

## SSLV100 - Hollow roll in plane strains

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### Summarized:

This test makes it possible to validate the elements of plane strain on the following features:

- imposed distributed pressure
- , stiffness matrix
- , displacements:
  - by d.o.f.,
  - face of element.

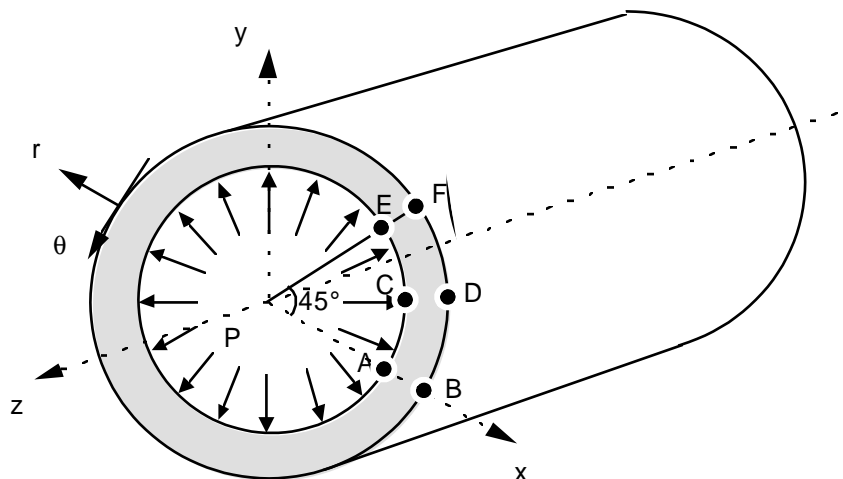
It understands 4 modelizations.

The 3 first correspond to elements of the different type (linear and quadratic).

The last validates the displacements imposed by face (blocking of the normal component).

## 1 Problem of reference

### 1.1 Geometry



Rayon interne             $a = 0.1 \text{ m}$   
Rayon externe          $b = 0.2 \text{ m}$

Coordinated of the points:

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>
<i>x</i>	0.100	0.200	$0.1 \cos(22.5)$	$0.2 \cos(22.5)$	$1/\sqrt{2}$	$\sqrt{2}$
<i>y</i>	0.	0.	$0.1 \sin(22.5)$	$0.2 \sin(22.5)$	$1/\sqrt{2}$	$\sqrt{2}$
<i>z</i>	0	0.	0.	0.	0.	0.

### 1.2 Properties of the materials

$$E = 210^5 \text{ Mpa}$$

$$\nu = 0.3$$

### 1.3 Boundary conditions and loadings

internal Pressure:  $P = 60. \text{ MPa}$

## 2 Reference solution

### 2.1 Method of calculating used for the Analytical reference solution

$$\begin{aligned}\sigma_{zz} &= 2\nu P \frac{a^2}{b^2 - a^2} \\ \sigma_{rr} &= P \frac{a^2}{b^2 - a^2} \left[ 1 - \frac{b^2}{r^2} \right] \\ \sigma_{\theta\theta} &= P \frac{a^2}{b^2 - a^2} \left[ 1 + \frac{b^2}{r^2} \right] \\ \sigma_{r\theta} &= 0 \\ u_r &= \frac{P}{E} \frac{a^2}{b^2 - a^2} (1 + \nu) \left[ (1 - 2\nu) + \frac{b^2}{r^2} \right] r\end{aligned}$$

One obtains:

For $r=0.1$	$u_r = 5,72 \cdot 10^{-5}$	For $r=0.2$	$u_r = 3,64 \cdot 10^{-5}$
	$\sigma_{rr} = -60.$		$\sigma_{rr} = 0.$
	$\sigma_{\theta\theta} = 100.$		$\sigma_{\theta\theta} = 40.$
	$\sigma_{zz} = 12.$		$\sigma_{zz} = 12.$
	$\sigma_{r\theta} = 0.$		$\sigma_{r\theta} = 0.$

Transition in the system of Cartesian axes:

$$\begin{aligned}\sigma_{xx} &= \sigma_{rr} \cos^2 \theta + \sigma_{\theta\theta} \sin^2 \theta - 2 \sigma_{r\theta} \sin \theta \cos \theta \\ \sigma_{yy} &= \sigma_{rr} \sin^2 \theta + \sigma_{\theta\theta} \cos^2 \theta + 2 \sigma_{r\theta} \sin \theta \cos \theta \\ \sigma_{xy} &= \sigma_{rr} \sin \theta \cos \theta - \sigma_{\theta\theta} \sin \theta \cos \theta - 2 \sigma_{r\theta} (\cos^2 \theta - \sin^2 \theta)\end{aligned}$$

with:

- $\theta = 0^\circ$  at the points  $A$  and  $B$ ,
- $\theta = 22.5^\circ$  at the points  $C$  and  $D$ ,
- $\theta = 45^\circ$  at the points  $E$  and  $F$ .

