SSLS502 - Orthotropic cylinder subjected to line of load

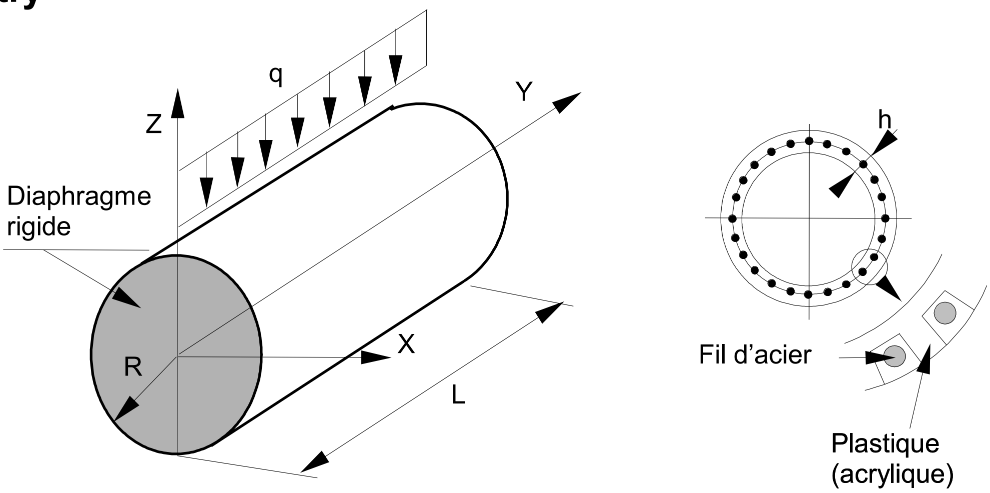
Summarized:

This test represents quasi-static computation, of a short orthotropic cylinder and an orthotropic long cylinder subjected to line of load. At their ends, the cylinders rest on rigid diaphragms. This benchmark makes it possible to validate the modelization finite elements DST with meshes the TRIA3 and QUAD4, an orthotropic homogeneous material.

Displacements and the forces obtained are compared with an experimental reference solution like to an analytical solution.

1 Problem of reference

1.1 Geometry



cylindre court :	L = 0.560 m R = 0.13595 m	h = 0.0061 m q = 2357.143 N/m
cylindre long :	L = 2.465 m R = 0.13595 m	h = 0.0061 m q = 896.552 N/m

1.2 Properties of the material

the material constituting the cylinder is homogeneous orthotropic. The axes of orthotropy correspond to the curvilinear directions x and y .

$$[H_{membrane}]=h[H] \; ; \; [H_{membrane-flexion}]=[0] \; ; \; [H_{flexion}]=h^3[H]/12$$

$$H_{11}=3.0644\times10^9 N/m^2 \quad ; \quad H_{12}=1.1048\times10^9 N/m^2 \quad ; \quad H_{13}=0$$
$$H_{22}=18.597\times10^9 N/m^2 \quad ; \quad H_{23}=0 \quad ; \quad H_{33}=1.250\, x\, 10^9 N/m^2$$

1.3 Boundary conditions and loadings

- Boundary conditions: The ends of the cylinder rest on rigid diaphragms

- **Modelizations A and b:** Force per unit length: $q=-2357.143\, N/m$
- **Modelizations C and D**: Force per unit length: $q=-896.552.\, N/m$

1.4 Initial conditions

Without Reference solution

2 object

2.1 Method of calculating used for the reference solution

We will use for this test two reference solutions, one experimental, resulting from works from Schwaighofer and Microys [bib2], the other drawn from works of Batoz in theory of the deep shells [bib1].

2.2 Results of reference

the results of reference are the following:

Roll short (A and B)	Batoz [bib1]	Experiment [bib2]
Displacement w at the point F	$0.35\,10^{-4}\,m$	$0.6\,10^{-4}\,m$
Displacement w at the point C	$-0.7\,10-3\,m$	$-0.6\,10^{-3}\,m$
Displacement w at the point D	$0.25\,10^{-4}\,m$	$0.1\,10^{-3}\,m$
Forced σ_{xx} at the point F	$-0.35\,MPa$	$-0.325\,MPa$
Forced σ_{yy} at the point F	$0.50\,MPa$	$0.60\,MPa$

Rolls long (C and D)	Batoz [bib1]	Experiment [bib2]
Displacement w at the point F	$1.32\,10^{-3}\,m$	$1.35\,10^{-3}\,m$
Displacement w at the point C	$-2.45\,10^{-3}\,m$	$-2.46\,10^{-3}\,m$
Displacement w at the point D	$-0.35\,10^{-3}\,m$	$-0.51\,10^{-3}\,m$
Forced σ_{xx} at the point F	$1.68\,MPa$	$1.9\,MPa$
Forced σ_{yy} at the point F	$1.8\,MPa$	$1.55\,MPa$

2.3 Uncertainties on the solution

Approximately 5% with regard to the solution of Batoz, undoubtedly much more – approximately 30% - for the experimental solution.

