SSLV04 - Hollow roll in plane stresses

Summarized:

This test is drawn from Guide VPCS (test SSLV04/89) and has as an aim a hollow roll charged in internal pressure.

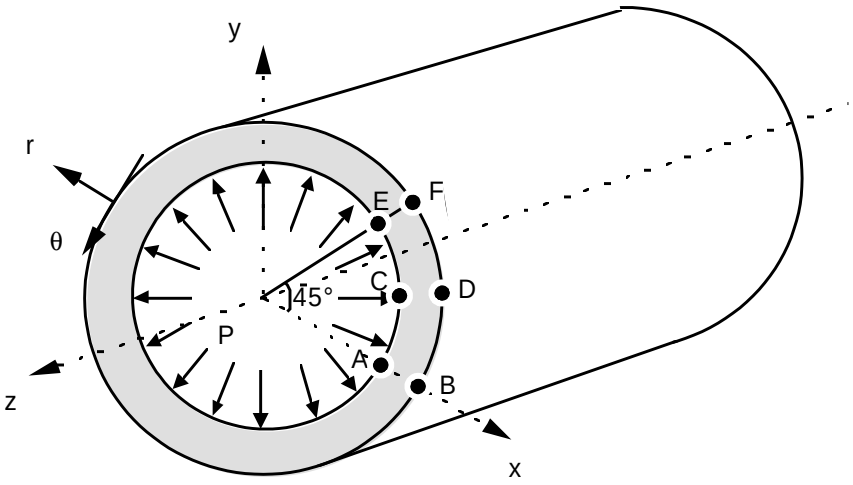
With this three-dimensional problem is dealt with various modelizations:
•in 3D : 9 modelizations (pentahedral, hexahedrons, tetrahedrons and pyramids, degrees 1 and 2),
•in 2D plane stresses: 4 modelizations (triangles and quadrangles degrees 1 and 2, quadrangles with 9 nodes),
•into 2D axisymmetric: 3 modelizations (triangles and quadrangles degrees 1 and 2, quadrangles with 9 nodes).

The features tested are:
•distributed pressure,
•basic effect (with fixed or variable pressure),
•imposed displacements,
•nodal stiffness matrixes
•, strains and nodal stresses
•, reactions (modelization K).

There are 16 modelizations.

1 Problem of reference

1.1 Geometry



Rayon interne a = 0.1 m
Rayon externe b = 0.2 m

Coordinated of the points:

	A	B	C	D	E	F
x	0.100	0.200	$0.1\cos(22.5)$	$0.2\cos(22.5)$	$\sqrt{2}/2$	$\sqrt{2}$
y	0.	0.	$0.1\sin(22.5)$	$0.2\sin(22.5)$	$\sqrt{2}/2$	$\sqrt{2}$
z	0	0.	0.	0.	0.	0.

1.2 Material properties

the Young modulus of the material is equal to $E=2.10^5\ MPa$.
The Poisson's ratio is equal to $\nu=0.3$.

1.3 Boundary conditions and loadings

internal Pressure:
$P=60\ MPa$

Pressure interns variable (modelization P only):
P vary linearly $60\ MPa$ with $t=1.\text{s}$ $120\ MPa$ with $t=2.\text{s}$

2 Reference solution

2.1 Method of calculating used for the reference solution

In plane stress (cylinder on free board at the ends)

$$\begin{aligned}
\sigma_{zz} &= 0 \\
\sigma_{rr} &= P\frac{a^2}{b^2-a^2}\left[1-\frac{b^2}{r^2}\right] \\
\sigma_{\theta\theta} &= P\frac{a^2}{b^2-a^2}\left[1+\frac{b^2}{r^2}\right] \\
\sigma_{r\theta} &= 0 \\
u_r &= \frac{P}{E}\frac{a^2}{b^2-a^2}\left[(1-\nu)+(1+\nu)\frac{b^2}{r^2}\right]r \\
\varepsilon_{rr} &= \frac{P}{E}\frac{a^2}{b^2-a^2}\left[(1-\nu)-(1+\nu)\frac{b^2}{r^2}\right] \\
\varepsilon_{\theta\theta} &= \frac{u_r}{r}
\end{aligned}$$

One obtains:

for $r=0.1$:

$$\begin{aligned}
u_r &= 59.10^{-6} \\
\sigma_{rr} &= -60. \\
\sigma_{\theta\theta} &= 100. \\
\sigma_{zz} &= \sigma_{r\theta} = 0. \\
\varepsilon_{rr} &= -45.10^{-5} \\
\varepsilon_{\theta\theta} &= 59.10^{-5}
\end{aligned}$$

; for $r=0.2$:

$$\begin{aligned}
u_r &= 40.10^{-6} \\
\sigma_{rr} &= 0. \\
\sigma_{\theta\theta} &= 40. \\
\sigma_{zz} &= \sigma_{r\theta} = 0. \\
\varepsilon_{rr} &= -6.10^{-5} \\
\varepsilon_{\theta\theta} &= 2.10^{-4}
\end{aligned}$$

Transition in the system of Cartesian axes:

$$\begin{aligned}
\sigma_{xx} &= \sigma_{rr}\cos^2\theta+\sigma_{\theta\theta}\sin^2\theta-2\sigma_{r\theta}\sin\theta\cos\theta \\
\sigma_{yy} &= \sigma_{rr}\sin^2\theta+\sigma_{\theta\theta}\cos^2\theta+2\sigma_{r\theta}\sin\theta\cos\theta \\
\sigma_{xy} &= \sigma_{rr}\sin\theta\cos\theta-\sigma_{\theta\theta}\sin\theta\cos\theta-2\sigma_{r\theta}\left(\cos^2\theta-\sin^2\theta\right)
\end{aligned}$$

with:

- $\theta=0°$ at the points A and B ,
- $\theta=22.5°$ at the points C and D ,
- $\theta=45°$ at the points E and F .

2.2 Results of reference

Displacements (u,v) and stresses $\left(\sigma_{xx},\sigma_{yy},\sigma_{zz},\sigma_{xy}\right)$ with the points A, B, C, D, E, F .

2.3 Bibliographical references

1 Guides VPCS. SSLV04/89

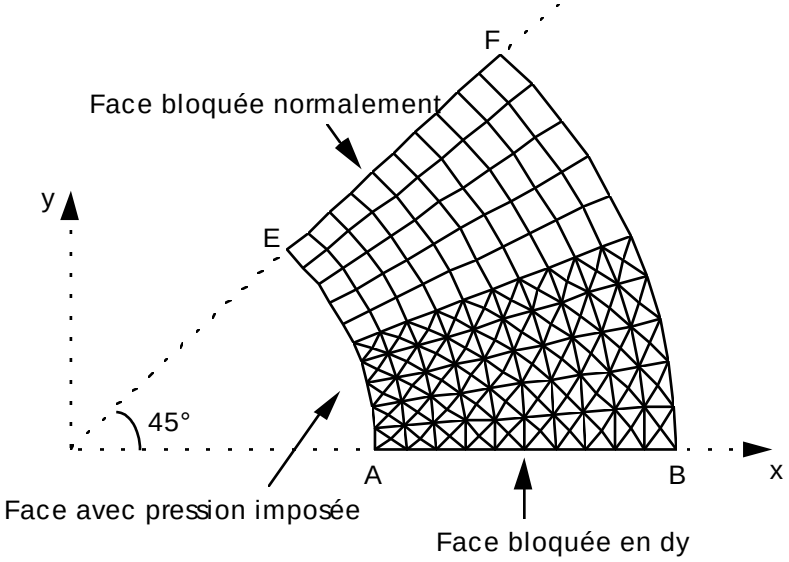
2 Y.C. FUNG. Foundations of solid mechanics. Prentice-hall, Inc. Englewood Cliffs. NJ. 1965 p. 243 to 245.

3 J. COURBON. Strength of materials p 649

3 Modelization A

3.1 Characteristic of the modelization

Elements 3D (PENTA6 and HEXA8).
Mesh obtained by extrusion from a mesh $2D$ resembling the mesh below (30 elements in the radial direction with progressive coarsening and $15+15$ elements in the circumferential direction).



Along the axis Z : 1 layer of elements
total Thickness: 0.01m

Limiting conditions:

the node is outside the field of definition with a right profile of the EXCLU type
node: $F \quad u_z=0$
face AB blocked opposite dy
EF blocked normally
pressure on the face AE $p=60.$

Names the nodes: $A=N993$ $B=N1443$ $C=N1$
$D=N31$ $E=N496$ $F=N495$

3.2 Characteristics of the mesh

Many nodes: 1922
Number of meshes and types: 900 PENTA6, 450 HEXA8 and 90 QUAD4 (sides internal skin).

3.3 Quantities tested and results

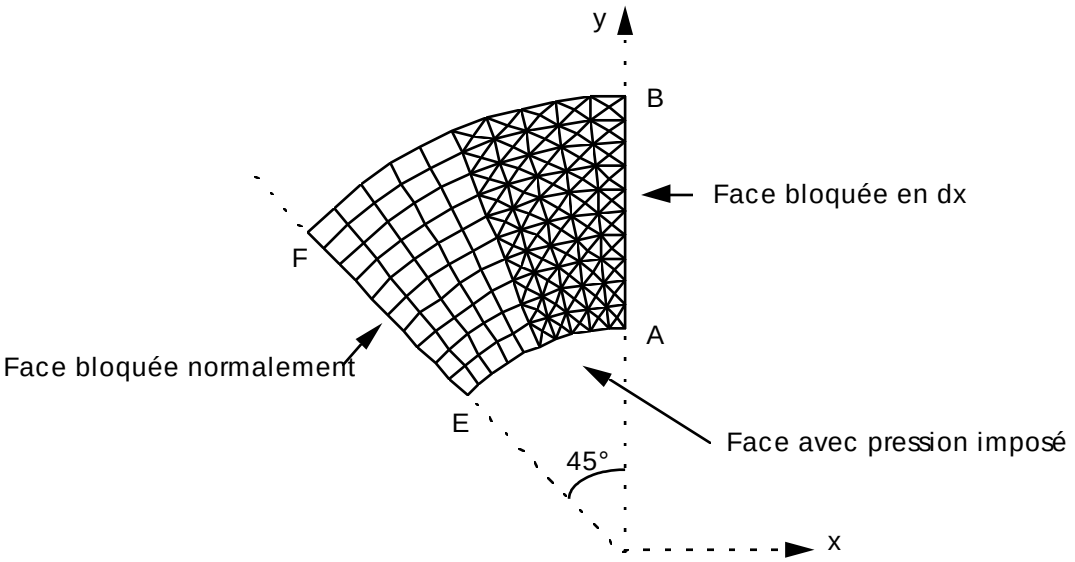
Localization	Quantity	Reference	*Aster*	% difference
A	u	5.9 10-5	5.8950 10-5	0.08
	v	0.	*eps*	-
	σ_{xx}	– 60.	– 59.2225	1.30
	σ_{vv}	100.	100.4159	0.42
	σ_{zz}	0.	0.3093	-
	σ_{xv}	0.	– 1.0442	-
	ε_{xx}	– 4.5 10-4	– 4.472 10-4	0.62
	ε_{vv}	5.9 10-4	5.904 10-4	0.08
	ε_{xv}	0.	$-6.788\ 10^{-5}$	-
B	u	$4\ 10^{-5}$	$3.9959\ 10^{-5}$	0.10
	v	0.	*eps*	-
	σ_{xx}	0.	– 1.7246	-
	σ_{vv}	40.	39.2451	1.89
	σ_{zz}	0.	– 0.3761	-
	σ_{xv}	0.	– 0.2659	-
	ε_{xx}	– 0.6 10-4	– 6.692 10-5	11.54
	ε_{vv}	2. 10-4	1.994 10-4	0.31
	ε_{xv}	0.	– 1.728 10-6	-
E	u	4.17193 10-5	4.1708 10-5	0.03
	v	4.17193 10-5	4.1708 10-5	0.03
	σ_{xx}	20.	19.0824	4.59
	σ_{vv}	20.	21.1394	5.70
	σ_{zz}	0.	0.0870	-
	σ_{xv}	– 80.	– 79.8831	0.15
	ε_{xx}	0.7 10-4	0.636 10-4	9.18
	ε_{vv}	0.7 10-4	0.769 10-4	9.92
	ε_{xv}	$-5.2\ 10^{-4}$	$-5.192\ 10^{-4}$	0.15
F	u	$2.82843\ 10^{-5}$	$2.8302\ 10^{-5}$	0.06
	v	2.82843 10-5	2.8302 10-5	0.06
	σ_{xx}	20.	18.9528	5.24
	σ_{vv}	20.	19.9104	0.45
	σ_{zz}	0.	0.1198	-
	σ_{xv}	– 20.	– 20.1809	0.90
	ε_{xx}	0.7 10-4	0.647 10-4	7.54
	ε_{vv}	0.7 10-4	0.709 10-4	1.35
	ε_{xv}	– 1.3 10-4	– 1.312 10-4	0.90