### 2.2 Results of reference

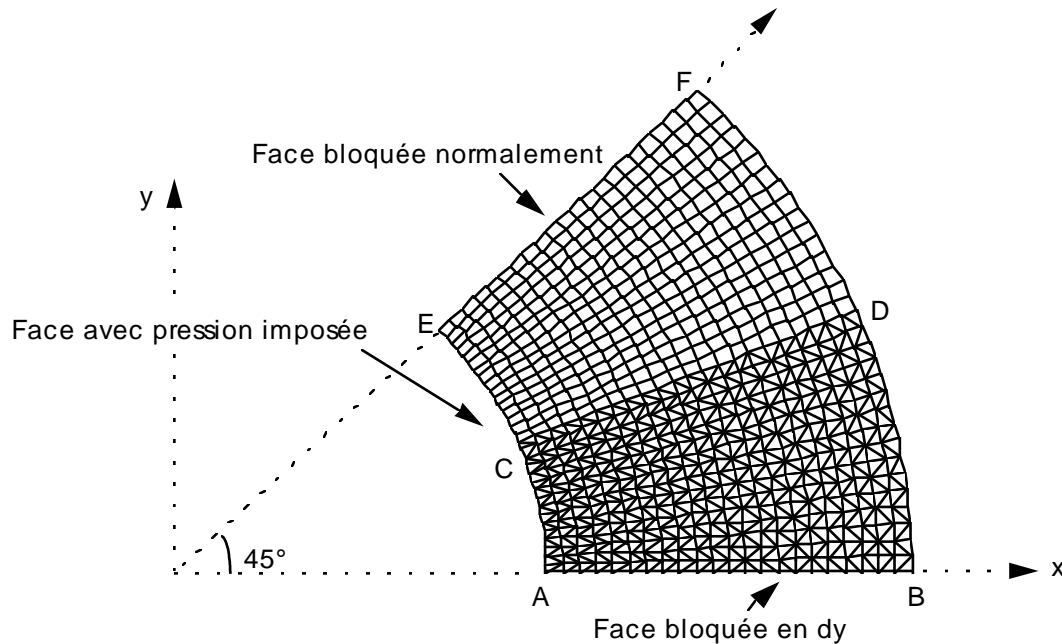
Displacements  $(u, v)$  and stresses  $(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy})$  with the points  $A, B, C, D, E, F$ .

### 2.3 Bibliographical references

- [1] Y.C. FUNG. Foundations of solid mechanics. Prentice-hall, Inc. Englewood Cliffs. NJ. 1965 p. 243 to 245.

## 3 Modelization A

### 3.1 Characteristic of the modelization: D-plane (QUAD4 + TRIA3)



limiting Conditions:

side <i>AB</i>	DDL_IMPO = (GROUP_NO = bordAB DY = 0. )
side <i>EF</i>	FACE_IMPO = (GROUP_MA = faceEFDNOR = 0. )
pressure on the face <i>AE</i>	PRES_REP = (GROUP_MA = faceAE NEAR = 60. )

Names of the nodes:	<i>A</i> = N23	<i>B</i> = N1	<i>C</i> = N391
	<i>D</i> = N369	<i>E</i> = N451	<i>F</i> = 751