2.4 Bibliographical references

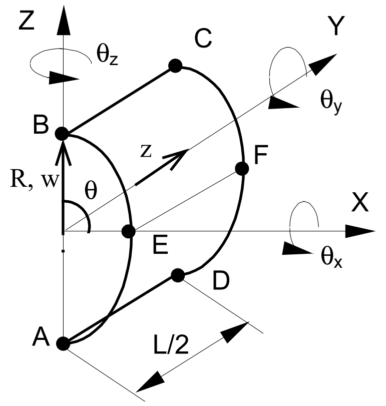
1) BATOZ J.L., DHATT G.: Modelization of structures by finite elements, Flight 3, Shells, HERMES.

2) SCHWAIGHOFER J., MICROYS H.F.: Orthotropic Cylindrical shells under line load, Newspaper of applied Mechanics, June 1979, Flight 46.

3) GEOFFROY P., Development and evaluating of a finite element for the static nonlinear analysis and dynamics of thin shells, Thesis of Doctor Engineer, University of Technology of Compiegne, 4/27/83.

3 Modelization A

3.1 Characteristic of the modelization



Modélisation DST (on modélise un demi cylindre)

- 8 éléments dans la direction circonférentielle
- 12 éléments dans le sens longitudinal

- Conditions aux limites : Côté AB : $u = w = \theta_y = 0$
- Conditions de symétrie : Côtés AD et BC : $u = \theta_y = \theta_z = 0$
  Côté DC : $v = \theta_x = \theta_z = 0$

- Force par unité de longueur côté BC : q/2 = –1178.5715 N/m

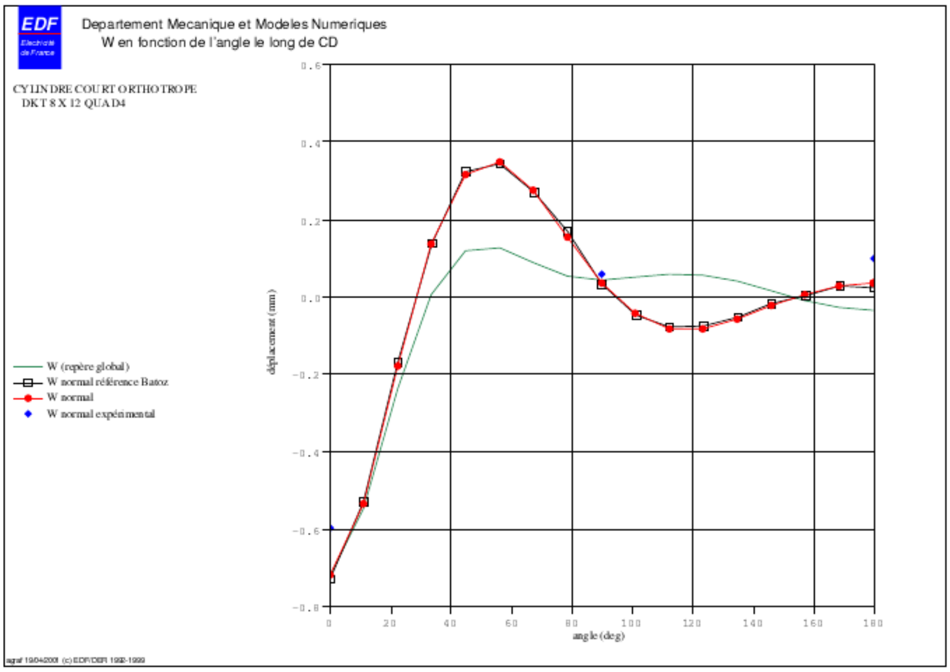
3.2 Characteristics of the mesh

Many nodes: 224
Number of meshes and type: 192 QUAD4

3.3 Values tested

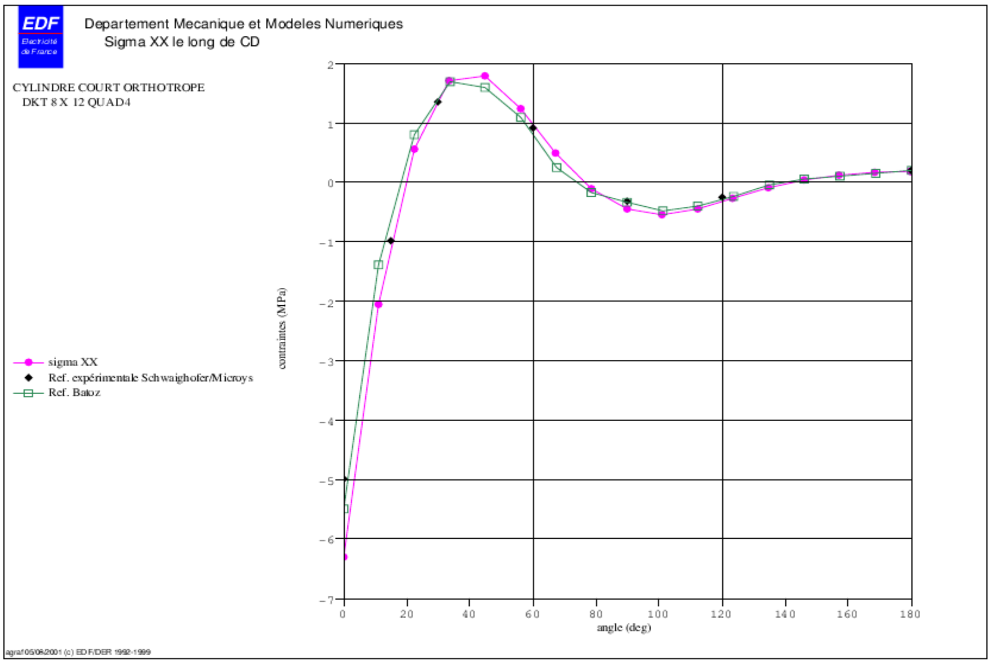
numerical	Identification Reference [bib1]	experimental Reference [bib2]	*Aster*	% differences
Displacement w to item F	$0.35\,10^{-4}\,m$	$0.6\,10^{-4}\,m$	$0.373\,10^{-4}\,m$	6.703 [bib1] – 37.757 [bib2]
Displacement w as in point C	$-0.7\,10^{-3}\,m$	$-0.6\,10^{-3}\,m$	$-0.721\,10^{-3}$	3.033 [bib1] 20.205 [bib2]
Displacement w as in point D	$0.25\,10^{-4}\,m$	$0.1\,10^{-3}\,m$	$0.369\,10^{-4}$	47.689 [bib1] -63.078 [bib2]
Forced $SIXX$ as in point F	$-0.350\,MPa$	$-0.325\,MPa$	$-0.480\,MPa$	37.339 [bib1] 47.904 [bib2]
Forced $SIYY$ as in point F	$0.500\,MPa$	$0.600\,MPa$	$0.490\,MPa$	-1.901 [bib1] -18.259 [bib2]