Localization	Quantity	Reference	Tolerance
Nets M1380, not A			
SIRO_ELEM	SIG_NX	60.0	2.0%
Normal stress with the face of the element	SIG_NY	0.0.1.0.0.0	
	SIG_NZ		0.01
	SIG_N	-60.0	2.0%
SIRO_ELEM	SIG_TX	0.0	0.01
Shear stress in the plane of the element	SIG_TY	0.0	1.0%
	SIG_TZ	0.0.1.0	
SIRO_ELEM	SIG_T1X	0.0	0.01
First value of the stress	SIG_T1Y	0.0	0.01
tangential in the plane of the element	SIG_T1Z	0.0.1.0.0.0	
	SIG_T1		1.0
SIRO_ELEM	SIG_T2X	0.0.2.0	
Second value of the tangential	SIG_T2Y	stress	-100.0
1.0% in the plane of the element	SIG_T2Z	0.0	0.01
	SIG_T2	100.0	1.0%
Nets M1351, not E			
SIRO_ELEM	SIG_NX	42.426	1.5%
Normal stress with the face of the element	SIG_NY	42.426	2.0%
	SIG_NZ	0.0	0.01
	SIG_N	-60.0	1.0%
SIRO_ELEM	SIG_TX	0.0	1.0
Shear stress in the plane of the element	SIG_TY	0.0	1.0.0.0.0.2
	SIG_TZ		
SIRO_ELEM	SIG_T1X	0.0	0.01
First value of the stress	SIG_T1Y	0.0	0.01
tangential in the plane of the element	SIG_T1Z	0.0.0.1.0.0	
	SIG_T1		0.2
SIRO_ELEM	SIG_T2X	70.711	1.5%
Second value of the tangential	SIG_T2Y	stress	-70.711
1.5% in the plane of the element	SIG_T2Z	0.0	0.01
	SIG_T2	100.0	1.0%

4 Modelization B

4.1 Characteristic of the modelization

Elements 3D (PENTA15 and HEXA20).
Mesh obtained by extrusion starting from the mesh $2D$ below (modelization F)



Along the axis Z : 2 layers of elements
total Thickness: $0.01\mathrm{m}$

Limiting conditions:

node $F = u_z = 0$
face AB blocked opposite dx
EF blocked normally
pressure on the face AE $p = 60.$

Names the nodes: $A = NO2$ $B = NO361$ $C = NO121$
$D = NO584$ $E = NO155$ $F = NO503$

4.2 Characteristics of the mesh

Many nodes: 2115
Number of meshes and types: 400 PENTA15, 100 HEXA20 40 QUAD8 (sides skin interns)

4.3 Quantities tested and results

Localization	Quantity	Reference	*Aster*	% difference
A	u	0.	eps	-
	v	5.9 10-5	5.8944 10-5	– 0.09
	σ_{xx}	100.	99.6056	– 0.39
	σ_{yy}	– 60.	– 59.4473	– 0.92
	σ_{zz}	0.	0.0196	-
	σ_{xy}	0.	0.2481	-
	ε_{xx}	5.9 10-4	5.87 10-4	– 0.48
	ε_{yy}	– 4.5 10-4	– 4.47 10-4	– 0.74
	ε_{xy}	0.	$1.61\ 10^{-6}$	-
B	u	0.	eps	-
	v	4 10-5	3.9974 10-5	– 0.07
	σ_{xx}	40.	39.9711	0.07
	σ_{yy}	0.	0.0781	-
	σ_{zz}	0.	5.7992 10-3	-
	σ_{xy}	0.	– 0.0182	-
	ε_{xx}	2. 10-4	1.997 10-4	– 0.13
	ε_{yy}	– 0.6. 10-4	– 0.596 10-4	– 0.67
	ε_{xy}	0.	– 1.1810-7	-
E	u	4.17193 10-5	4.1680 10-5	– 0.09
	v	4.17193 10-5	4.1680 10-5	– 0.09
	σ_{xx}	20.	20.0515	0.26
	σ_{yy}	20.	20.0264	0.13
	σ_{zz}	0.	– 0.0155	-
	σ_{xy}	80.	79.7918	– 0.26
	ε_{xx}	0.7 10-4	0.702 10-4	0.34
	ε_{yy}	0.7 10-4	0.701 10-4	0.11
	ε_{xy}	$5.2\ 10^{-4}$	$5.19\ 10^{-4}$	– 0.26
F	u	$-2.82843\ 10^{-5}$	$-2.82656\ 10^{-5}$	– 0,07
	v	2.82843 10-5	2.82656 10-5	– 0.07
	σ_{xx}	20.	20.0099	0.05
	σ_{yy}	20.	19.9980	– 0.01
	σ_{zz}	0.	– 3.90 10-4	-
	σ_{xy}	20.	20.0122	0.06
	ε_{xx}	0.7 10-4	0.7005 10-4	0.08
	ε_{yy}	0.7 10-4	0.6997 10-4	– 0.03
	ε_{xy}	– 1.3 10-4	-1.3008 10-4	0.06

Localization	Quantity	Reference	Tolerance
Nets MA751, not A			
SIRO_ELEM	SIG_NX	0.0	3.
Normal stress with the face of the element	SIG_NY	60.0	0.5%
	SIG_NZ	0.0	0.01
SIRO_ELEM	SIG_TX	0.0.0.1	
Shear stress in the plane of the element	SIG_TY	0.0.0.1	
	SIG_TZ	0.0.0.1	
SIRO_ELEM	SIG_T1X	0.0	0.01
First value of the stress	SIG_T1Y	0.0	0.01
tangential in the plane of the element	SIG_T1Z	0.0	0.05
SIRO_ELEM	SIG_T2X	100.0	0.15%
Second value of tangential	SIG_T2Y	stress	
0.0.4.5 in the plane of the element	SIG_T2Z	0.0	0.20
Mesh MA769, not E			
SIRO_ELEM	SIG_NX	-42.426	6.0%
Normal stress with the face of the element	SIG_NY	42.426	4.0%
	SIG_NZ	0.0	0.01
SIRO_ELEM	SIG_TX	0.0	0.01
Shear stress in the plane of the element	SIG_TY	0.0	0.01
	SIG_TZ	0.0	0.03
SIRO_ELEM	SIG_T1X	0.0	0.01
First value of the stress	SIG_T1Y	0.0	0.01
tangential in the plane of the element	SIG_T1Z	0.0	0.01
SIRO_ELEM	SIG_T2X	70.711	3.5%
Second value of tangential	SIG_T2Y	stress	70.711
4.5% in the plane of the element	SIG_T2Z	0.0.0.1	

5 Modelization C

5.1 Characteristics of the modelization

Elements 3D (TETRA4).
AB is on the axis OX

Cutting:	21 equidistant nodes on the equidistant AB segments CD , EF and 21 nodes on the arcs ACE and BDF
Along the axis Z :	1 layer of elements
total Thickness:	$0.01\mathrm{m}$

Limiting conditions:

the node is outside the field of definition with a right profile of the EXCLU type node: F	$u_z=0$
face AB blocked opposite dy	
EF blocked normally	
pressure on the face AE	$p=60.$

Names the nodes: $A=N165$ $B=N4$ $C=N209$
$D=N82$ $E=N244$ $F=N1068$

5.2 Characteristics of the mesh

Many nodes: 1115
Number of meshes and types: 3724 TETRA4 and 1760 TRIA3 (sides skin interns)

5.3 Quantities tested and results

Localization	Quantity	Reference	*Aster*	% difference
A	u	5.9 10-5	5.8901 10-5	– 0.17
	v	0.	eps	-
	σ_{xx}	– 60.	– 57.2290	– 4.62
	σ_{vv}	100.	97.8711	– 2.13
	σ_{zz}	0.	0.0568	-
	σ_{xv}	0.	– 2.6589	-
	ε_{xx}	– 4.5 10-4	– 4.33 10-4	– 3.77
	ε_{vv}	5.9 10-4	5.75 10-4	– 2.52
	ε_{xv}	0.	$-1.73\ 10^{-5}$	-
B	u	$4\ 10^{-5}$	$3.9878\ 10^{-5}$	– 0.30
	v	0.	eps	-
	σ_{xx}	0.	– 1.5296	-
	σ_{vv}	40.	40.9839	2.46
	σ_{zz}	0.	– 0.1006	-
	σ_{xv}	0.	– 0.8513	-
	ε_{xx}	– 0.6 10-4	– 6.897 10-4	14.95
	ε_{vv}	2. 10-4	2.074 10-4	3.68
	ε_{xv}	0.	– 5.534 10-5	-
E	u	4.17193 10-5	4.1655 10-5	– 0.15
	v	4.17193 10-5	4.1655 10-5	– 0.15
	σ_{xx}	20.	17.9096	– 10.45
	σ_{vv}	20.	21.8929	9.46
	σ_{zz}	0.	– 0.3679	-
	σ_{xv}	– 80.	– 77.6897	– 2.89
	ε_{xx}	0.7 10-4	0.573 10-4	– 18.20
	ε_{vv}	0.7 10-4	0.832 10-4	18.79
	ε_{xv}	$-5.2\ 10^{-4}$	– 5.050 10-4	– 2.89
F	u	$2.82843\ 10^{-5}$	$2.8251\ 10^{-5}$	– 0.12
	v	2.82843 10-5	2.8251 10-5	– 0.12
	σ_{xx}	20.	18.4444	– 7.78
	σ_{vv}	20.	19.8876	– 0.56
	σ_{zz}	0.	– 0.3910	-
	σ_{xv}	– 20.	– 20.1631	0.81
	ε_{xx}	0.7 10-4	0.630 10-4	– 10.05
	ε_{vv}	0.7 10-4	0.723 10-4	3.35
	ε_{xv}	– 1.3 10-4	– 1.311 10-4	0.81

