SSLS503 - Plate stratified in skew-symmetric bending Summarized stacking simply

supported:

This test represents a quasi-static computation of a laminated plate, in skew-symmetric bending stacking, simply supported, subjected to a pressure uniformly distributed.

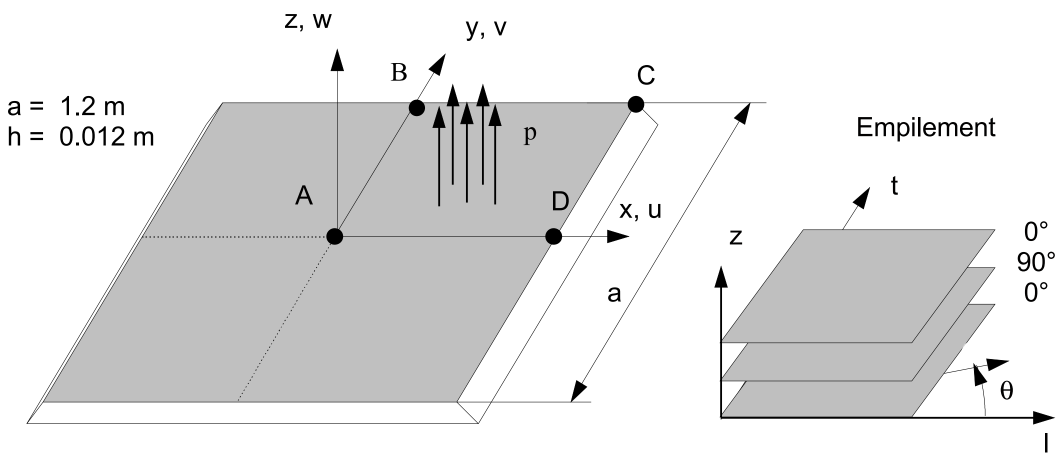
The 4 modelizations make it possible to validate:

- the modelizations finite elements DKT (QUAD4, TRIA3) and DST (QUAD4, TRIA3) in the case of a composite (3 different layers of directional sense),
- the shearing stresses transverse.

Displacements and the forced obtained are compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



1.2 Properties of the material

the properties of the material constituting the plate are the following ones:

One-way (U):

$E_l = 4.10^{10}\,Pa$ $\quad$ $E_t = 0.16\,10^{10}\,Pa$ $\quad$ ($l \Leftrightarrow x$; $t \Leftrightarrow y$)

$G_{lt} = G_{lz} = 8.10^8\,Pa$ $\quad$ $G_{tz} = 3.2\,10^8\,Pa$

$\nu_{lt} = 0.25$

Stacking:

- Directional sense: $[0/90/0]$
- Nature: $[U/U/U]$
- Thickness: $[h/3/h/3/h/3]$

1.3 Boundary conditions and loadings

- the plate is simply leaned on its contour
- Pressure uniformly distributed : $p = 3000\,Pa$

1.4 Initial conditions

Without Reference solution

2 object

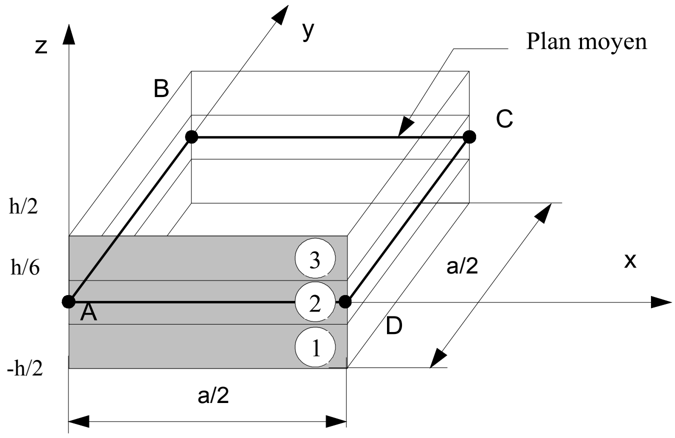
2.1 Method of calculating used for the reference solution

Displacement: analytical solution obtained by decomposition in series of the form:

$$w=\sum_i \sum_j w_{ij} \sin\left(\frac{i\pi x}{a}\right) \sin\left(\frac{j\pi y}{b}\right)$$

Stresses: numerical solution [bib1], [bib2]