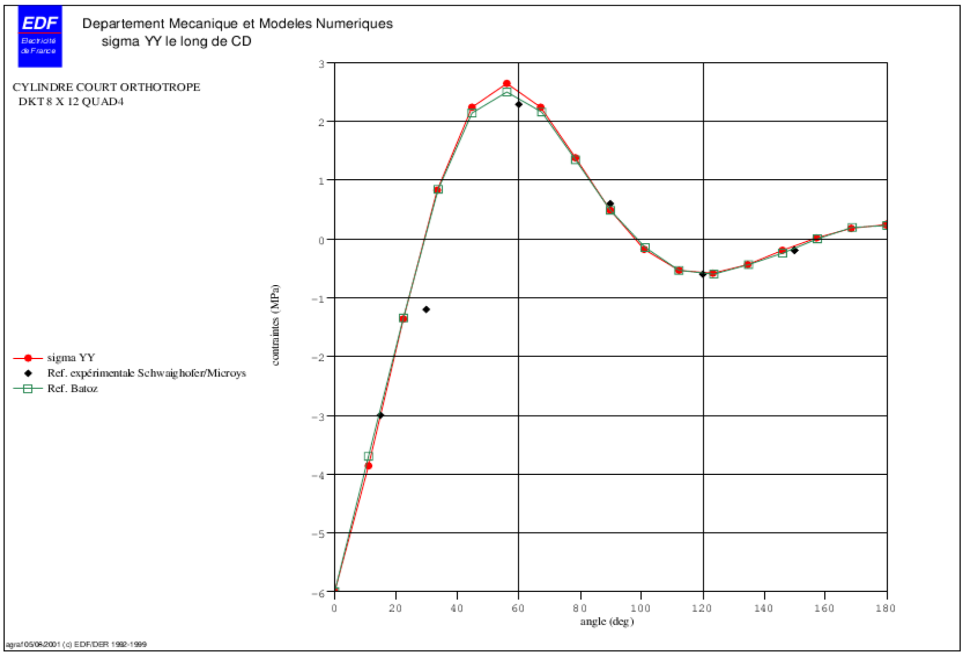
3.4 Value of normal displacement W along CD



One can note that beyond the variations observed on the points tested C F D, the normal displacement calculated along CD is close to the solution in theory "deep shells" adopted by Batoz [bib1]. One can charge the errors relating to the points F and D the low value of displacement (about $10^{-5} m$).

3.5 Value of the stresses along CD





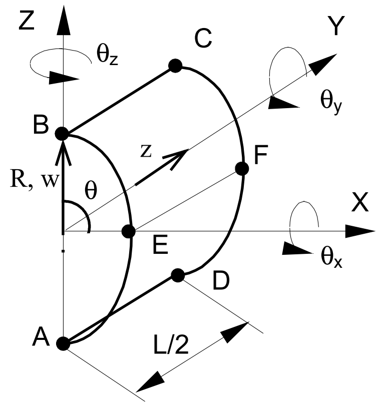
One can note that the stresses calculated along CD are overall in agreement with the solution in theory "deep shells" adopted by Batoz [bib1].

3.6 Remarks

- the values of coefficients CISA_L and CISA_T are not available. As the structure is mean ($h/R=0.045$), it is supposed that the effects of the transverse shears are negligible, we thus imposed CISA_L=CISA_T= 10^{10} .
- Normal w displacement (figure of [§4.2]) is expressed in the local cylindrical coordinate system (R, θ, z) , it acts of normal displacement to the shell element. The displacement w tested with [§4.1] as for him is expressed in the total reference (following displacement z).