Localization	Quantity	Reference	Tolerance
Nets M5444, not A			
SIRO_ELEM	SIG_NX	60.0	8.0%
Normal stress with the face of the element	SIG_NY	0.0.1.5.0.0	
	SIG_NZ		0.1
SIRO_ELEM	SIG_TX	0.0.0.1	
Shear stress in the plane of the element	SIG_TY	0.0	1.0
	SIG_TZ	0.0.1.0	
SIRO_ELEM	SIG_T1X	0.0	0.01
First value of the stress	SIG_T1Y	0.0	0.01
tangential in the plane of the element	SIG_T1Z	0.0.1.5	
SIRO_ELEM	SIG_T2X	0.0.2.5	
Second value of tangential	SIG_T2Y	stress	-100.0
2.0% in the plane of the element	SIG_T2Z	0.0	0.20
M5404 Mesh, not E			
SIRO_ELEM	SIG_NX	42.426	6.0%
Normal stress with the face of the element	SIG_NY	42.426	9.0%
	SIG_NZ	0.0	1.00
SIRO_ELEM	SIG_TX	0.0	1.0
Shear stress in the plane of the element	SIG_TY	0.0	1.0
	SIG_TZ	0.0.1.0	
SIRO_ELEM	SIG_T1X	0.0	0.01
First value of stress	SIG_T1Y	0.0	0.01
tangential in the plane of the element	SIG_T1Z	0.0	1.50
SIRO_ELEM	SIG_T2X	-70.711	2.0%
Second value of tangential	SIG_T2Y	stress	70.711
4.0% in the plane of the element	SIG_T2Z	0.0	0.10

6 Modelization D

6.1 Characteristic of the modelization

Element 3D (TETRA10).

AB is on the axis OX

Cutting:	11 equidistant nodes on the equidistant AB segments CD , EF and 11 nodes on the arcs ACE and BDF
Along the axis Z :	1 layer of elements
total Thickness:	$0.01\mathrm{m}$

Limiting conditions:

the node is outside the field of definition with a right profile of the EXCLU type node: F	
$u_z=0$	
face AB blocked opposite dy	
EF blocked normally	
pressure on the face AE	$p=60.$

Names the nodes:	$A=N184$	$B=N4$	$C=N207$
	$D=N50$	$E=N22$	$F=N726$

6.2 Characteristics of the mesh

Many nodes: 1395
Number of meshes and types: 652 `TETRA10` and 480 `TRIA6` (sides skin interns)

6.3 Quantities tested and results

Localization	Quantity	Reference	*Aster*	% difference
A	u	5.9 10-5	5.8974 10-5	– 0.04
	v	0.	*eps*	-
	σ_{xx}	– 60.	– 60.3816	0.64
	σ_{vv}	100.	99.1907	– 0.81
	σ_{zz}	0.	– 0.9707	-
	σ_{xv}	0.	– 0.2979	-
	ε_{xx}	– 4.5 10-4	– 4.49 10-4	– 0.17
	ε_{vv}	5.9 10-4	5.88 10-4	– 0.34
	ε_{xv}	0.	$-1.94\ 10^{-6}$	-
B	u	$4\ 10^{-5}$	$3.9989\ 10^{-5}$	– 0.03
	v	0.	*eps*	-
	σ_{xx}	0.	0.0388	-
	σ_{vv}	40.	40.0725	0.18
	σ_{zz}	0.	– 0.0046	-
	σ_{xv}	0.	0.1634	-
	ε_{xx}	– 0.6 10-4	– 0.599 10-4	– 0.15
	ε_{vv}	2. 10-4	2.003 10-4	0.16
	ε_{xv}	0.	1.062 10-6	-
E	u	4.17193 10-5	4.17021 10-5	0.04
	v	4.17193 10-5	4.17021 10-5	0.04
	σ_{xx}	20.	19.1178	– 4.41
	σ_{vv}	20.	19.6399	– 1.80
	σ_{zz}	0.	– 1.0206	-
	σ_{xv}	– 80.	– 79.7804	– 0.27
	ε_{xx}	0.7 10-4	0.677 10-4	– 3.34
	ε_{vv}	0.7 10-4	0.711 10-4	1.50
	ε_{xv}	$-5.2\ 10^{-4}$	$-5.186\ 10^{-4}$	– 0.27
F	u	$2.82843\ 10^{-5}$	$2.82718\ 10^{-5}$	– 0.04
	v	2.82843 10-5	2.82718 10-5	– 0.04
	σ_{xx}	20.	20.1903	0.95
	σ_{vv}	20.	19.9023	– 0.49
	σ_{zz}	0.	– 0.0016	-
	σ_{xv}	– 20.	– 20.0570	0.28
	ε_{xx}	0.7 10-4	0.711 10-4	1.57
	ε_{vv}	0.7 10-4	0.692 10-4	– 1.10
	ε_{xv}	– 1.3 10-4	– 1.304 10-4	0.28

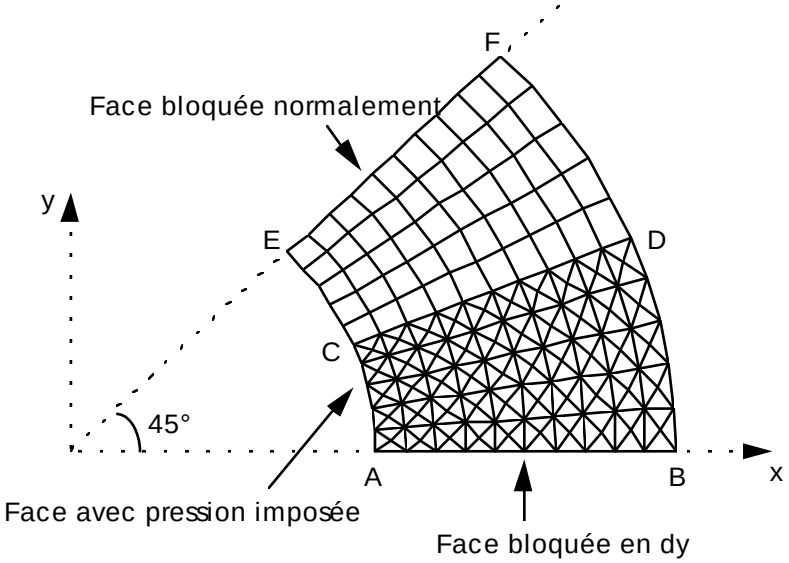
Localization	Quantity	Reference	Tolerance
Nets M1111, not A			
SIRO_ELEM	SIG_NX	60.0	1.0%
Normal stress with the face of the element	SIG_NY	0.0.3.0.0.0	
	SIG_NZ		1.0
SIRO_ELEM	SIG_TX	0.0	1.0
Shear stress in the plane of the element	SIG_TY	0.0.2.5	
	SIG_TZ	0.0.1.0	
SIRO_ELEM	SIG_T1X	0.0	0.01
First value principal of the stress	SIG_T1Y	0.0	0.01
tangential in the plane of the element	SIG_T1Z	0.0.1.0	
SIRO_ELEM	SIG_T2X	0.0.4.5	
Second value principal of the tangential	SIG_T2Y	stress	-100.0
1.0% in the plane of element	SIG_T2Z	0.0.0.1	
Nets M1093, not E			
SIRO_ELEM	SIG_NX	42.426	14%
Normal stress with the face of the element	SIG_NY	42.426	5.0%
	SIG_NZ	0.0.1.0	
SIRO_ELEM	SIG_TX	0.0	2.0
Shear stress in the plane of the element	SIG_TY	0.0	2.0
	SIG_TZ	0.0.1.0	
SIRO_ELEM	SIG_T1X	0.0	0.01
First value principal stress	SIG_T1Y	0.0	0.01
tangential in the plane of the element	SIG_T1Z	0.0.1.5	
SIRO_ELEM	SIG_T2X	-70.711	5.0%
Second principal value of tangential	SIG_T2Y	stress	70.711
4.0% in the plane of the element	SIG_T2Z	0.0.2.0	

7 Modelization E

7.1 Characteristic of the modelization

Elements C_PLAN (TRIA3 + QUAD4)

Mesh $2D$ resembling the mesh below (30 elements in the radial direction with progressive coarsening and $15+15$ elements in the circumferential direction).