### 3.2 Characteristics of the mesh

Many nodes: 759

Number of meshes and types: 704 TRIA3, 352 QUAD4

## 3.3 Quantities tested and results

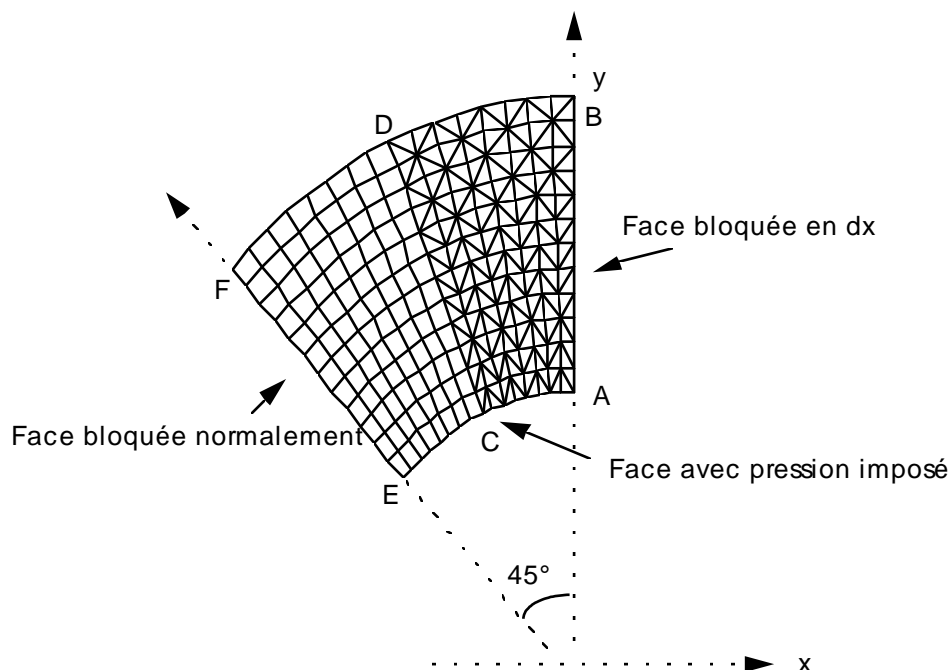
Place	Quantity	Reference	Aster	% difference	tolerance
A	$u$	5.72 10-5	5.7155 10-5	- 0.08	10-2
	$v$	0.	epsilon	-	
	$\sigma_{xx}$	- 60.	- 56.3770	- 6.04	
	$\sigma_{yy}$	100.	96.3917	- 3.61	
	$\sigma_{zz}$	12.	12.0044	0.04	
	$\sigma_{xy}$	0.	- 0.9563	-	
C	$u$	5.28459 10-5	5.2832 10-5	- 0.03	10-2
	$v$	2.18895 10-5	2.1777 10-5	- 0.51	
	$\sigma_{xx}$	- 36.56854	- 33.5312	- 8.31	
	$\sigma_{yy}$	76.56854	76.9335	0.48	
	$\sigma_{zz}$	12.	13.0207	8.51	
	$\sigma_{xy}$	- 56.56854	- 53.7445	- 4.99	
E	$u$	4.04465 10-5	4.0400 10-5	- 0.11	10-2
	$v$	4.04465 10-5	4.0400 10-5	- 0.11	
	$\sigma_{xx}$	20.	23.4926	17.46	
	$\sigma_{yy}$	20.	25.4141	27.07	
	$\sigma_{zz}$	12.	14.6720	22.27	
	$\sigma_{xy}$	- 80.	- 78.3081	- 2.11	
B	$u$	3.640 10-5	3.6405 10-5	0.01	10-2
	$v$	0.	epsilon	-	
	$\sigma_{xx}$	0.	- 0.4064	-	
	$\sigma_{yy}$	40.	39.8759	- 0.31	
	$\sigma_{zz}$	12.	11.8408	- 1.33	
	$\sigma_{xy}$	0.	- 0.4447	-	
D	$u$	3.36292 10-5	3.3603 10-5	- 0.08	10-2
	$v$	1.39297 10-5	1.3945 10-5	0.11	
	$\sigma_{xx}$	5.85786	5.2229	- 10.84	
	$\sigma_{yy}$	34.14214	33.8961	- 0.72	
	$\sigma_{zz}$	12.	11.7357	- 2.20	
	$\sigma_{xy}$	- 14.14214	- 14.1755	0.24	
F	$u$	2.57387 10-5	2.5710 10-5	- 0.11	10-2
	$v$	2.57387 10-5	2.5710 10-5	- 0.11	
	$\sigma_{xx}$	20.	19.1238	- 4.38	
	$\sigma_{yy}$	20.	19.6156	- 1.92	
	$\sigma_{zz}$	12.	11.6218	- 3.15	
	$\sigma_{xy}$	- 20.	- 20.1797	0.90	

## 3.4 Remarks

the increase in the error, when one passes from  $AB$  to  $CD$  then  $EF$ , is ascribable with the mesh (density in elements QUAD4 lower than that in TRIA3).