4 Modelization B

4.1 Characteristic of the modelization



Modélisation DST (on modélise un demi cylindre)

- 8 éléments dans la direction circonférentielle
- 12 éléments dans le sens longitudinal

- Conditions aux limites : Côté AB : $u = w = \theta_y = 0$
- Conditions de symétrie : Côtés AD et BC : $u = \theta_y = \theta_z = 0$
  Côté DC : $v = \theta_x = \theta_z = 0$

- Force par unité de longueur côté BC : q/2 = –1178.5715 N/m

4.2 Characteristics of the mesh

Many nodes: 224
Number of meshes and type: 384 TRIA3

4.3 Values tested

Identification	**Reference [bib1]**	**Reference [bib2]**	***Aster***	**% differences**
Displacement w to the item F	$0.35\,10^{-4}m$	$0.6\,10^{-4}m$	$0.383\,10^{-4}$	9.571 [bib1] 36.084 [bib2]
Displacement w to the item C	$-0.710^{-3}m$	$-0.610^{-3}m$	-7.13810^{-4}	1.985 [bib1] 18.982 [bib2]
Displacement w to the item D	$0.25\,10^{-4}m$	$0.1\,10^{-3}m$	$0.350\,10^{-4}$	40.368 [bib1] – 64.908 [bib2]
Forced $SIXX$ to the item F	$-0.350\,MPa$	$-0.325\,MPa$	$-0.470\,MPa$	34.348 [bib1] 44.682 [bib2]
Forced $SIYY$ to the point F	$0.500\,MPa$	$0.600\,MPa$	$0.400\,MPa$	– 19.929 [bib1] – 33.274 [bib2]

4.4 Remarks

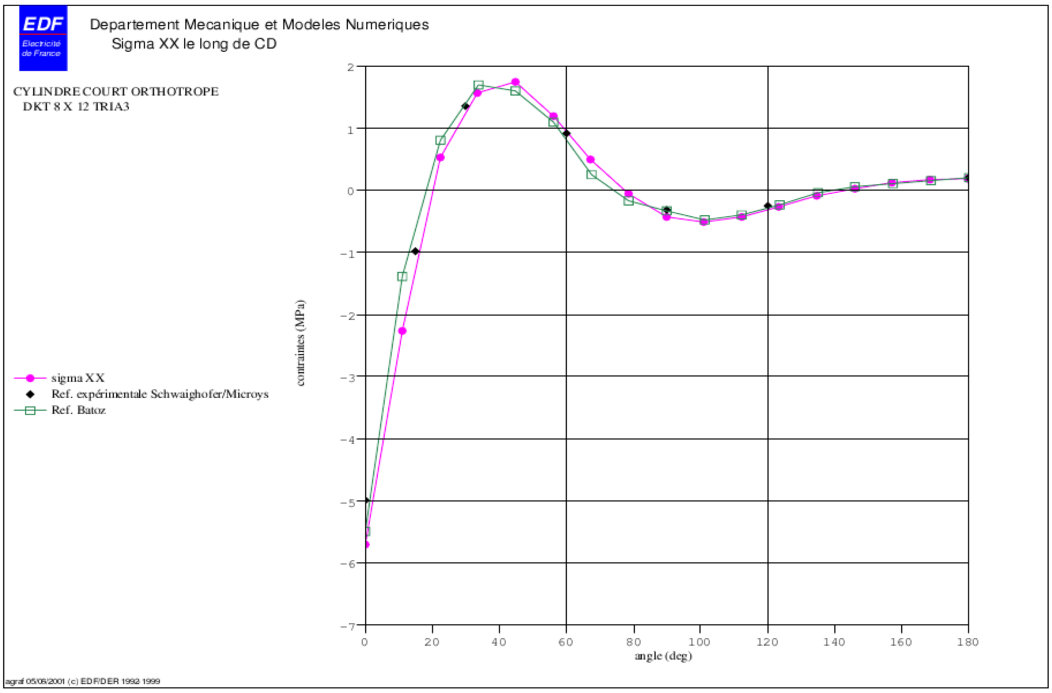
- the values of coefficients CISA_L and CISA_T are not available. As the structure is mean ($h/R=0.045$), it is supposed that the effects of the transverse shears are negligible, we thus imposed CISA_L=CISA_T= 10^{10} .