Limiting conditions:

side AB blocked in dy
side EF normally blocked
pressure on AE $p=60.$

Names of the nodes: $A=N1$ $B=N451$ $C=N496$
$D=N495$ $E=N990$ $F=N989$

7.2 Characteristics of the mesh

Many nodes: 961
Number of meshes and types: 900 TRIA3, 450 QUAD4

7.3

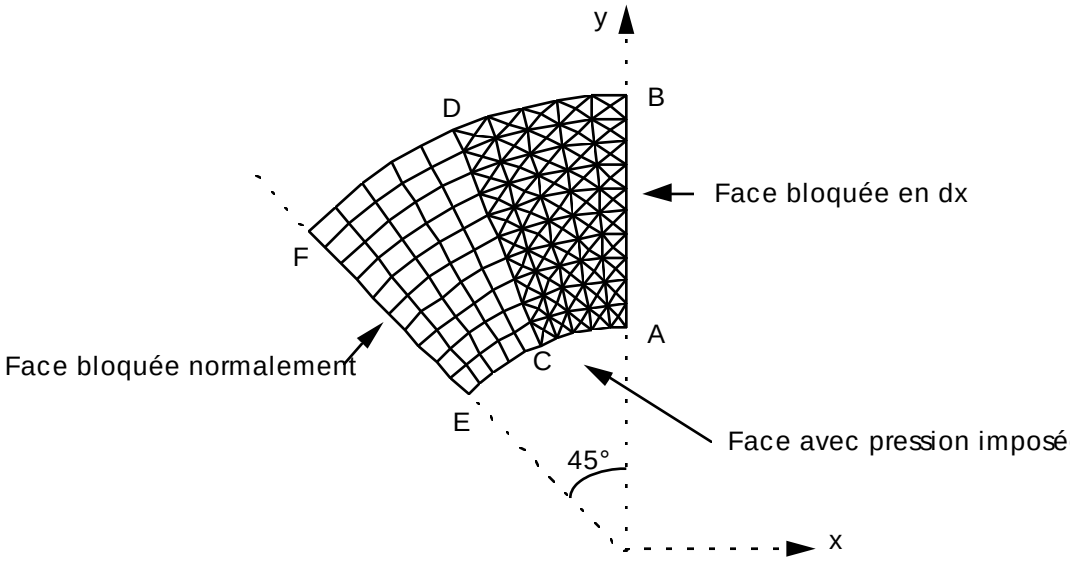
7.4 Quantities tested and results

Localization	Quantity	Reference	*Aster*	% difference
A	u	5.9 10-5	5.8957 10-5	– 0.07
	v	0.	eps	-
	σ_{xx}	– 60.	– 59.3645	– 1.06
	σ_{yy}	100.	100.2653	0.26
	σ_{zz}	0.	0.	-
	σ_{xy}	0.	– 1.0472	-
	ε_{xx}	– 4.5 10-4	– 4.472 10-4	– 0.62
	ε_{yy}	5.9 10-4	5.904 10-4	0.06
	ε_{xy}	0.	$-6.807\ 10^{-6}$	-
B	u	$4.\ 10^{-5}$	$3.9965\ 10^{-5}$	– 0.09
	v	0.	eps	-
	σ_{xx}	0.	– 1.4986	-
	σ_{yy}	40.	39.4415	– 1.40
	σ_{zz}	0.	0.	-
	σ_{xy}	0.	– 0.2658	-
	ε_{xx}	– 0.6 10-4	– 0.667 10-5	11.09
	ε_{yy}	2. 10-4	1.995 10-4	– 0.27
	ε_{xy}	0.	– 1.728 10-6	-
E	u	4.17193 10-5	4.17101 10-5	– 0.02
	v	4.17193 10-5	4.17101 10-5	– 0.02
	σ_{xx}	20.	19.0706	– 4.65
	σ_{yy}	20.	21.1354	5.68
	σ_{zz}	0.	0.	-
	σ_{xy}	– 80.	– 79.8720	– 0.16
	ε_{xx}	0.7 10-4	0.636 10-4	– 9.07
	ε_{yy}	0.7 10-4	0.771 10-4	10.10
	ε_{xy}	$-5.2\ 10^{-4}$	$-5.192\ 10^{-4}$	– 0.16
F	u	$2.82843\ 10^{-5}$	$2.82996\ 10^{-5}$	0.05
	v	2.82843 10-5	2.82996 10-5	0.05
	σ_{xx}	20.	18.9626	– 5.19
	σ_{yy}	20.	19.8483	– 0.76
	σ_{zz}	0.	0.	-
	σ_{xy}	– 20.	– 20.2466	1.23
	ε_{xx}	0.7 10-4	0.650 10-4	– 7.08
	ε_{yy}	0.7 10-4	0.708 10-4	1.14
	ε_{xy}	– 1.3 10-4	– 1.316 10-4	1.23

8 Modelization F

8.1 Characteristic of the modelization

C_plan `Elements` (`QUAD8` + `TRIA6`)



limiting Conditions:

side AB blocked in dx
side EF normally blocked
pressure on AE $p=60.$

Names of the nodes: $A=N2$ $B=N361$ $C=N121$
$D=N584$ $E=N155$ $F=N503$

8.2 Characteristics of the mesh

Many nodes: 591
Number of meshes and types: 200 `TRIA6`, 50 `QUAD8`

8.3

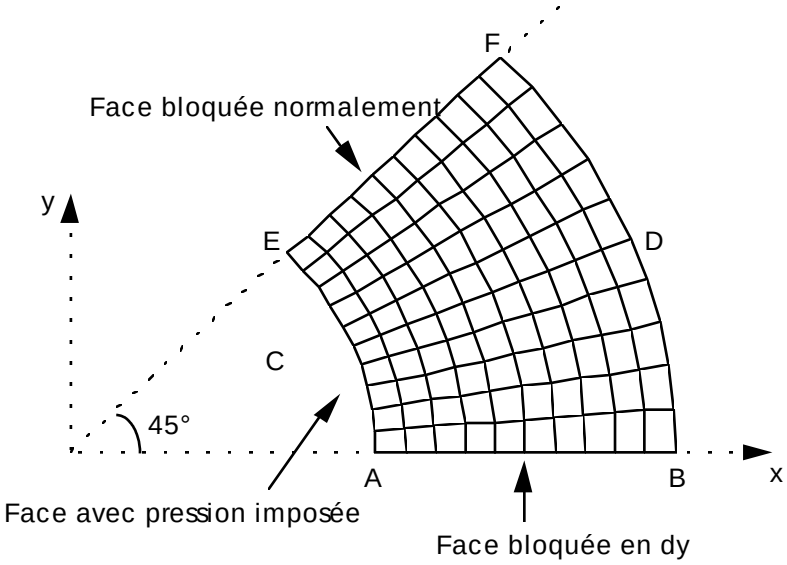
8.4 Quantities tested and results

Localization	Quantity	Reference	*Aster*	% difference
A	u	0.	0.	-
	v	5.9 10-5	5.8945 10-5	– 0.09
	σ_{xx}	100.	99.6095	– 0.39
	σ_{yy}	– 60.	– 59.4620	– 0.90
	σ_{zz}	0.	0.	-
	σ_{xy}	0.	0.2441	-
	ε_{xx}	5.9 10-4	5.872 10-4	– 0.47
	ε_{yy}	– 4.5 10-4	– 4.467 10-4	– 0.73
	ε_{xy}	0.	$1.586\ 10^{-6}$	-
B	u	0.	eps	-
	v	4 10-5	3.9974 10-5	– 0.07
	σ_{xx}	40.	39.9774	– 0.06
	σ_{yy}	0.	0.0786	-
	σ_{zz}	0.	0.	-
	σ_{xy}	0.	– 0.0181	-
	ε_{xx}	2. 10-4	1.998 10-4	– 0.11
	ε_{yy}	– 0.6 10-4	– 0.596 10-4	– 0.67
	ε_{xy}	0.	– 1.176 10-7	-
E	u	-4.17193 10-5	-4.16814 10-5	– 0.09
	v	4.17193 10-5	4.16814 10-5	– 0.09
	σ_{xx}	20.	20.0024	0.01
	σ_{yy}	20.	20.0045	0.02
	σ_{zz}	0.	0.	-
	σ_{xy}	80.	79.8164	– 0.23
	ε_{xx}	0.7 10-4	0.7001 10-4	0.01
	ε_{yy}	0.7 10-4	0.7002 10-4	0.03
	ε_{xy}	$5.2\ 10^{-4}$	$5.188\ 10^{-4}$	– 0.23
F	u	$-2.82843\ 10^{-5}$	$-2.82655\ 10^{-5}$	– 0.07
	v	2.82843 10-5	2.82655 10-5	– 0.07
	σ_{xx}	20.	20.0083	0.04
	σ_{yy}	20.	19.9915	– 0.04
	σ_{zz}	0.	0.	-
	σ_{xy}	20.	20.0138	0.07
	ε_{xx}	0.7 10-4	0.7005 10-4	0.08
	ε_{yy}	0.7 10-4	0.6995 10-4	– 0.08
	ε_{xy}	– 1.3 10-4	1.3009 10-4	0.07

9 Modelization G

9.1 Characteristic of the modelization

Modelization C_PLAN (QUAD9)



limiting Conditions:

side AB blocked in dy
side EF normally blocked
pressure on AE $p=60.$

Names of the nodes: $A=N1$ $B=N347$ $C=N21$
$D=N432$ $E=N39$ $F=N229$

9.2 Characteristics of the mesh

Many nodes: 441
Number of meshes and types: 100 QUAD9

9.3

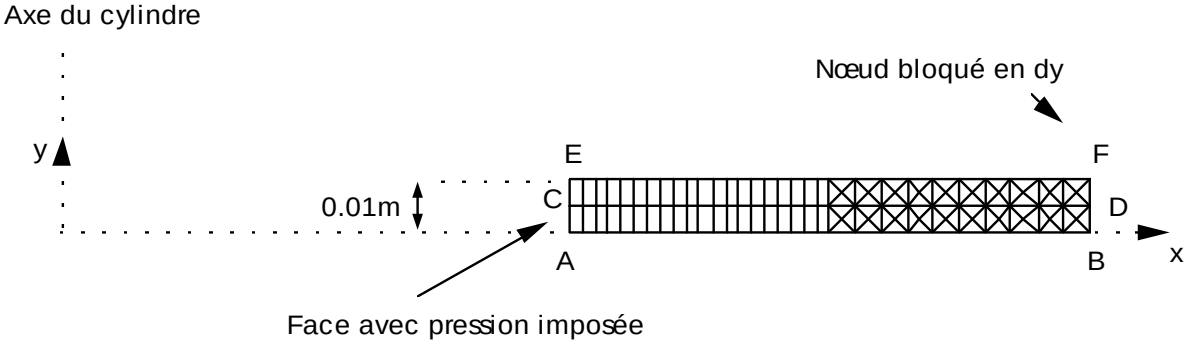
9.4 Quantities tested and results

Localization	Quantity	Reference	*Aster*	% difference
A	u	5.9 10-5	5.9000 10-5	0.00
	v	0.	eps	-
	σ_{xx}	– 60.	– 59.8354	– 0.27
	σ_{vv}	100.	99.8409	– 0.16
	σ_{zz}	0.	0.	-
	σ_{xv}	0.	0.0283	-
	ε_{xx}	– 4.5 10-4	– 4.489 10-4	-0.24
	ε_{vv}	5.9 10-4	5.890 10-4	– 0.18
	ε_{xv}	0.	$-1.839\ 10^{-7}$	-
B	u	$4\ 10^{-5}$	$3.9999\ 10^{-5}$	– 0.001
	v	0.	eps	-
	σ_{xx}	0.	-0.0189	-
	σ_{vv}	40.	40.0182	0.05
	σ_{zz}	0.	0.	-
	σ_{xv}	0.	– 3.6815 10-3	-
	ε_{xx}	– 0.6 10-4	– 0.601 10-4	0.20
	ε_{vv}	2. 10-4	2.001 10-4	0.06
	ε_{xv}	0.	– 2.393 10-8	-
E	u	4.17193 10-5	4.17195 10-5	0.00
	v	4.17193 10-5	4.17195 10-5	0.00
	σ_{xx}	20.	19.9745	– 0.13
	σ_{vv}	20.	20.0311	0.16
	σ_{zz}	0.	0.	-
	σ_{xv}	– 80.	– 79.8382	– 0.20
	ε_{xx}	0.7 10-4	0.698 10-4	– 0.25
	ε_{vv}	0.7 10-4	0.702 10-4	0.28
	ε_{xv}	$-5.2\ 10^{-4}$	$-5.189\ 10^{-4}$	– 0.20
F	u	$2.82843\ 10^{-5}$	$2.82839\ 10^{-5}$	– 0.001
	v	2.82843 10-5	2.82839 10-5	– 0.001
	σ_{xx}	20.	19.9960	– 0.02
	σ_{vv}	20.	20.0034	0.02
	σ_{zz}	0.	0.	-
	σ_{xv}	– 20.	– 20.0185	0.09
	ε_{xx}	0.7 10-4	0.6997 10-4	– 0.04
	ε_{vv}	0.7 10-4	0.7002 10-4	0.03
	ε_{xv}	– 1.3 10-4	– 1.301 10-4	0.09