## 4 Modelization B

### 4.1 Characteristic of the modelization: D-plane (QUAD8 + TRIA6)



limiting Conditions:

side *AB*

DDL\_IMPO = (GROUP\_NO = bordAB DY = 0. )

side *EF*

FACE\_IMPO = (GROUP\_MA = faceEFDNOR = 0. )

pressure on *AE*

PRES\_REP = (GROUP\_MA = faceAE NEAR = 60. )

Names of the nodes:

*A* = N2

*B* = N48

*C* = N401

*D* = N424

*E* = N606

*F* = N494

### 4.2 Characteristics of the mesh

Many nodes: 729

Number of meshes and types: 192 TRIA6, 96 QUAD8

## 4.3 Quantities tested and results

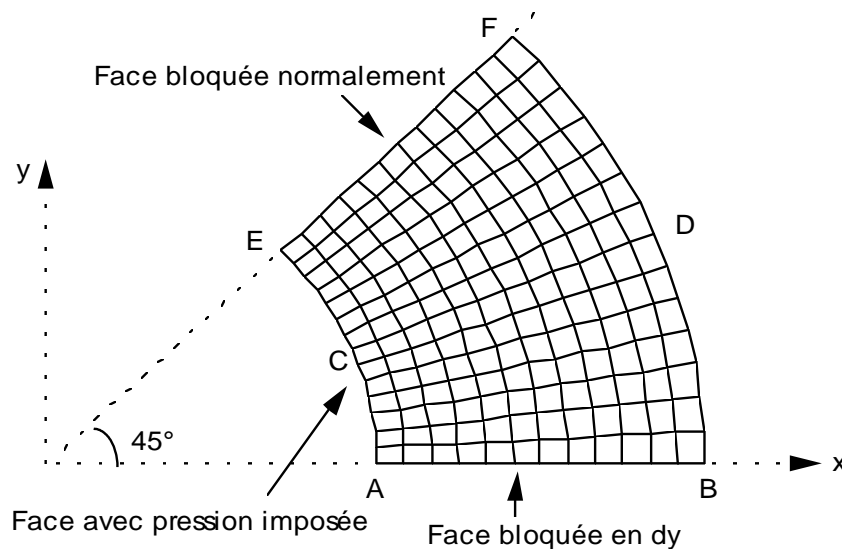
Place	Quantity	Reference	Aster	% difference	tolerance
A	$u$	0.	eps	-	10-2
	$v$	5.72 10-5	5.7155 10-5	-0.04	
	$\sigma_{xx}$	100.	99.7100	-0.29	
	$\sigma_{yy}$	-60.	-59.7725	-0.38	
	$\sigma_{zz}$	12.	11.9813	-0.16	
	$\sigma_{xy}$	0.	0.2643	-	
C	$u$	-2.18895 10-5	-2.1881 10-5	-0.04	10-2
	$v$	5.28459 10-5	5.2826 10-5	-0.04	
	$\sigma_{xx}$	76.56854	76.7005	0.17	
	$\sigma_{yy}$	-36.56854	-36.4500	-0.32	
	$\sigma_{zz}$	12.	12.0751	0.63	
	$\sigma_{xy}$	-56.56854	56.2844	-0.50	
E	$u$	-4.04465 10-5	-4.0432 10-5	-0.04	10-2
	$v$	4.04465 10-5	4.0432 10-5	-0.04	
	$\sigma_{xx}$	20.	20.0083	4.10-4	
	$\sigma_{yy}$	20.	19.9988	-6.10-5	
	$\sigma_{zz}$	12.	12.0021	2.10-4	
	$\sigma_{xy}$	80.	79.8176	-0.23	
B	$u$	0.	eps	-	10-2
	$v$	3.640 10-5	3.6390 10-5	-0.03	
	$\sigma_{xx}$	40.	39.9924	-0.02	
	$\sigma_{yy}$	0.	-0.001338	-	
	$\sigma_{zz}$	12.	11.9973	-0.02	
	$\sigma_{xy}$	0.	-0.04083	-	
D	$u$	-1.39297 10-5	-1.3926 10-5	-0.03	10-2
	$v$	3.36292 10-5	3.3619 10-5	-0.03	
	$\sigma_{xx}$	34.14214	34.1361	-0.02	
	$\sigma_{yy}$	5.85786	5.8948	0.63	
	$\sigma_{zz}$	12.	12.0093	0.08	
	$\sigma_{xy}$	14.14214	14.1596	0.12	
F	$u$	-2.57387 10-5	-2.5731 10-5	-0.03	10-2
	$v$	2.57387 10-5	2.5731 10-5	-0.03	
	$\sigma_{xx}$	20.	20.0000	3.10-6	
	$\sigma_{yy}$	20.	19.9996	-2.10-5	
	$\sigma_{zz}$	12.	11.9999	-7.10-6	
	$\sigma_{xy}$	20.	19.9975	-1.10-4	