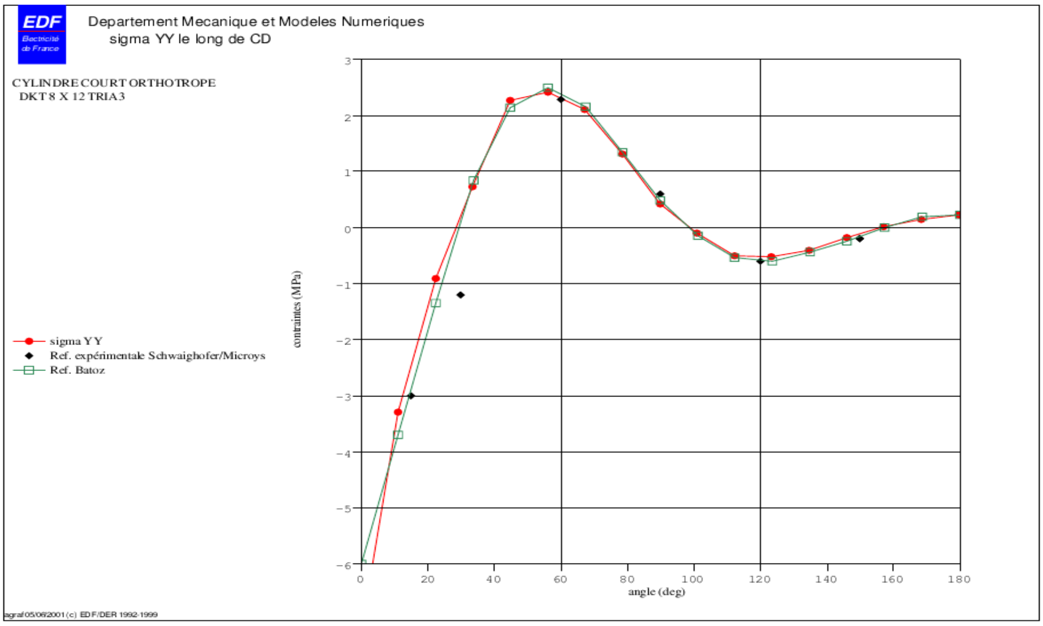
- Displacement W normal is expressed in the local cylindrical coordinate system (R, θ, z), it acts of normal displacement to the shell element.

4.5 Value of normal displacement along CD

the results got with a mesh TRIA3 are very close to those obtained by mesh QUAD4.

4.6 Value of the stresses along CD

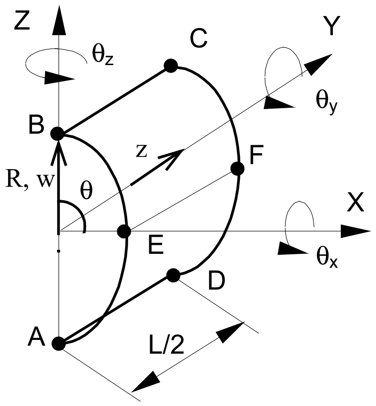




the profiles of the stresses obtained by the modelization B with TRIA3 are as a whole close to the solutions of Batoz.

5 Modelization C

5.1 Characteristic of the modelization



Modélisation DST (on modélise un demi cylindre)

- 8 éléments dans la direction circonférentielle
- 12 éléments dans le sens longitudinal

- Conditions aux limites : Côté AB : $u = w = \theta_y = 0$
- Conditions de symétrie : Côtés AD et BC : $u = \theta_y = \theta_z = 0$
  Côté DC : $v = \theta_x = \theta_z = 0$

- Force par unité de longueur côté BC : q/2 = –448.276 N/m

5.2 Characteristics of the mesh

Many nodes: 224
Number of meshes and type: 384 TRIA3

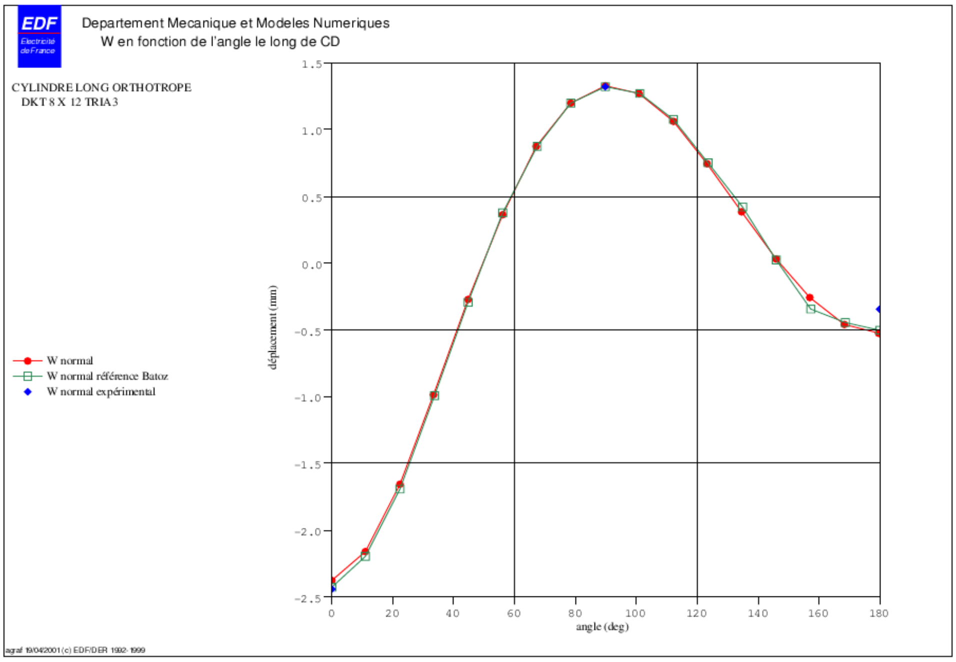
5.3 Values tested

numerical	**Identification Reference [bib1]**	**experimental Reference [bib2]**	***Aster***	**% difference**
Displacement w to item F	$1.325\,10^{-3}\,m$	$1.35\,10^{-3}\,m$	$1.327\,10^{-3}\,m$	0.154 [bib1] – 1.701 [bib2]
Displacement w at the point C	$-2.45\,10^{-3}\,m$	$-2.46\,10^{-3}\,m$	$-2.379\,10^{-3}\,m$	– 2.881 [bib1] – 3.275 [bib2]
Displacement w as in point D	$-0.51\,10^{-3}\,m$	$-0.35\,10^{-3}\,m$	$-0.529\,10^{-3}\,m$	3.859 [bib1] 51.337 [bib2]
Forced $SIXX$ at the point F	$1.68\,MPa$	$1.9\,MPa$	$1.643\,MPa$	– 2.155 [bib1] – 13.484 [bib2]
Forced $SIYY$ at the point F	$1.8\,MPa$	$1.55\,MPa$	$1.782\,MPa$	– 0.986 [bib1] – 14.984 [bib2]