10 Modelization H

10.1 Characteristic of the modelization

Elements AXIS (TRIA3 + QUAD4)



limiting Conditions:

node F blocked in dy
pressure on AE $p=60.$

Names of the nodes: $A=N111$ $B=N1$ $C=N112$
$D=N3$ $E=N113$ $F=N4$

10.2 Characteristics of the mesh

Many nodes: 113
Number of meshes and types: 40 QUAD4, 80 TRIA3

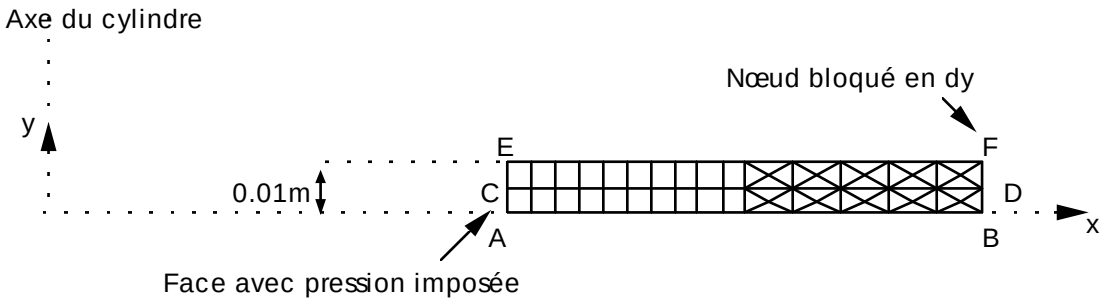
10.3 Quantities tested and results

Localization	Quantity	Reference	*Aster*	% difference
A	u	5.9 10-5	5.8992 10-5	– 0.01
	v	0.	-	-
	σ_{xx}	– 60.	– 56.6060	– 5.66
	σ_{vv}	0.	1.0383	-
	σ_{zz}	100.	101.2924	1.29
	σ_{xv}	0.	– 1.1635	-
	ε_{xx}	– 4.5 10-4	– 4.36 10-4	– 2.99
	ε_{vv}	0.	– 6.18 10-5	-
	ε_{zz}	5.9 10-4	5.898 10-4	– 0.03
	ε_{xv}	0.	$-1.06\ 10^{-6}$	-
B	u	$4\ 10^{-5}$	$3.9997\ 10^{-5}$	– 0.01
	v	0.	-	-
	σ_{xx}	0.	– 0.8951	-
	σ_{vv}	0.	– 0.4106	-
	σ_{zz}	40.	39.6001	– 1.00
	σ_{xv}	0.	– 0.1281	-
	ε_{xx}	– 0.6 10-4	– 0.632 10-4	5.43
	ε_{vv}	0.	– 6.011 10-5	-
	ε_{zz}	2. 10-4	1.999 10-4	– 0.02
	ε_{xv}	0.	– 8.325 10-7	-
E	u	5.9 10-5	5.8992 10-5	– 0.01
	v	0.	-	-
	σ_{xx}	– 60.	– 56.6060	– 5.66
	σ_{vv}	0.	1.0383	-
	σ_{zz}	100.	101.2924	1.29
	σ_{xv}	0.	1.1635	-
	ε_{xx}	– 4.5 10-4	– 4.365 10-4	– 2.99
	ε_{vv}	0.	– 6.184 10-5	-
	ε_{zz}	5.9 10-4	5.898 10-4	– 0.03
	ε_{xv}	0.	$1.063\ 10^{-6}$	-
F	u	$4\ 10^{-5}$	$3.9997\ 10^{-5}$	– 0.01
	v	0.	-	-
	σ_{xx}	0.	– 0.4221	-
	σ_{vv}	0.	– 0.2280	-
	σ_{zz}	40.	39.8015	– 0.50
	σ_{xv}	0.	– 0.0020	-
	ε_{xx}	– 0.6 10-4	– 0.615 10-4	2.45
	ε_{vv}	0.	– 6.021 10-5	-
	ε_{zz}	2. 10-4	1.9998 10-4	– 0.01
	ε_{xv}	0.	– 1.280 10-8	-

11 Modelization I

11.1 Characteristic of the modelization

Elements AXIS (TRIA6 + QUAD8)



limiting Conditions:

Node F blocked in dy
pressure on AE $p=60.$

Names of the nodes: $A=N8$ $B=N174$ $C=N5$
$D=N170$ $E=N3$ $F=N159$

11.2 Characteristics of the mesh

Many nodes: 175
Number of meshes and types: 20 QUAD8, 40 TRIA6

11.3 Quantities tested and results

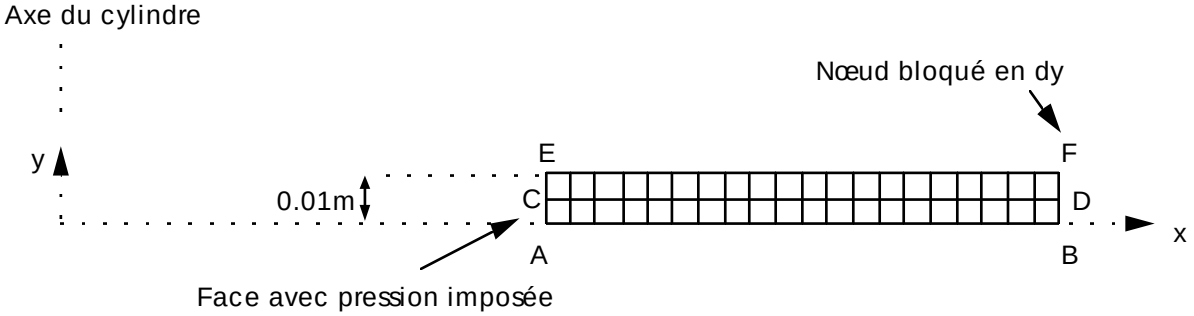
Localization	Quantity	Reference	*Aster*	% difference
A	u	5.9 10-5	5.9000 10-5	0.00
	v	0.	-	-
	σ_{xx}	– 60.	– 59.8976	– 0.17
	σ_{vv}	0.	– 0.0024	-
	σ_{zz}	100.	99.9089	– 0.09
	σ_{xv}	0.	– 0.0137	-
	ε_{xx}	– 4.5 10-4	– 4.493 10-4	– 0.14
	ε_{vv}	0.	– 6.003 10-5	-
	ε_{zz}	5.9 10-4	5.894 10-4	– 0.10
	ε_{xv}	0.	$-8.895\ 10^{-8}$	-
B	u	$4\ 10^{-5}$	$4.0000\ 10^{-5}$	0.00
	v	0.	-	-
	σ_{xx}	0.	0.0308	-
	σ_{vv}	0.	– 0.0020	-
	σ_{zz}	40.	39.9738	– 0.07
	σ_{xv}	0.	0.0131	-
	ε_{xx}	– 0.6 10-4	– 0.598 10-4	– 0.33
	ε_{vv}	0.	– 6.002 10-5	-
	ε_{zz}	2. 10-4	1.998 10-4	– 0.09
	ε_{xv}	0.	8.495 10-8	-
E	u	5.9 10-5	5.9000 10-5	0.00
	v	0.	-	-
	σ_{xx}	– 60.	– 59.8976	– 0.17
	σ_{vv}	0.	– 0.0024	-
	σ_{zz}	100.	99.9089	– 0.09
	σ_{xv}	0.	0.0137	-
	ε_{xx}	– 4.5 10-4	– 4.493 10-4	– 0.14
	ε_{vv}	0.	– 6.003 10-5	-
	ε_{zz}	5.9 10-4	5.894 10-4	– 0.10
	ε_{xv}	0.	$8.895\ 10^{-8}$	-
F	u	$4\ 10^{-5}$	$4.0000\ 10^{-5}$	0.00
	v	0.	-	-
	σ_{xx}	0.	0.0308	-
	σ_{vv}	0.	– 0.0020	-
	σ_{zz}	40.	39.9738	– 0.07
	σ_{xv}	0.	– 0.0131	-
	ε_{xx}	– 0.6 10-4	– 0.598 10-4	– 0.33
	ε_{vv}	0.	– 6.002 10-5	-
	ε_{zz}	2. 10-4	1.998 10-4	– 0.09
	ε_{xv}	0.	– 8.495 10-8	it

has compared to the end of this test a loading in rotation into axisymmetric pure with the same loading as a Fourier mode 0. One finds many identical results.