2.2 Results of reference



the results of reference are the following:

$w(0,0,0)$	$0.01507\,m$	Displacement w in the center of the plate (not A),
$SIXX(0,0,h/2)$	$2.4216\,10^7\,Pa$	Forced σ_{xx} on the skin higher of the layer 3 ($z=h/2$) than the center of the plate (not A),
$SIYY(0,0,h/6)$ layer with $90°$	$5.7810\,10^6\,Pa$	Stress σ_{yy} on the skin higher of the layer 2 ($z=h/6$) than the center of the plate (not A),
$SIXY(a/2,a/2,h/2)$	$1.2825\,10^6\,Pa$	Forced σ_{xy} at the point C on the higher skin of the layer 3 ,
$SIXZ(a/2,0,0)$	$2.3526\,10^5\,Pa$	Stress σ_{xz} at the point D on the average skin of the layer 2 ($z=0$),
$SIYZ(0,a/2,0)$	$8.8950\,10^4\,Pa$	Forced σ_{yz} at the point B on the average skin of the layer 2 ($z=0$),

2.3 Uncertainties on the solution

- the reference solution is given for a number of terms in the series equal to 25.
- The factor of correction of the transverse shears used is of 5/6.
- With an important slenderness ($a/h=100$), the transverse level of shears is weak and thus difficult to obtain with accuracy. There exists an uncertainty then on the values of stress σ_{ij} calculated during the validation of the test $VPCS$, the differences obtained by the software on the components of the shears are about 10% .

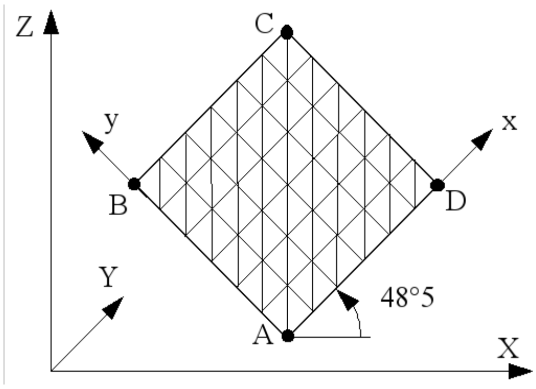
2.4 Bibliographical references

1) VPCS: Software package of composite structural analysis; Examples of validation. Review of the composites and advanced materials, Volume 5 - number except series 1995. Hermes edition.

2) PUTCHA, N.S. and REDDY, J.N. : A mixed shear flexible finite element for the analysis of laminated punts, computer meth. in applied mech. Eng. 44 (1984).

3 Modelization A

3.1 Characteristic of the modelization



Modelization `DKT` (`TRIA3`)

- the plate is located in the plane $Y=0.5$
- Not A $(0.4;0.5;0.25)$
- Boundary conditions:
  - ◦Side BC : $v=0$
  - ◦Side CD : $v=0$
- Conditions of symmetry: (local coordinate system)
  - ◦Side AB : $u=\theta_y=0$
  - ◦Side AD : $v=\theta_x=0$

3.2 Characteristics of the mesh

Many nodes: 49
Number of meshes and types : 72 `TRIA3`

3.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Toléranceformule $(\%)$
$w(0,0,0)$	`ANALYTIQUE`	0.01507	1.1
$SIXX(0,0,h/2)$	`SOURCE_EXTERNE`	2.4216 107.2.1	
$SIYY(0,0,h/6)$ layer to $90°$	`SOURCE_EXTERNE`	$5.7810\ 10^{6.2.7}$	
$SIXY(a/2,a/2,h/2)$	`SOURCE_EXTERNE`	$1.2825\ 10^{6.4.6}$	
$SIXZ(a/2,0,0)$	`SOURCE_EXTERNE`	– 2.3526 105	37
$SIYZ(0,a/2,0)$	`SOURCE_EXTERNE`	8.8950 104.3.1	

3.4 Remarks

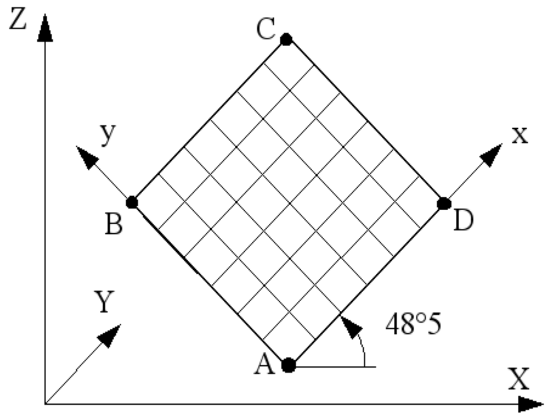
the stresses are expressed in orthotropic reference defined by `ANGL_REP` (`AFFE_CARA_ELEM`), and the norm of the element.