5.4 Remarks

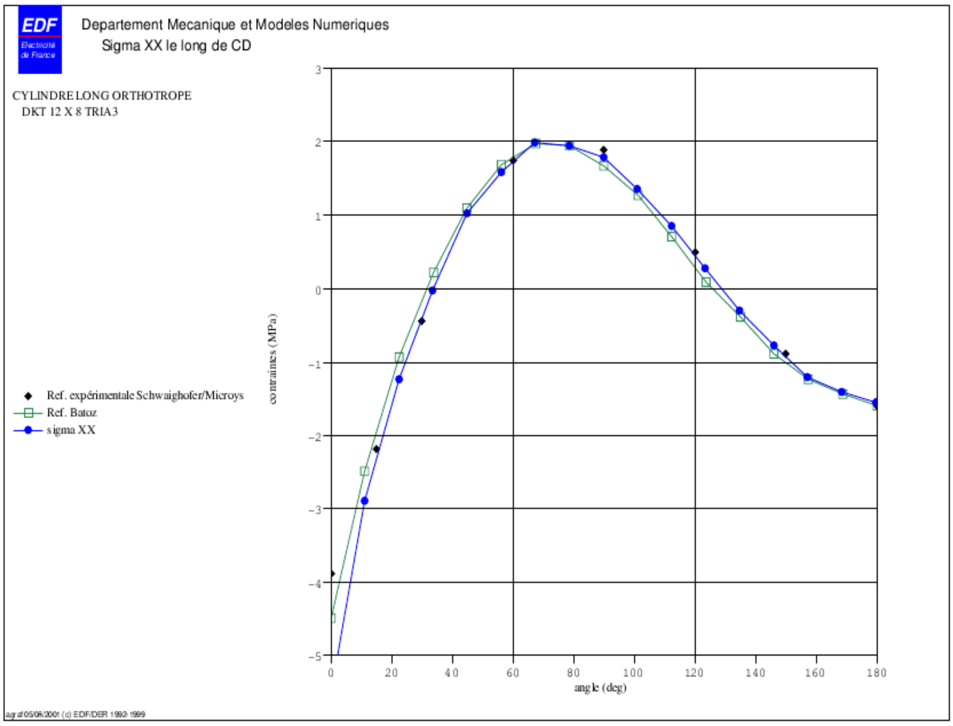
- the value of coefficients CISA_L and CISA_T are not available. As the structure is mean ($h/R=0.045$), it is supposed that the effects of the transverse shears are negligible, we thus imposed CISA_L=CISA_T= 10^{10} .
- Normal w displacement is expressed in the local cylindrical coordinate system (R,θ,z) , it acts of normal displacement to the shell element.

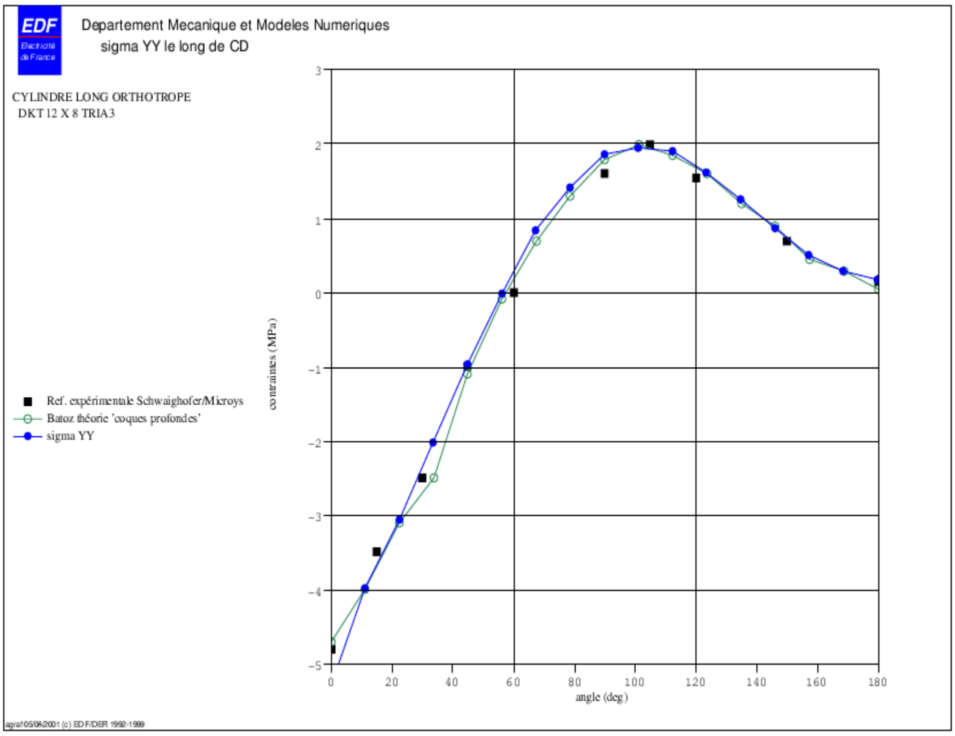
5.5 Value of normal displacement along CD



One can note that beyond the variation observed on the experimental value at the point D, the normal displacement calculated along CD is very close to the solution in theory “deep shells” adopted by Batoz [bib1].