12 Modelization J

12.1 Characteristic of the modelization

Elements AXIS (QUAD9)



limiting Conditions:

node F blocked in dy
pressure on AE $p=60.$

Names of the nodes: $A=N196$ $B=N1$ $C=N200$
$D=N5$ $E=N202$ $F=N7$

12.2 Characteristics of the mesh

Many nodes: 205
Number of meshes and types: 40 QUAD9

12.3 Quantities tested and results

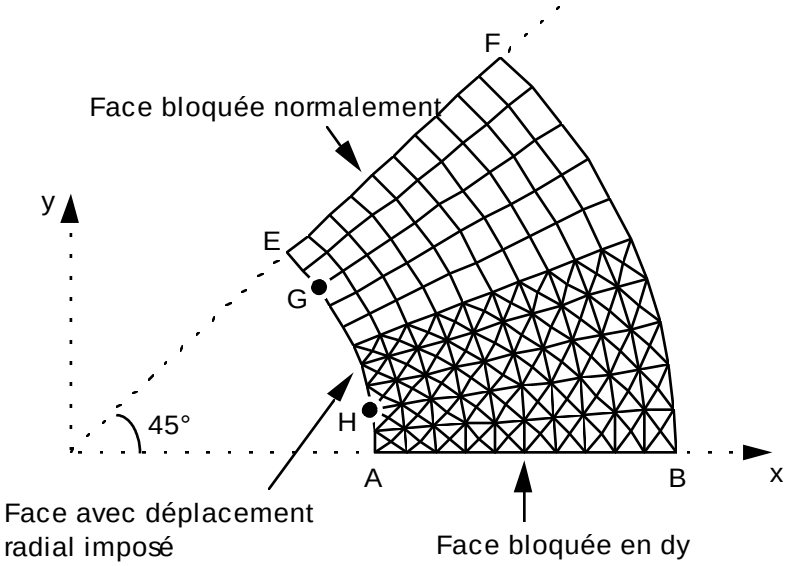
Localization	Quantity	Reference	*Aster*	% difference
A	u	5.9 10-5	5.9000 10-5	0.00
	v	0.	-	-
	σ_{xx}	– 60.	– 59.8997	– 0.17
	σ_{yy}	0.	– 0.0035	-
	σ_{zz}	100.	99.9080	– 0.09
	σ_{xy}	0.	– 0.0141	-
	ε_{xx}	– 4.5 10-4	– 4.494 10-4	– 0.14
	ε_{yy}	0.	– 6.003 10-5	-
	ε_{zz}	5.9 10-4	5.894 10-4	– 0.10
	ε_{xy}	0.	$-9.156\ 10^{-8}$	-
B	u	$4.\ 10^{-5}$	$4.0000\ 10^{-5}$	0.00
	v	0.	-	-
	σ_{xx}	0.	0.0070	-
	σ_{yy}	0.	– 0.0001	-
	σ_{zz}	40.	39.9936	– 0.02
	σ_{xy}	0.	0.0010	-
	ε_{xx}	– 0.6 10-4	– 0.5996 10-4	– 0.07
	ε_{yy}	0.	– 6.000 10-5	-
	ε_{zz}	2. 10-4	1.9996 10-4	– 0.02
	ε_{xy}	0.	6.748 10-9	-
E	u	5.9 10-5	5.9000 10-5	0.00
	v	0.	-	-
	σ_{xx}	– 60.	– 59.8997	– 0.17
	σ_{yy}	0.	– 0.0035	-
	σ_{zz}	100.	99.9080	– 0.09
	σ_{xy}	0.	0.0141	-
	ε_{xx}	– 4.5 10-4	– 4.494 10-4	– 0.14
	ε_{yy}	0.	– 6.003 10-5	-
	ε_{zz}	5.9 10-4	5.894 10-4	– 0.10
	ε_{xy}	0.	$-9.156\ 10^{-8}$	-
F	u	$4.\ 10^{-5}$	$4.0000\ 10^{-5}$	0.00
	v	0.	-	-
	σ_{xx}	0.	0.0070	-
	σ_{yy}	0.	– 0.0001	-
	σ_{zz}	40.	39.9936	– 0.02
	σ_{xy}	0.	– 0.0010	-
	ε_{xx}	– 0.6 10-4	– 0.5996 10-4	– 0.07
	ε_{yy}	0.	– 6.000 10-5	-
	ε_{zz}	2. 10-4	1.9996 10-4	– 0.02
	ε_{xy}	0.	– 6.748 10-9	-

13 Modelization K

13.1 Characteristic of the modelization

Elements 3D (PENTA6 and HEXA8)

Mesh obtained by extrusion starting from the mesh $2D$ below (modelization E)



Along the axis Z : 2 layers of elements
total Thickness: 0.01m

Limiting conditions:

the node is outside the field of definition with a right profile of the EXCLU type node: $F \quad u_z=0$
face AB blocked opposite dy
EF blocked normally
face AE radial displacement imposed on $5.9E-5m$

Names the nodes:	$A=No1$	$C=No36$	$D=No166$
plane $z=0.005$	$A2=No172$	$C2=No242$	$D2=No5025$
plane $z=0.01$	$A3=No173$	$C3=No243$	$D3=No503$
Names of the nodes:	$E=No41$	$H=No9$	$G=No38$
plane $z=0.005$	$E2=No252$	$H2=No188$	$G2=No246$
plane $z=0.01$	$E3=No253$	$H3=No189$	$G3=No247$

13.2 Characteristics of the mesh

Many nodes: 513
Number of meshes and types: 400 PENTA6, 100 HEXA8 40 QUAD4 (sides skin interns)

13.3 Remarks

the loading are here in imposed displacement, contrary to the other modelizations. The reactions are tested.

13.4 Quantities tested and results

Localization	Quantity	Reference	*Aster*	% difference
C	Fx	1.0884 E-3	1.0953 E-3	0.64
	Fy	4.5084 E-4	4.5836 E-4	1.67
C_2	Fx	2.1768 E-3	2.1571 E-3	– 0.91
	Fy	9.0170 E-4	9.1304 E-4	1.26
C_3	Fx	1.0884 E-3	1.0953 E-3	0.64
	Fy	4.5084 E-4	4.5836 E-4	1.67
H	Fx	1.1636 E-3	1.1709 E-3	0.63
	Fy	1.8429 E-4	1.8527 E-4	0.53
G	Fx	1.0045 E-3	1.0144 E-3	0.99
	Fy	6.1550 E-4	6.2117 E-4	0.92
H_2	Fx	2.3272 E-3	2.3173 E-3	– 0.43
	Fy	3.6858 E-4	3.6669 E-4	– 0.51
G_2	Fx	2.0090 E-3	1.9951 E-3	– 0.69
	Fy	1.2310 E-3	1.2214 E-3	– 0.78

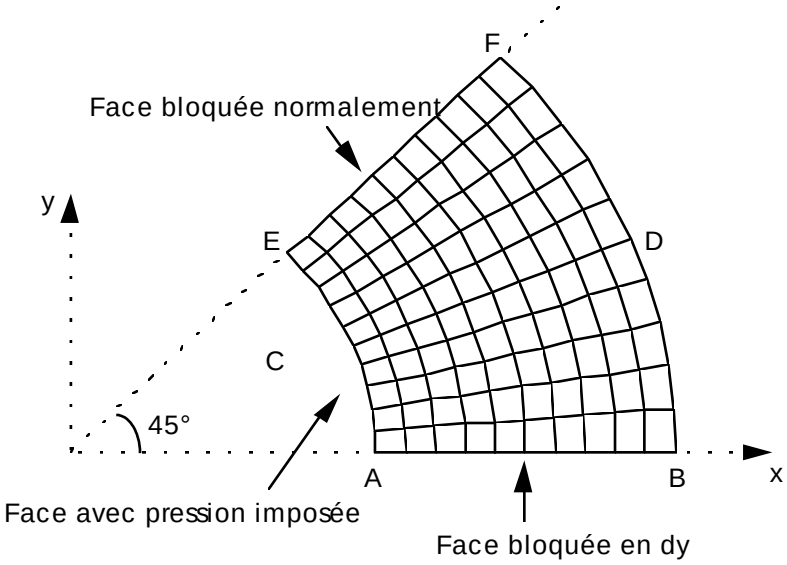
13.5 Remarks

It is checked that the nodal forces of reactions are null in all the nodes, except on the nodes of surface AE and surfaces EF and AB .

14 Modelization L

14.1 Characteristic of the modelization

Elements 3D (PYRAM5)



Along the axis Z : each parallelepiped is cut out in 6 pyramids
total Thickness: 0.01m

Limiting conditions:

the node is outside the field of definition with a right profile of the EXCLU type node: $F \quad u_z=0$
face AB blocked opposite dy
EF blocked normally
pressure on the face AE $p=60.$

Names the nodes: $A=N267$ $B=N142$ $E=N29$ $F=N1$

14.2 Characteristics of the mesh

Many nodes: 342
Number of meshes and types: 600 PYRAM5 620 QUAD4 (sides skin interns)

14.3

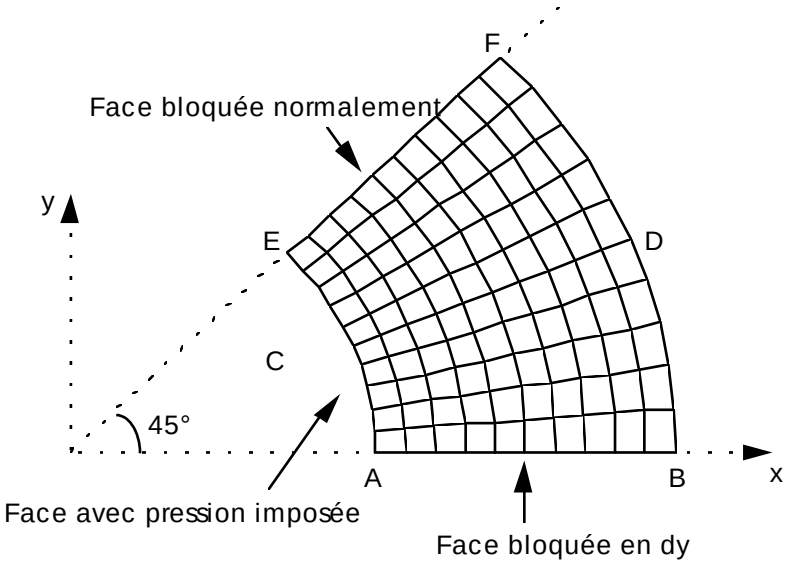
14.4 Quantities tested and results

Identification	Reference	*Aster*	% difference
u - node (A)	5.9 10^{-5}	5.8873 10^{-5}	– 0.21
v - node (A)	0.	eps	-
σ_{xx} - node (A)	– 60.	– 52.9567	– 11.74
σ_{yy} - node (A)	100.	91.7830	– 8.22
σ_{zz} - node (A)	0.	– 1.1206	-
σ_{xy} - node (A)	0.	– 4.5121	-
ε_{xx} - node (A)	– 4.5 10^{-4}	– 4.008 10^{-4}	– 10.94
ε_{yy} - node (A)	5.9 10^{-4}	5.400 10^{-4}	– 8.47
ε_{xy} - node (A)	0.	– 2.933 10^{-5}	-
u - node (B)	4. 10^{-5}	3.9936 10^{-5}	– 0.16
v - node (B)	0.	eps	-
σ_{xx} - node (B)	0.	– 0.7670	-
σ_{yy} - node (B)	40.	39.5319	– 1.17
σ_{zz} - node (B)	0.	– 0.3115	-
σ_{xy} - node (B)	0.	– 1.5858	-
ε_{xx} - node (B)	– 0.6 10^{-4}	– 0.627 10^{-4}	4.44
ε_{yy} - node (B)	2. 10^{-4}	1.993 10^{-4}	– 0.36
ε_{xy} - node (B)	0.	– 1.031 10^{-5}	-
u - node (E)	4.17193 10^{-5}	4.16293 10^{-5}	– 0.21
v - node (E)	4.17193 10^{-5}	4.16293 10^{-5}	– 0.21
σ_{xx} - node (E)	20.	19.3586	– 3.21
σ_{yy} - node (E)	20.	31.5151	57.57
σ_{zz} - node (E)	0.	2.5686	-
σ_{xy} - node (E)	– 80.	– 77.2309	– 3.46
ε_{xx} - node (E)	0.7 10^{-4}	0.457 10^{-4}	– 34.76
ε_{yy} - node (E)	0.7 10^{-4}	1.247 10^{-4}	78.12
ε_{xy} - node (E)	– 5.2 10^{-4}	– 5.020 10^{-4}	– 3.46
u - node (F)	2.82843 10^{-5}	2.82393 10^{-5}	– 0.16
v - node (F)	2.82843 10^{-5}	2.82393 10^{-5}	– 0.16
σ_{xx} - node (F)	20.	18.9523	– 5.24
σ_{yy} - node (F)	20.	20.9510	4.75
σ_{zz} - node (F)	0.	0.0035	-
σ_{xy} - node (F)	– 20.	– 20.9897	4.95
ε_{xx} - node (F)	0.7 10^{-4}	0.633 10^{-4}	– 9.60
ε_{yy} - node (F)	0.7 10-4	0.763 10-4	8.96
ε_{xy} - node (F)	– 1.3 10-4	– 1.364 10-4	4.95