## 4.4 Remarks

the evolution of the error induced by the following mesh  $AB$  ,  $CD$  or  $EF$  , is clearly attenuated compared to the modelization A.

## 5 Modélisation C

### 5.1 Characteristic of the modelization: D-plane (QUAD9)



limiting Conditions:

side *AB*

DDL\_IMPO = (GROUP\_NO = bordAB DY = 0. )

side *EF*

FACE\_IMPO = (GROUP\_MA = faceEF DNOR = 0. )

pressure on *AE*

PRES\_REP = (GROUP\_MA = faceAE NEAR = 60. )

Names of the nodes:

*A* = N1

*B* = N47

*C* = N351

*D* = N374

*E* = N569

*F* = N423

### 5.2 Characteristics of the mesh

Many nodes: 725

Number of meshes and types: 168 QUAD9



## 5.3 Quantities tested and results

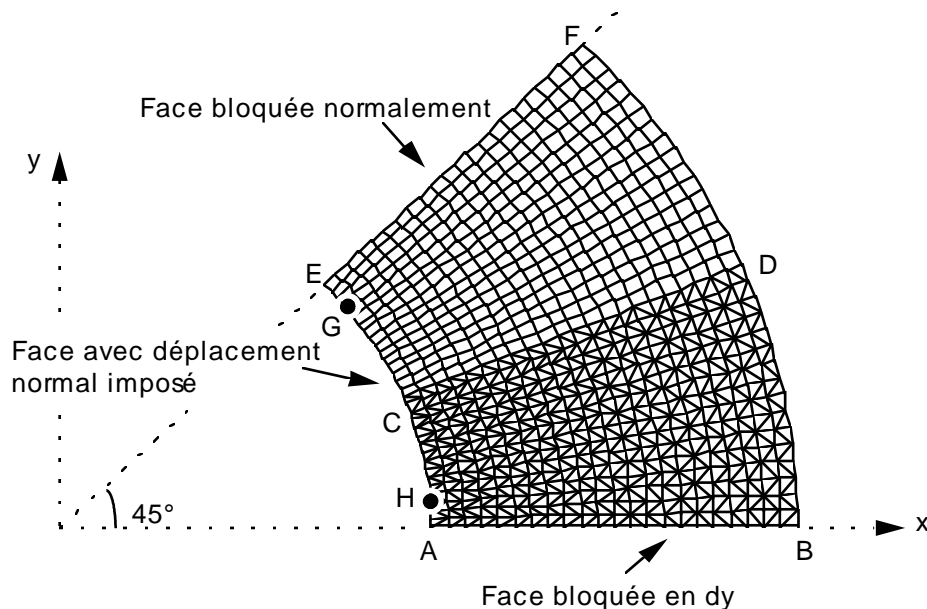
Place	Quantity	Reference	Aster	% difference	tolerance
A	$u$	5.72 10-5	5.7173 10-5	- 0.05	10-2
	$v$	0.	epsilon	-	
	$\sigma_{xx}$	- 60.	- 56.8334	- 0.27	
	$\sigma_{yy}$	100.	99.84	- 0.16	
	$\sigma_{zz}$	12.	12.0023	0.02	
	$\sigma_{xy}$	0.	- 0.00272	-	
C	$u$	5.28459 10-5	5.2821 10-5	- 0.05	10-2
	$v$	2.18895 10-5	2.1879 10-5	- 0.05	
	$\sigma_{xx}$	- 36.56854	- 36.45	0.32	
	$\sigma_{yy}$	76.56854	76.46	- 0.14	
	$\sigma_{zz}$	12.	12.003	0.02	
	$\sigma_{xy}$	- 56.56854	- 56.45	0.2	
E	$u$	4.04465 10-5	4.0427 10-5	- 0.05	10-2
	$v$	4.04465 10-5	4.0427 10-5	- 0.05	
	$\sigma_{xx}$	20.	19.996	- 2. 10-4	
	$\sigma_{yy}$	20.	20.011	5.5 10-4	
	$\sigma_{zz}$	12.	12.002	2. 10-4	
	$\sigma_{xy}$	- 80.	- 79.837	0.2	
B	$u$	3.640 10-5	3.6386 10-5	0.04	10-2
	$v$	0.	epsilon	-	
	$\sigma_{xx}$	0.	- 2.7 10-3	-	
	$\sigma_{yy}$	40.	40.0011	- 4. 10-4	
	$\sigma_{zz}$	12.	11.9995	- 4. 10-4	
	$\sigma_{xy}$	0.	- 4.8 10-4	-	
D	$u$	3.36292 10-5	3.3617 10-5	- 0.04	10-2
	$v$	1.39297 10-5	1.3924 10-5	- 0.04	
	$\sigma_{xx}$	5.85786	5.8557	- 0.03	
	$\sigma_{yy}$	34.14214	34.143	2.5 10-5	
	$\sigma_{zz}$	12.	11.9996	3. 10-5	
	$\sigma_{xy}$	- 14.14214	- 14.1435	- 1. 10-4	
F	$u$	2.57387 10-5	2.5729 10-5	- 0.04	10-2
	$v$	2.57387 10-5	2.5729 10-5	- 0.04	
	$\sigma_{xx}$	20.	19.999	- 3. 10-5	
	$\sigma_{yy}$	20.	20.0002	1. 10-5	
	$\sigma_{zz}$	12.	11.9999	- 9. 10-6	
	$\sigma_{xy}$	- 20.	- 20.0025	- 0.01	