5.6 Value of the stresses along CD

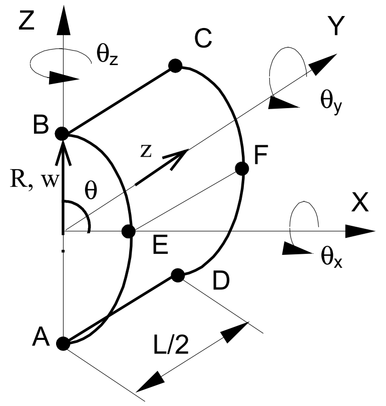




the profiles of the stresses calculated by the code are overall in agreement with works of Batoz.

6 Modelization D

6.1 Characteristic of the modelization



Modélisation DST (on modélise un demi cylindre)

- 8 éléments dans la direction circonférentielle
- 12 éléments dans le sens longitudinal

- Conditions aux limites : Côté AB : $u = w = \theta_y = 0$
- Conditions de symétrie : Côtés AD et BC : $u = \theta_y = \theta_z = 0$
  Côté DC : $v = \theta_x = \theta_z = 0$

- Force par unité de longueur côté BC : q/2 = –448.276 N/m

6.2 Characteristics of the mesh

Many nodes: 224
Number of meshes and type: 192 QUAD4

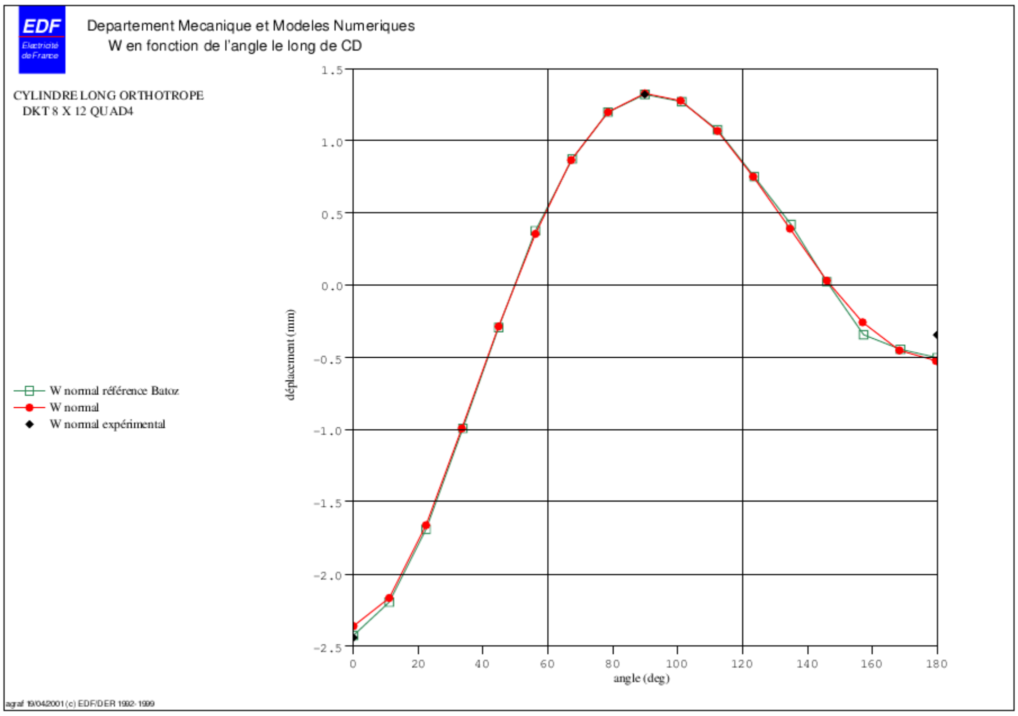
6.3 Values tested

Identification	Reference [bib1]	Reference [bib2]	*Aster*	% difference
Displacement w to item F	$1.325\,10^{-3}\,m$	$1.35\,10^{-3}\,m$	$1.329\,10^{-3}\,m$	0.365 [bib1] – 1.494 [bib2]
Displacement w to the point C	$-2.45\,10^{-3}\,m$	$-2.46\,10^{-3}\,m$	$-2.369\,10^{-3}\,m$	– 3.274 [bib1] – 3.667 [bib2]
Displacement w as in point D	$-0.51\,10^{-3}\,m$	$-0.35\,10^{-3}\,m$	$-0.528\,10^{-3}\,m$	3.634 [bib1] 51.009 [bib2]
Forced $SIXX$ as in point F	$1.68\,MPa$	$1.9\,MPa$	$1.79\,MPa$	6.616 [bib1] – 5.729 [bib2]
Forced $SIYY$ as in point F	$1.8\,MPa$	$1.55\,MPa$	$1.84\,MPa$	2.465 [bib1] 18.991 [bib2]