15 Modelization M

15.1 Characteristic of the modelization

Elements 3D (PYRAM13)



Along the axis Z : each parallelepiped is cut out in 6 pyramids
total Thickness: 0.01m

Limiting conditions:

the node is outside the field of definition with a right profile of the EXCLU type
node: F $u_z=0$
face AB blocked opposite dy
EF blocked normally
pressure on the face AE $p=60.$

Names the nodes: $A=N1403$ $B=N734$ $E=N152$ $F=N4$

15.2 Characteristics of the mesh

Many nodes: 1703
Number of meshes and types: 600 PYRAM13 620 QUAD8 (sides skin interns)

15.3

15.4 Quantities tested and results

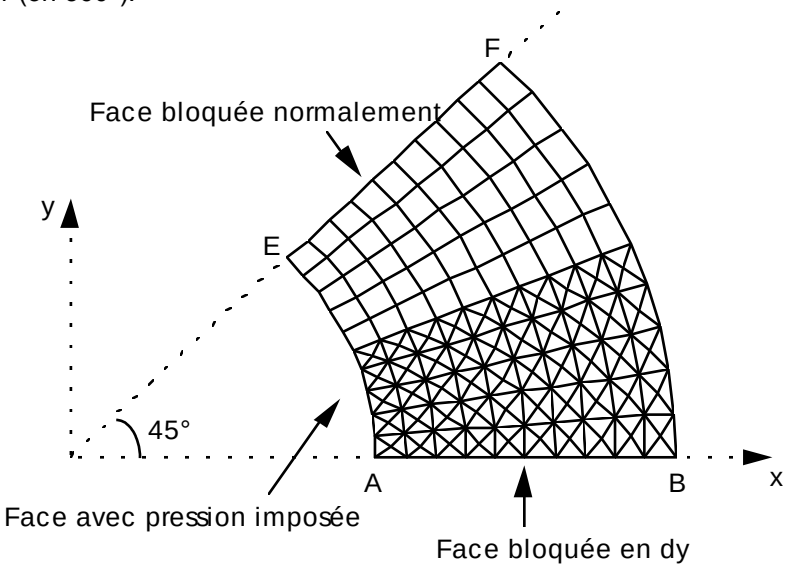
Identification	Reference	*Aster*	% difference
u - node (A)	$5.9\ 10^{-5}$	$5.8999\ 10^{-5}$	– 0.002
v - node (A)	0.	eps	-
σ_{xx} - node (A)	– 60.	– 59.9880	– 0.02
σ_{yy} - node (A)	100.	100.1277	0.13
σ_{zz} - node (A)	0.	0.0425	-
σ_{xy} - node (A)	0.	– 0.0913	-
ε_{xx} - node (A)	$-4.5\ 10^{-4}$	$-4.502\ 10^{-4}$	0.04
ε_{yy} - node (A)	$5.9\ 10^{-4}$	$5.906\ 10^{-4}$	0.09
ε_{xy} - node (A)	0.	$-5.934\ 10^{-7}$	-
u - node (B)	$4.\ 10^{-5}$	$4.0000\ 10^{-5}$	0.00
v - node (B)	0.	eps	-
σ_{xx} - node (B)	0.	– 0.0276	-
σ_{yy} - node (B)	40.	40.0331	0.08
σ_{zz} - node (B)	0.	0.0024	-
σ_{xy} - node (B)	0.	0.0126	-
ε_{xx} - node (B)	$-0.6\ 10^{-4}$	$-0.602\ 10^{-4}$	0.32
ε_{yy} - node (B)	$2.\ 10^{-4}$	$2.002\ 10^{-4}$	0.10
ε_{xy} - node (B)	0.	$8.177\ 10^{-8}$	-
u - node (E)	$4.17193\ 10^{-5}$	$4.17183\ 10^{-5}$	– 0.002
v - node (E)	$4.17193\ 10^{-5}$	$4.17183\ 10^{-5}$	– 0.002
σ_{xx} - node (E)	20.	19.9787	-0.11
σ_{yy} - node (E)	20.	20.1612	0.81
σ_{zz} - node (E)	0.	0.0425	-
σ_{xy} - node (E)	– 80.	– 80.0580	0.07
ε_{xx} - node (E)	$0.7\ 10^{-4}$	$0.696\ 10^{-4}$	– 0.59
ε_{yy} - node (E)	$0.7\ 10^{-4}$	$0.708\ 10^{-4}$	1.11
ε_{xy} - node (E)	$-5.2\ 10^{-4}$	$-5.204\ 10^{-4}$	0.07
u - node (F)	$2.82843\ 10^{-5}$	$2.82844\ 10^{-5}$	0.00
v - node (F)	$2.82843\ 10^{-5}$	$2.82844\ 10^{-5}$	0.00
σ_{xx} - node (F)	20.	20.0224	0.11
σ_{yy} - node (F)	20.	19.9901	– 0.05
σ_{zz} - node (F)	0.	0.0031	-
σ_{xy} - node (F)	– 20.	– 19.9818	– 0.09
ε_{xx} - node (F)	$0.7\ 10^{-4}$	$0.701\ 10^{-5}$	0.17
ε_{yy} - node (F)	0.7 10-4	0.699 10-4	– 0.13
ε_{xy} - node (F)	– 1.3 10-5	– 1.299 10-5	– 0.09

16 Modelization N

16.1 Characteristic of the modelization

Elements 3D (PENTA15 and HEXA20)

Mesh obtained by extrusion from a mesh $2D$ resembling the mesh below (8 elements in the radial direction, $4+4$ elements in the direction circumferential) and duplicated to have a complete section of the cylinder (on 360°).



Along the axis Z : 1 layer of elements
total Thickness: 0.01m

Limiting conditions:

face AB blocked opposite dy
EF blocked normally

pressure on the face AE $p=60.$
basic effect on the sections $p=60.$

Names of the nodes: $A=N5349$ $B=N6092$ $C=N433$
$D=N441$ $E=N2180$ $F=N1632$

16.2 Characteristics of the mesh

Many nodes: 8832
Number of meshes and types: 1024 PENTA15, 512 HEXA20, 1176 QUAD8 and 2048 TRIA6.

16.3 Remarks

Contrary to the preceding modelizations, one takes into account here the basic effect applying to the sections at the ends of the cylinder.

16.4

16.5 Quantities tested and results

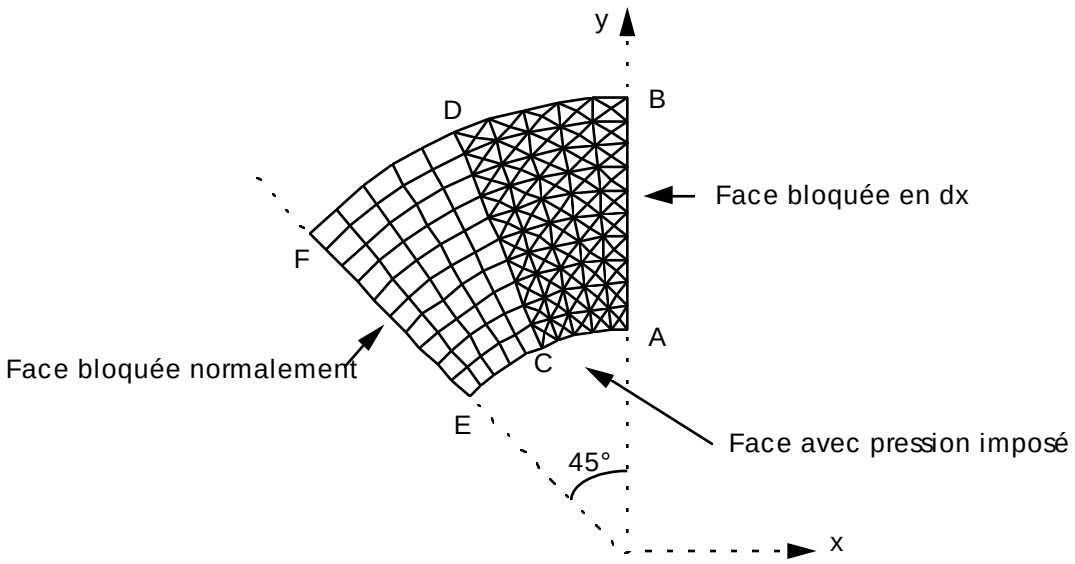
Identification	Reference	*Aster*	% difference
u - node (A)	$5.6\ 10^{-5}$	$5.6000\ 10^{-5}$	0.00
v - node (A)	0.	eps	-
σ_{xx} - node (A)	– 60.	-59.5972	– 0.67
σ_{yy} - node (A)	100.	99.5187	– 0.48
σ_{zz} - node (A)	20.	19.9584	– 0.21
σ_{xy} - node (A)	0.	eps	-
u - node (B)	$3.4\ 10^{-5}$	$3.4000\ 10^{-5}$	0.00
v - node (B)	0.	eps	-
σ_{xx} - node (B)	0.	$2.0877\ 10^{-2}$	-
σ_{yy} - node (B)	40.	39.9685	– 0.08
σ_{zz} - node (B)	20.	19.9946	– 0.03
σ_{xy} - node (B)	0.	eps	-
σ_{xx} - node (E)	20.	20.3287	1.64
σ_{yy} - node (E)	20.	20.3287	1.64
σ_{zz} - node (E)	20.	20.1739	0.87
σ_{xy} - node (E)	– 80.	-79.9775	– 0.03
σ_{xx} - node (F)	20.	20.0176	0.09
σ_{yy} - node (F)	20.	20.0176	0.09
σ_{zz} - node (F)	20.	20.0072	0.04
σ_{xy} - node (F)	– 20.	-20.0027	0.01

17 Modelization O

17.1 Characteristic of the modelization

Elements C_PLAN (QUAD8 + TRIA6)

Mesh $2D$ resembling the mesh below (8 elements in the radial direction, $4+4$ elements in the direction circumferential) and duplicated to have a complete section of the cylinder (on 360°).



