant pressure, and that the 3 various loads detailed above are transmitted to 3 CALC_G different *via* key word EXCIT.

6.1 Quantities tested and results

the values tested are:

- total rate of energy restitution G ,
- local rate of energy restitution g with the node A of the crack tip (in $s=0$).

The choice of contours for the method theta is that of contour n°2 of the modelization D (contour C_2).

G The total one calculated here corresponds to the quarter of G reference defined previously because the loading is symmetric.

Standard	identification of reference	Value of reference	Tolerance
G pressure function	"ANALYTIQUE"	1,44 10-3	1.00%
G pressure function	"NON_REGRESSION"	1,449052 10-3	10-4%
G constant force	"AUTRE_ASTER"	1,449052 10-3	$10^{-4\%}$
G force function	"AUTRE_ASTER"	1,449052 10-3	$10^{-4\%}$
$g(A)$ pressure function	"ANALYTIQUE"	7,16 10-5	3.00%
$g(A)$ pressure function	"NON_REGRESSION"	6,98287 10-5	10-4%
$g(A)$ constant force	"AUTRE_ASTER"	6,98287 10-5	$10^{-4\%}$
$g(A)$ force function	"AUTRE_ASTER"	6,98287 10-5	$10^{-4\%}$

7 Modelization F

7.1 Characteristic of the modelization

In this modelization, the crack is not with a grid. The method X-FEM is used.

Taking into account symmetries of the problem, it is possible to represent only one eighth of structure (as that is done in the modelization A). However, with the method X-FEM, it is not possible to represent a crack which is located in a symmetry plane (on edge of the modelled field). One thus models in this modelization a quarter of structure, i.e. a portion of $90°$ ellipse.

The mesh is composed of meshes HEXA8, uniformly distributed along the axes X and Y divided into geometric progression along the axis Z so that in the plane $Z=0$, meshes are approximately cubes of with dimensions $10\,mm$.

Conditions of symmetry are applied to the sides in $X=0$ and $Y=0$. The rigid mode following the axis Z is blocked by blocking the following displacement Z of the point located in $(0{,}0,-1250\,mm)$.

Loading: Unit pressure distributed on the two normal sides of the block:

$P=1\,MPa$ in the plane $Z=\pm 1250\,mm$.

7.2 Characteristics of the mesh

Many nodes: 21000
Number of meshes and types: 13000 PENTA6 and 12500 HEXA8 (linear mesh)

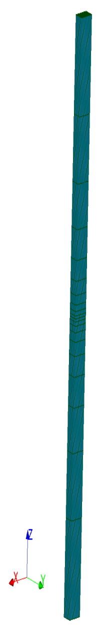


Figure 7.2-1 : initial mesh, overall picture

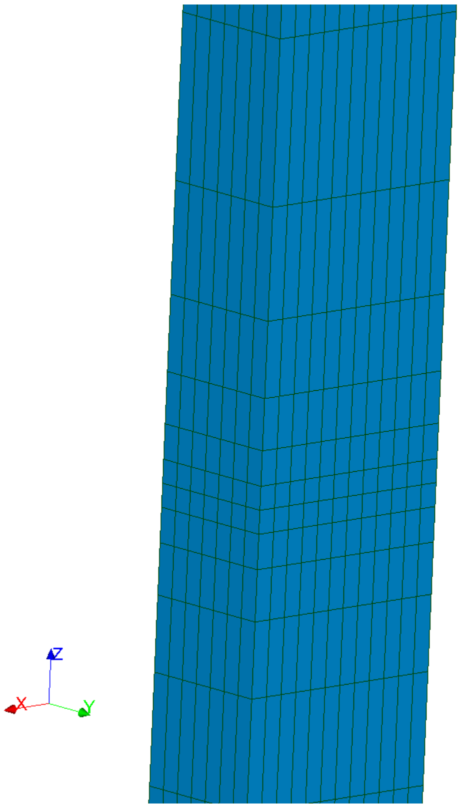


Figure 7.2-2: initial mesh, zoom in medium plane

As this initial mesh is well too coarse for a precise computation of the stress intensity factors along the bottom of crack, an automatic procedure of refinement of meshes close to the crack tip is used, as recommended in documentation [U2.05.02].

The size targets meshes wished is $b/9$. That will induce 5 successive refinements. The size of meshes of the mesh thus refined is then $h=0{,}39\,mm$.

