SSLV104 - Beam in rotation

Summarized:

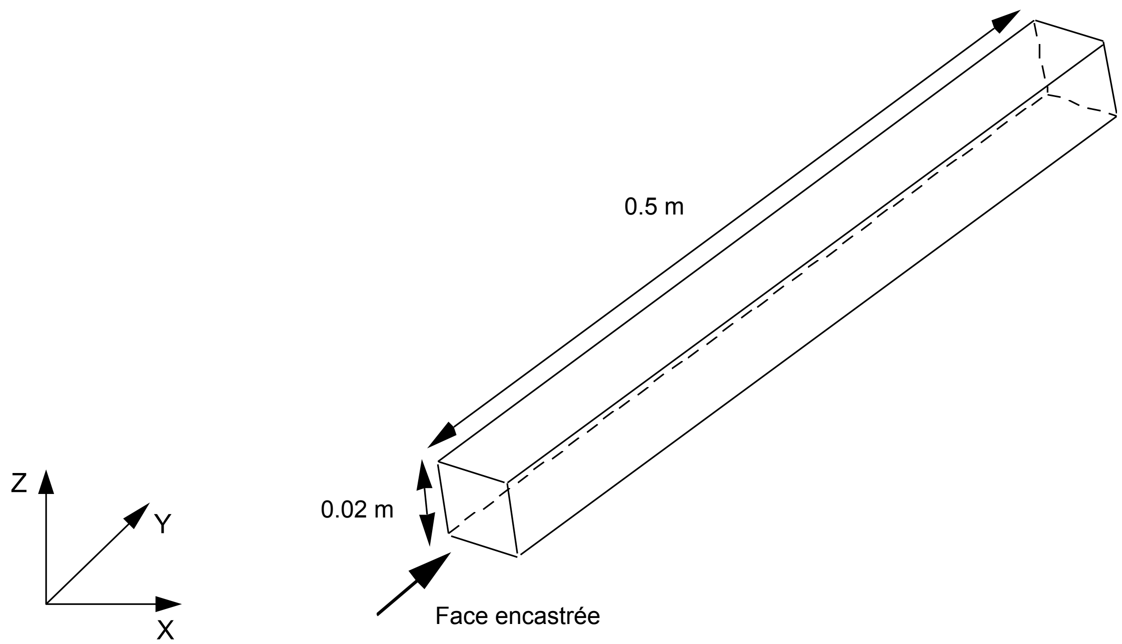
This test makes it possible to validate the linear elastic design of a slender beam subjected to a rotation of one of its ends. Four modelizations are tested: elements `3D` (HEXA20), elements `COQUE_3D` (QUAD9 and TRIA7) and elements `D_PLAN` (QUAD8). That tests the inertia forces of rotation, without taking account of the elementary terms of centrifugal stiffening (cf [V3.04.105]).

The reference solution is analytical (1D). The results coincide perfectly with the reference solution.

1 Problem of reference

1.1 Geometry

directed slender Beam carried in space by the axis of directing vector $(1,1,1)$.



Square section of area: $4.0\,10^{-4}m^2$

Length of beam: $0.5\,m$

1.2 Material properties

$E=2.10^{11}Pa$

$\nu=0$

$\rho=7800\,kg/m^3$

$\text{A\_CIS}=0.8333$ (factor of correction of transverse shears equal to $5/6$ for a theory of the type Reissner thin shell)

1.3 Boundary conditions and loadings

free clamped Beam in rotation around an axis perpendicular to its greater dimension and passing by the center of the clamped face.

Component of the vector rotation: $(1,0,-1)$.

Rotational speed: $\omega=3000\,rd/s$.

The important value rotational speed does not have anything physics.

2 Reference solution

2.1 Method of calculating used for the reference solution

In the local coordinate system of beam:

$$\frac{\partial^2 U_x}{\partial x^2}+\frac{\rho}{E}\omega^2 x=0 \quad \text{with} \quad \begin{array}{l} U_x(0)=0 \\ \dfrac{\partial U_x}{\partial x}(L)=\sigma_{xx}(L)=0 \end{array}$$

By integrating the preceding differential equation one obtains, in the reference of beam:

$$U_x(x)=\frac{\rho\,\omega^2}{2E}\left(x\,L^2-\frac{x^3}{3}\right) \quad U_y=U_z=0$$

Displacements of all points of the beam are thus written in the total reference:

$$U_x(X,Y,Z)=\frac{\rho\,\omega^2}{2\sqrt{3}\,E}\left(r\,L^2-\frac{r^3}{3}\right)$$
$$U_y(X,Y,Z)=\frac{\rho\,\omega^2}{2\sqrt{3}\,E}\left(r\,L^2-\frac{r^3}{3}\right)$$
$$U_z(X,Y,Z)=\frac{\rho\,\omega^2}{2\sqrt{3}\,E}\left(r\,L^2-\frac{r^3}{3}\right)$$
with $r=\sqrt{X^2+Y^2+Z^2}$