Limiting conditions:

side AB blocked in dx
side EF normally blocked
pressure on AE $p=60.$

Names of the nodes: $A=N249$ $B=N992$ $C=N1667$
$D=N1588$ $E=N3776$ $F=N3228$

17.2 Characteristics of the mesh

Many nodes: 3840
Number of meshes and types: 1026 TRIA6, 512 QUAD8

17.3

17.4 Quantities tested and results

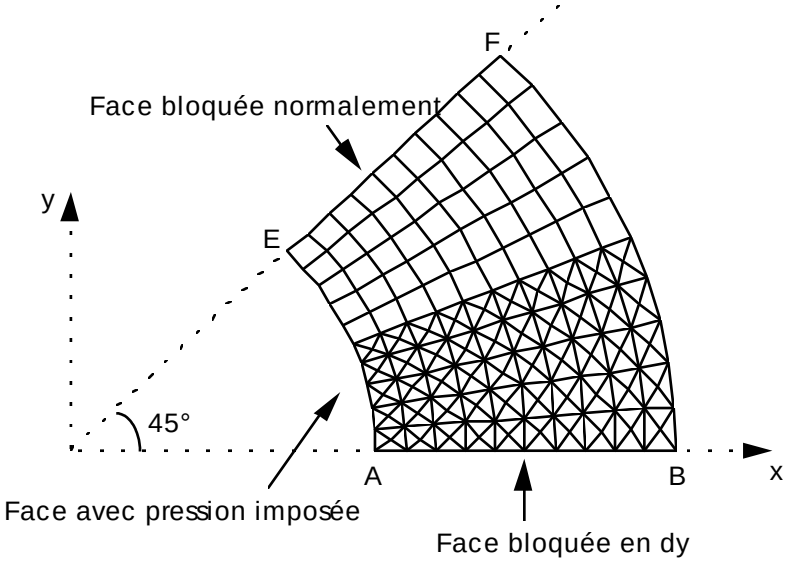
Identification	Reference	*Aster*	% difference
u - node (A)	5.9 10^{-5}	5.8999 10^{-5}	– 0.002
v - node (A)	0.	eps	-
σ_{xx} - node (A)	– 60.	-59.5340	– 0.78
σ_{vv} - node (A)	100.	99.5453	– 0.45
σ_{zz} - node (A)	0.	0.	-
σ_{xv} - node (A)	0.	eps	-
u - node (B)	4. 10^{-5}	3.99996 10^{-5}	0.00
v - node (B)	0.	0.	-
σ_{xx} - node (B)	0.	2.6874 10^{-2}	-
σ_{vv} - node (B)	40.	39.9716	– 0.07
σ_{zz} - node (B)	0.	0.	-
σ_{xv} - node (B)	0.	eps	-
u - node (E)	4.17193 10^{-5}	4.17215 10^{-5}	0.005
v - node (E)	4.17193 10^{-5}	4.17215 10^{-5}	0.005
σ_{xx} - node (E)	20.	20.2875	1.44
σ_{vv} - node (E)	20.	20.2875	1.44
σ_{zz} - node (E)	0.	0.	-
σ_{xv} - node (E)	– 80.	-79.9196	– 0.10
u - node (F)	2.82843 10^{-5}	2.82841 10^{-5}	– 0.001
v - node (F)	2.82843 10^{-5}	2.82841 10^{-5}	– 0.001
σ_{xx} - node (F)	20.	20.0167	0.08
σ_{vv} - node (F)	20.	20.0167	0.08
σ_{zz} - node (F)	0.	0.	-
σ_{xv} - node (F)	– 20.	-19.9993	– 0.004

18 Modelization P

18.1 Characteristic of the modelization

Elements 3D (PENTA15 and HEXA20) – even mesh that the modelization N

Mesh obtained by extrusion from a mesh $2\mathrm{D}$ resembling the mesh below (8 elements in the radial direction, $4+4$ elements in the direction circumferential) and duplicated to have a complete section of the cylinder (on 360°).



Along the axis Z : 1 layer of elements
total Thickness: $0.01\mathrm{m}$

Limiting conditions:

normally EF blocked face
face AB blocked in dy
pressure on the face AE fp
basic effect on the sections fp

With fp : linear pressure function of time being worth $60.$ with $t=1\mathrm{s}$ and $120.$ $t=2\mathrm{s}$

Names of the nodes: $A=N5349$ $B=N6092$ $C=N433$
$D=N441$ $E=N2180$ $F=N1632$

18.2 Characteristics of the mesh

Many nodes: 8832
Number of meshes and types: 1024 PENTA15, 512 HEXA20, 1176 QUAD8 and 2048 TRIA6.

18.3 Remarks

Contrary to the modelization N, one tests here a basic pressure and effect variables according to time. A linear variation of the pressure involves a linear variation of the stresses.

18.4

18.5 Quantities tested and results

Identification	Reference	*Aster*	% difference
U-node (A) to t=1. S	5.6 10-5	5.6000 10-5	0.00
V-node (A) with t=1. S	0.	eps	-
σ_{xx} - node (A) with t=1. S	– 60.	-59.5414	– 0.76
σ_{vv} - node (A) with t=1. S	100.	99.6183	– 0.38
σ_{zz} - node (A) with t=1. S	20.	19.9976	– 0.01
σ_{xv} - node (A) with t=1. S	0.	eps	-
σ_{xx} - node (A) with t=2. S	– 120.	-119.0829	– 0.76
σ_{vv} - node (A) with t=2. S	200.	199.2366	– 0.38
σ_{zz} - node (A) with t=2. S	40.	39.9952	– 0.01
σ_{xv} - node (A) with t=2. S	0.	eps	-
U-node (B) with t=1. S	3.4 10-5	3.4000 10-5	0.00
V-node (B) with t=1. S	0.	eps	-
σ_{xx} - node (B) with t=1. S	0.	2.6761 10-2	-
σ_{vv} - node (B) with t=1. S	40.	39.9740	– 0.06
σ_{zz} - node (B) with t=1. S	20.	19.9973	– 0.01
σ_{xv} - node (B) with t=1. S	0.	eps	-
σ_{xx} - node (B) with t=2. S	0.	5.3523 10-2	-
σ_{vv} - node (B) with t=2. S	80.	79.9480	– 0.06
σ_{zz} - node (B) with t=2. S	40.	39.9946	– 0.01
σ_{xv} - node (B) with t=2. S	0.	eps	-
σ_{xx} - node (E) with t=1. S	20.	20.3287	1.64
σ_{vv} - node (E) with t=1. S	20.	20.3287	1.64
σ_{zz} - node (E) with t=1. S	20.	20.1739	0.87
σ_{xv} - node (E) with t=1. S	– 80.	-79.9775	– 0.03
σ_{xx} - node (E) with t=2. S	40.	40.6575	1.64
σ_{vv} - node (E) with t=2. S	40.	40.6575	1.64
σ_{zz} - node (E) with t=2. S	40.	40.3479	0.87
σ_{xv} - node (E) with t=2. S	– 160.	-159.9550	– 0.03
σ_{xx} - node (F) with t=1. S	20.	20.0176	0.09
σ_{vv} - node (F) with t=1. S	20.	20.0176	0.09
σ_{zz} - node (F) with t=1. S	20.	20.0072	0.04
σ_{xv} - node (F) with t=1. S	– 20.	-20.0027	0.01
σ_{xx} - node (F) with t=2. S	40.	40.0351	0.09
σ_{vv} - node (F) with t=2. S	40.	40.0351	0.09
σ_{zz} - node (F) with t=2. S	40.	40.0144	0.04
σ_{xv} - node (F) with t=2. S	– 40.	-40.0054	0.01

19 Summary of the Summary

results of the errors max in %

3D	Localization	MOD A	MOD B	MOD C	MOD D	MOD L	MOD M	MOD N	MOD P
elem		pe6, h8	pe15, h20	te4	te10	py5	py13	pe15, h20	pe15, h20
geom		45°	45°	45°	45°	45°	45°	360°	360°
Nb No		1922	2115	1115	1395	342	1703	8832	8832
Dépl.	A, E	0.08	0.09	0.17	0.04	0.21	0.00	0.00	0.00
	B, F	0.10	0.07	0.30	0.04	0.16	0.00	0.00	0.00
σ_{xx}	A, E	4.59	0.39	10.45	4.41	11.74	0.11	1.64	1.64
	B, F	5.24	0.07	7.78	0.95	5.24	0.11	0.09	0.09
σ_{vv}	A, E	5.70	0.92	9.46	1.80	57.57	0.81	1.64	1.64
	B, F	1.89	0.01	2.46	0.49	4.75	0.08	0.09	0.09
σ_{zz}	A, E	Good	Good	Good	Good	Good	Good	0.87	0.87
	B, F	Good	Good	Good	Good	Good	Good	0.04	0.04
σ_{xv}	A, E	0.15	0.26	2.89	0.27	3.46	0.07	0.03	0.03
	B, F	0.90	0.06	0.81	0.28	4.95	0.09	0.01	0.01

C_PLAN	Localization	MOD E	MOD F	MOD G	MOD O
Type of elements		tria3, quad4	tria6, quad8	quad9	tria6, quad8
modelled Geometry		45°	45°	45°	360°
Many nodes		961.591.441			384
Displacements	A, E	0.07	0.09	0.00	0.01
	B, F	0.09	0.07	0.00	0.00
Stresses σ_{xx}	A, E	4.65	0.39	0.27	1.44
	B, F	5.19	0.06	0.02	0.08
Stresses σ_{yy}	A, E	5.68	0.90	0.16	1.44
	B, F	1.40	0.04	0.05	0.08
Stresses σ_{zz}	A, E	Good	Good	Good	Good
	B, F	Good	Good	Good	Good
Stresses σ_{xy}	A, E	0.16	0.23	0.20	0.10
	B, F	1.23	0.07	0.09	0.00

AXIS	Localization	MOD H	MOD I	MOD J
Standard of elements		tria3, quad4	tria6, quad8	quad9
Many nodes		113.175.205		
Displacements	A, E	0.01	0.00	0.00
	B, F	0.01	0.00	0.00
Stresses σ_{xx}	A, E	5.66	0.17	0.17
	B, F	Good	Good	Good
Stresses σ_{yy}	A, E	Good	Good	Good
	B, F	Good	Good	Good
Stresses σ_{zz}	A, E	1.29	0.09	0.09
	B, F	1.00	0.07	0.02
Stresses σ_{xy}	A, E	the Good	Good	Good
	B, F	Good	Good	Good

- results are more precise with elements of order 2.
- The problem is adapted more to an axisymmetric modelization. The results are better.
- The meshes remain insufficient 3D for the elements of order 1: stresses and strains of the modelizations A, C, E and L (especially for the modelization L in PYRAM5).
- The pyramids 3D give results similar to the other elements, with equivalent mesh.

- The modelizations N and P with basic effect and pressure constant or variables give of good results.