The mesh refined (that on which mechanical computation is carried out) has as characteristics:

- 26484 nodes
- 7720 TETRA4, 10650 PYRAM5 and 20080 HEXA8

This mesh induce 99 points along the crack tip and taking into account the conditions of blocking 118404 equations in the system to solve

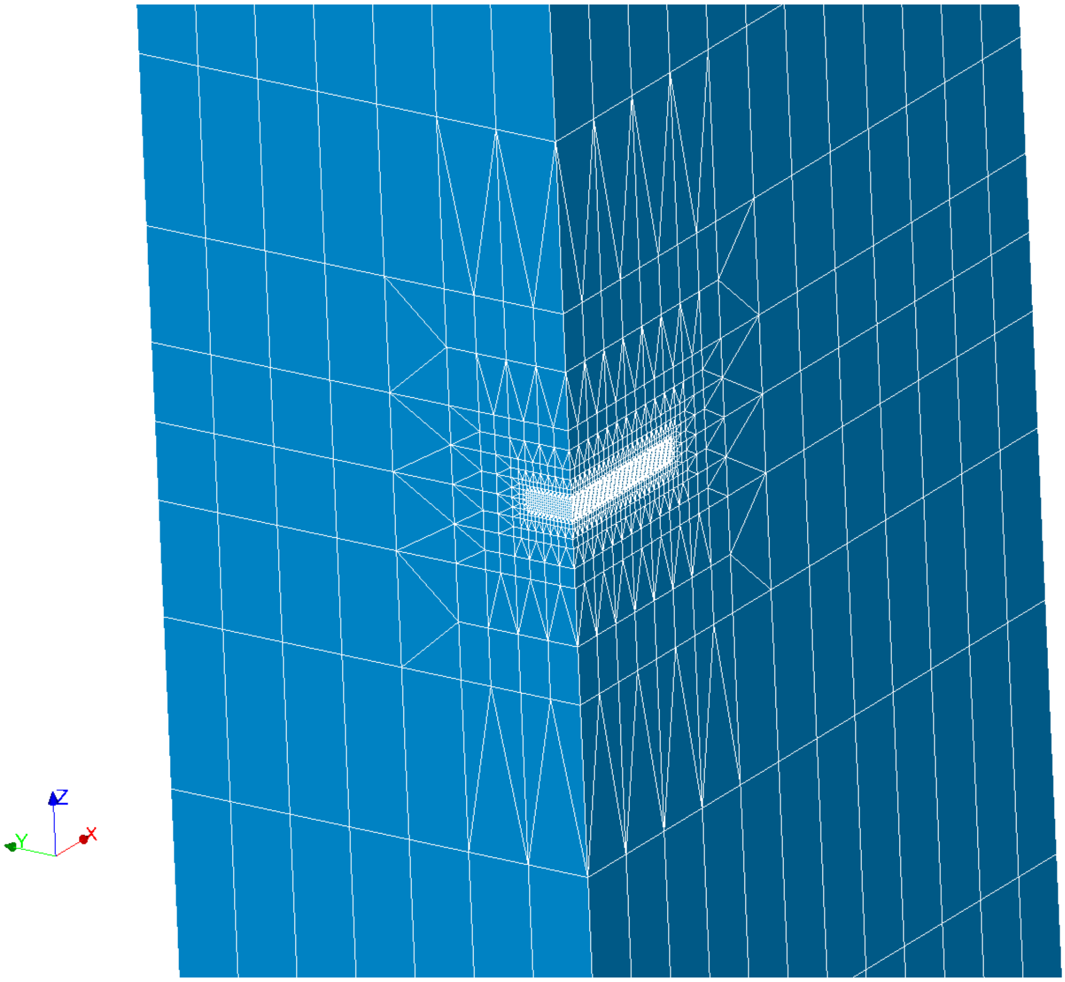


Figure 7.2-3: mesh refined, zoom on the zone close to the crack

7.3 Quantities tested and results

the values tested are the factors of intensity of the stresses $K1$ along the crack tip, calculated either by CALC_G, or by POST_K1_K2_K3.

For CALC_G, integration contour is worth $2h-4h$. The lissage by default (Legendre) is used.
For POST_K1_K2_K3, maximum curvilinear X-coordinate is worth $4h$. In order to reduce the computing times of POST_K1_K2_K3, one post-draft that on 20 points distributed uniformly along the crack tip.
Let us note that the computing time for the Legendre lissage of CALC_G is insensitive to this number.

One tests the values at the points A ($s=0$) and B ($s=26,7$).

Standard	identification of reference	Value of reference	Tolerance
CALC_G : $K1(A)$	"ANALYTIQUE"	1,9929	2.0%
CALC_G : $K1(B)$	"ANALYTIQUE"	4.068	2.0%
POST_K1_K2_K3 : $K1(A)$	"ANALYTIQUE"	1,9929	6.0%
POST_K1_K2_K3 : $K1(B)$	"ANALYTIQUE"	4.068	6.0%

For operator CALC_G, the lissages of the type LAGRANGE do not allow to have easily useable results; a lissage of the type LEGENDRE is thus to privilege.

8 Summary of the Computation

results of G or K room:

- for a crack with a grid, 3 methods (LEGENDRE, LAGRANGE and LAGRANGE_REGU) appreciably give the same results (less 5% of analytical error compared to the solution) except to the point B (not end of the ellipse on the large axis) where the Lagrange method is most precise;
- loading case: the values obtained with the voluminal loading are slightly higher than those obtained with imposed stresses (including for the values of G). The differences are tiny and due to numerical integrations different on the term from volume and the term of edge;
- the method X-FEM makes it possible to evaluate the factors of intensity of the stresses K on a mesh not fissured with an error lower than 10% .