The components $SIXX$, $SIYY$ and $SIYZ$ are the mean values of two the meshes convergent ones at the points A and C .

The variation obtained on $SIXZ$ is due unlike modelization of the transverse shears: in the reference, one uses a coefficient of transverse correction of shears of 5/6. In *Code_Aster*, one calculates the distribution of the shears in the thickness, presumedly parabolic in each layer.
The sign of $SIXZ$ is opposed to that of the reference solution.

4 Modelization B

4.1 Characteristic of the modelization



Modelization DKT (QUAD4)
- the plate is located in the plane $Y=0.5$
- Not A $(0.4\,;0.5\,;0.25)$
- Boundary conditions:
  - Side BC : $v=0$
  - Side CD : $v=0$
- Conditions of symmetry: (local coordinate system)
  - Side AB : $u=\theta_y=0$
  - Side AD : $v=\theta_x=0$

4.2 Characteristics of the mesh

Many nodes: 49
Number of meshes and types: 36 QUAD4

4.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Toléranceformu le $(\%)$
$w(0,0,0)$	ANALYTIQUE	0.01507	1.1
$SIXX(0,0,h/2)$	SOURCE_EXTERNE	2.4216 107.1.1	
$SIYY(0,0,h/6)$ lay down with $90°$	SOURCE_EXTERNE	5.7810 10 6.1.1	
$SIXY(a/2,a/2,h/2)$	SOURCE_EXTERNE	1.2825 10 6.5.1	
$SIXZ(a/2,0,0)$	SOURCE_EXTERNE	– 2.3526 105	16
$SIYZ(0,a/2,0)$	SOURCE_EXTERNE	8.8950 104.4.1	

4.4 Remarks

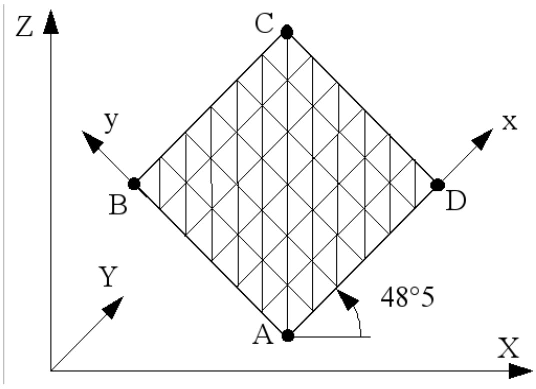
the components $SIXX$, $SIYY$ and $SIYZ$ are the mean values of two the meshes convergent ones at the points A and C .

The variation obtained on $SIXZ$ is due unlike modelization of the transverse shears: in the reference, one uses a coefficient of transverse correction of shears of 5/6. In *Code_Aster*, one calculates the distribution of the shears in the thickness, presumedly parabolic in each layer.

The sign of $SIXZ$ is opposed to that of the reference solution.

5 Modelization C

5.1 Characteristic of the modelization



Modelization DST (TRIA3)
- the plate is located in the plane $Y=0.5$
- Not A $(0.4;0.5;0.25)$
- Boundary conditions:
  - Side BC : $v=0$
  - Side CD : $v=0$
- Conditions of symmetry: (local coordinate system)
  - Side AB : $u=\theta_y=0$
  - Side AD : $v=\theta_x=0$

5.2 Characteristics of the mesh

Many nodes: 49
Number of meshes and types: 72 TRIA3

5.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Toléranceformu le $(\%)$
$w(0,0,0)$	ANALYTIQUE	0.01507	2.1
$SIXX(0,0,h/2)$	SOURCE_EXTERNE	2.4216 107.7.1	
$SIYY(0,0,h/6)$ lay down with $90°$	SOURCE_EXTERNE	5.7810 10 6	24.
$SIXY(a/2,a/2,h/2)$	SOURCE_EXTERNE	1.2825 10 6.4.1	
$SIXZ(a/2,0,0)$	SOURCE_EXTERNE	– 2.3526 105	37
$SIYZ(0,a/2,0)$	SOURCE_EXTERNE	8.8950 104	26