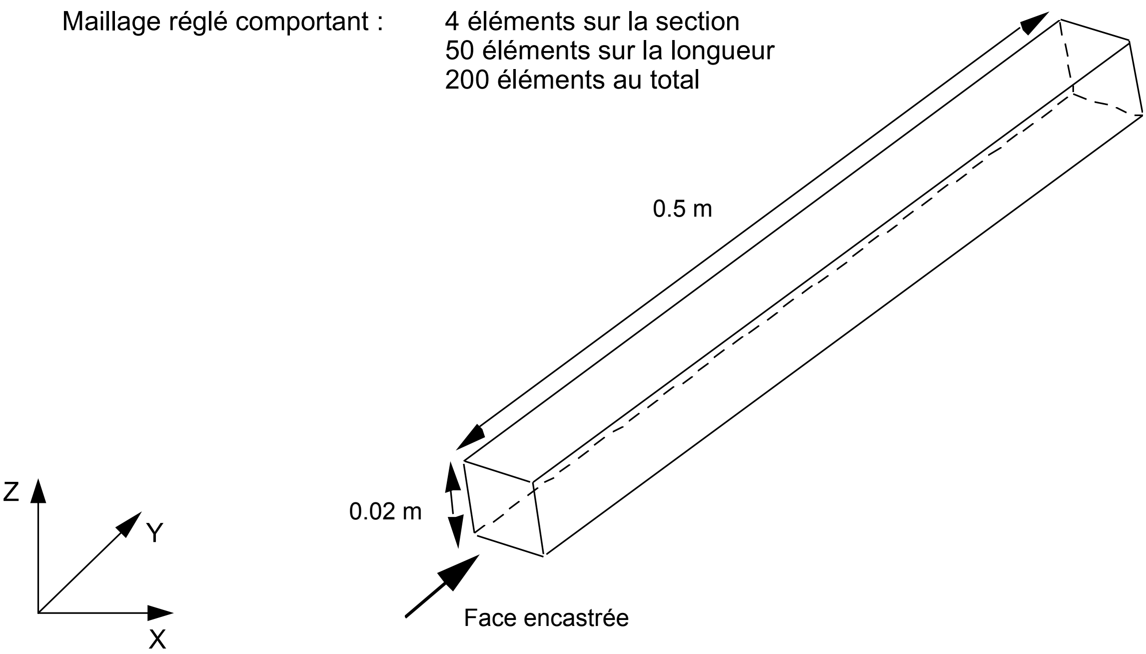
2.2 Results of reference

Values of three displacements to the center of the section furthest away from the rotational axis.

3 Modelization A

3.1 Characteristic of the modelization

Elements 3D (HEXA20)