## 5.4 Remarks

the evolution of the error induced by the following mesh  $AB$  ,  $CD$  or  $EF$  , is clearly attenuated compared to the modelization A.

## 6 Modélisation D

### 6.1 Characteristic of the modelization: D-plane (QUAD4 + TRIA3)



limiting Conditions:

side  $AB$  D DL\_IMPO: (GROUP\_NO = bordAB DY = 0. )  
 side  $EF$  FACE\_IMPO: (GROUP\_MA = faceEF DNOR = 0. )  
 on  $AE$  normal displacement imposed on  $5.72 E-5 m$   
 FACE\_IMPO: (GROUP\_MA = faceAE DNOR = -5.72 E-5)

Names of the nodes:  $A = N23$   $B = N1$   $C = N391$   
 $D = N369$   $E = N451$   $F = N751$   
 $H = N92$   $G = N447$

### 6.2 Characteristics of the mesh

Many nodes: 759

Number of meshes and types: 704 TRIA3, 352 QUAD4

## 6.3 Quantities tested and results

Localization	Quantity	Reference	Aster	% difference
C	$F_x$	0.1360	0.14069	3.45
	$F_y$	0.056	0.05827	4.06
H	$F_x$	0.14686	0.13608	7.34
	$F_y$	0.0108	0.0100	7.04
G	$F_x$	0.1138	0.114	0.19
	$F_y$	0.093	0.0936	0.61

## 6.4 Remarks

One checks that the nodal forces of reaction are null in all the nodes, except on the nodes of line  $AE$  ,  $EF$  and  $AB$  .

## 7 Summary of the results

### D\_plan

Summary of the errors max in %		modelization		
		A	B	C
Displacements	A, B	0.08	0.04	0.05
	C, D	0.51	0.04	0.05
	E, F	0.11	0.04	0.05
Stresses $\sigma_{xx}$	A, B	6.04	0.29	0.27
	C, D	10.84	0.17	0.32
	E, F	17.46	4.10-4	2.10-4
Stresses $\sigma_{yy}$	A, B	3.61	0.38	0.16
	C, D	0.72	0.63	0.14
	E, F	27.07	2.10-5	5.5.10-4
Stresses $\sigma_{zz}$	A, B	1.33	0.16	0.02
	C, D	8.51	0.63	0.02
	E, F	22.27	2.10-4	2.10-4
Stresses $\sigma_{xy}$	A, B	-	-	-
	C, D	4.99	0.50	0.2.0.2
	E, F	2.11	0.23	

These 3 modelizations appreciably have the same number of nodes; the results got with elements of order 1 (modelization A in TRIA3 and QUAD4) are definitely less precise, in particular on the internal wall.