5.4 Remarks

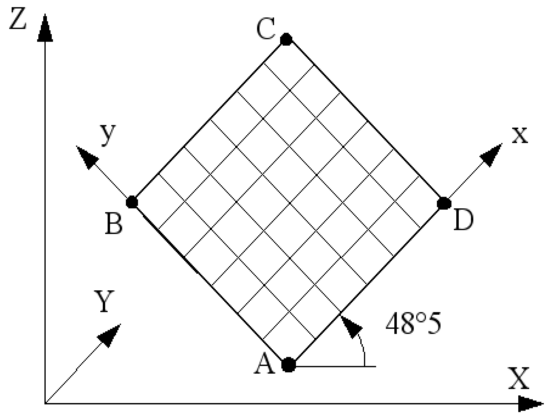
the components $SIXX$, $SIYY$ and $SIYZ$ are the mean values of two the meshes convergent ones at the points A and C .

The variation obtained on $SIXZ$ is due unlike modelization of the transverse shears: in the reference, one uses a coefficient of transverse correction of shears of 5/6. In *Code_Aster*, one calculates the distribution of the shears in the thickness, presumedly parabolic in each layer.
The sign of $SIXZ$ is opposed to that of the reference solution.

The other variations are probably due to the anisotropy of the triangular mesh.

6 Modelization D

6.1 Characteristic of the modelization



Modelization DST (QUAD4)
- the plate is located in the plane $Y=0.5$
- Not A $(0.4;0.5;0.25)$
- Boundary conditions:
  - Side BC : $v=0$
  - Side CD : $v=0$
- Conditions of symmetry: (local coordinate system)
  - Side AB : $u=\theta_y=0$
  - Side AD : $v=\theta_x=0$

6.2 Characteristics of the mesh

Many nodes: 49
Number of meshes and types: 36 QUAD4

6.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Toléranceformu le $(\%)$
$w(0,0,0)$	ANALYTIQUE	0.01507	1.1
$SIXX(0,0,h/2)$	SOURCE_EXTERNE	2.4216 107.1.1	
$SIYY(0,0,h/6)$ lay down with $90°$	SOURCE_EXTERNE	5.7810 10 6.1.1	
$SIXY(a/2,a/2,h/2)$	SOURCE_EXTERNE	1.2825 10 6.7.1	
$SIXZ(a/2,0,0)$	SOURCE_EXTERNE	– 2.3526 105	15.
$SIYZ(0,a/2,0)$	SOURCE_EXTERNE	8.8950 104.2.1	

6.4 Remarks

the components $SIXX$, $SIYY$ and $SIYZ$ are the mean values of two the meshes convergent ones at the points A and C .

The variation obtained on $SIXZ$ is due unlike modelization of the transverse shears: in the reference, one uses a coefficient of transverse correction of shears of 5/6. In *Code_Aster*, one calculates the distribution of the shears in the thickness, presumedly parabolic in each layer.
The sign of $SIXZ$ is opposed to that of the reference solution.

7 Summary of the Displacements

- **results** : some is the modelization used (DKT or DST) the results are satisfactory, the maximum error is lower than 0.7% .
- **Plane stresses** : the results are more precise with the modelization DKT, the error is lower than 1% except for $SIXY$ (QUAD4) where the error is of 5% . For modelization DST the error is higher ($<8\%$) with an important variation on $SIXX$ (28%) for mesh TRIA3.
- **Transverse shears** : some is the modelization used (DKT or DST) the results got with the quadrangular meshes are closer to the reference solution than those obtained with triangular meshes. In the first case the error on the component $SIXZ$ is lower than 15% , and the error on $SIYZ$ is lower than 3% , while in the second case, the error on $SIXZ$ is of 35% and that on $SIYZ$ is understood enters 2% and 24% . Except the least good accuracy of the triangular meshes because of their anisotropy, the variation which remains with quadrangular meshes is due unlike modelization of the transverse shears: in the reference, one uses a coefficient of transverse correction of shears of 5/6. In Code_Aster, one calculates the distribution of the shears in the thickness, presumedly parabolic in each layer.