3.2 Characteristic of the mesh

Many nodes: 1521

Number of meshes and types: 200 HEXA20

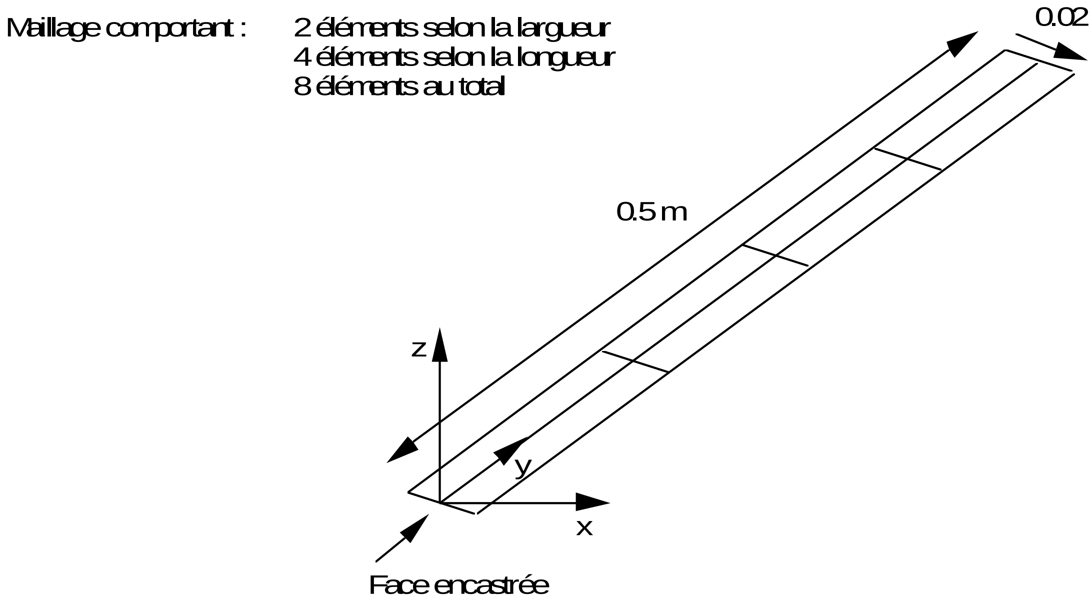
3.3 Quantities tested and results

Identification	Reference
DX in L	8.44 10-3
DY in L	8.44 10-3
DZ in L	8.44 10-3

4 Modelization B

4.1 Characteristic of the modelization

Shell elements `MEC3QU9H`



4.2 Characteristic of the mesh

Many nodes: 45

Number of meshes and types: 8 QUAD9

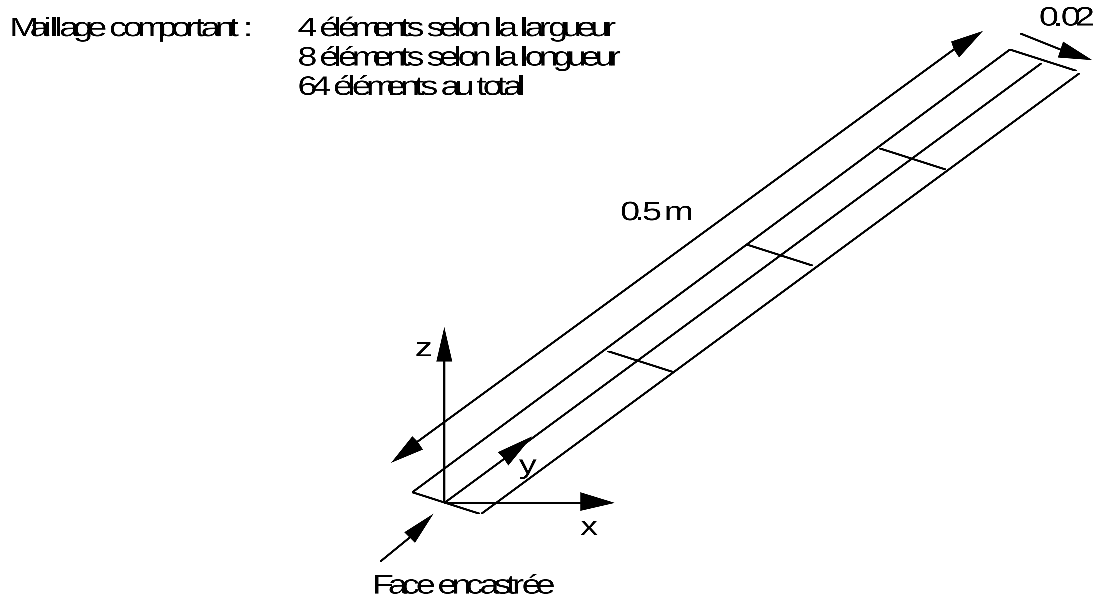
4.3 Quantities tested and results

Identification	Reference
DX in L	8.44 10-3
DY in L	8.44 10-3
DZ in L	8.44 10-3

5 Modelization C

5.1 Characteristic of the modelization

Shell elements **MEC3TR7H**



5.2 Characteristic of the mesh

Many nodes: 217

Number of meshes and types: 64 TRIA7

5.3 Quantities tested and results

Identification	Reference
DX in L	8.44 10-3
DY in L	8.44 10-3
DZ in L	8.44 10-3

6 Modelization D

6.1 Characteristic of the modelization

Elements D_PLAN `MEDPQU8`

Mesh comprising 2 elements according to the width, 50 elements according to the length.
100 elements on the whole.
In 2D, displacements of the beam are written :

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$$U_x(X,Y)=\frac{\rho\omega^2}{2\sqrt{2}E}\left(rL^2-\frac{r^3}{3}\right)$$

$$U_y(X,Y)=\frac{\rho\omega^2}{2\sqrt{2}E}\left(rL^2-\frac{r^3}{3}\right)$$

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with $r=\sqrt{X^2+Y^2}$

6.2 Characteristics of the mesh

Many nodes: 405

Number of meshes and types: 100 QUAD8

6.3 Quantities tested and results

Identification	Reference
DX into L	10.341 10-3
DY into L	10.341 10-3

7 Summary of the results

the coincidence of the results with the analytical solution makes it possible to validate the loading due to the inertia forces of rotation.

The modelization `COQUE_3D` with `MEC3QU9H` gives the solution with very few elements.

One will refer to test SSLV105 [V3.04.105] 3D to evaluate the effect of the centrifugal stiffening for the element, HEXA20.