6.4 Remarks

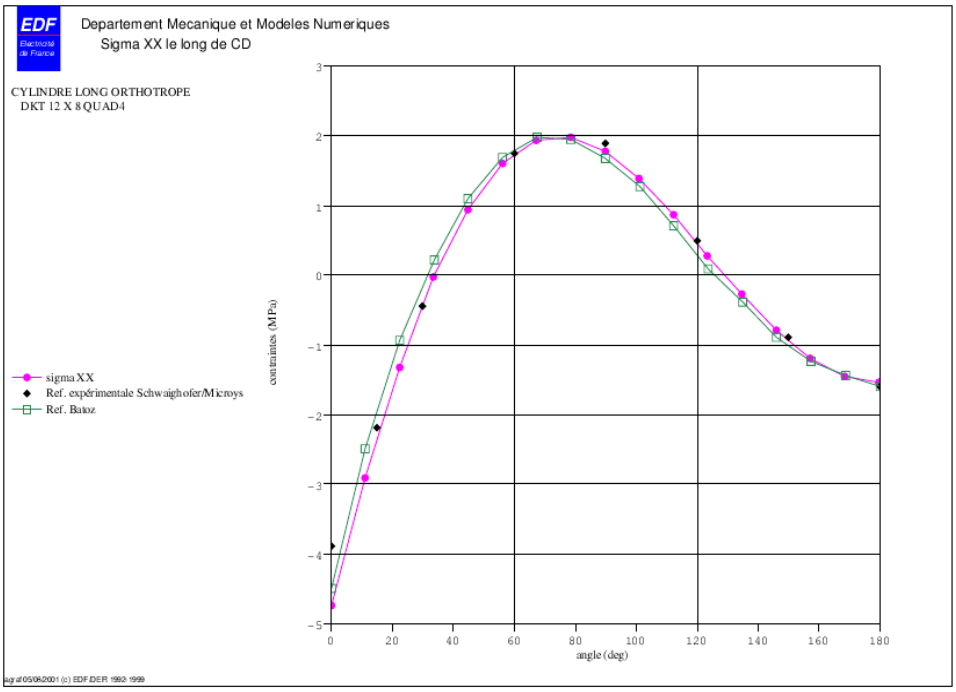
- the value of coefficients CISA_L and CISA_T are not available. As the structure is mean ($h/R=0.045$), it is supposed that the effects of the transverse shears are negligible, we thus imposed CISA_L=CISA_T= 10^{10} .
- Normal w displacement is expressed in the local cylindrical coordinate system (R,θ,z) , it acts of normal displacement to the shell element. Displacement w tested is that of the total reference (following displacement z).

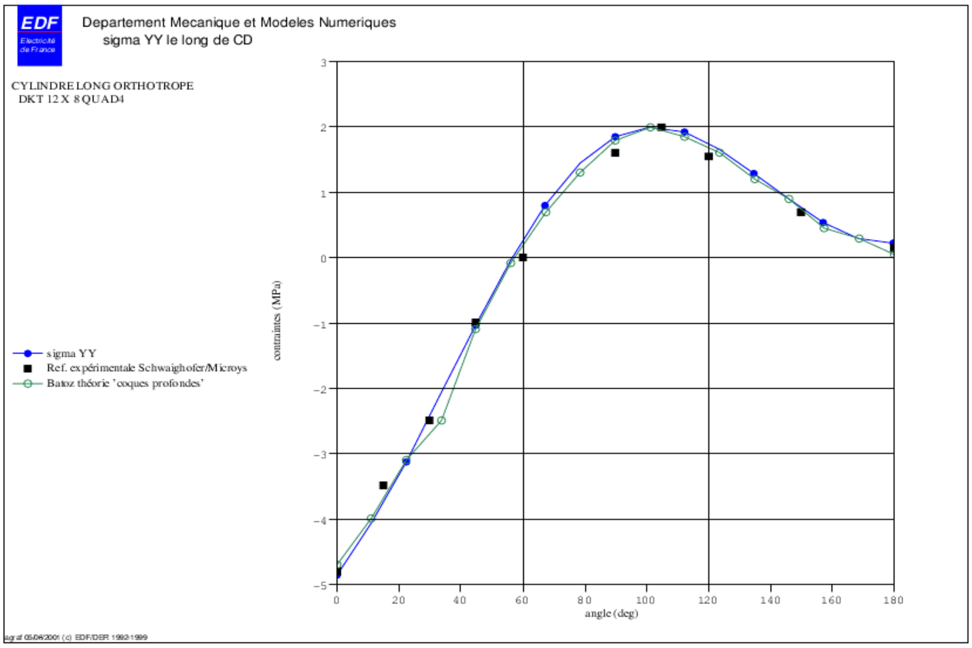
6.5 Value of displacement along CD



One can note that beyond the variation observed on the experimental value at the point D , the normal displacement calculated along CD is very close to the solution in theory "deep shells" adopted by Batoz [bib1].

6.6 Values of the stresses along CD





7 Summary of the results

the results are as a whole satisfactory. The specific variations which appear at the points tested, in particular the point D, seem due to the experimental uncertainty, undoubtedly reinforced by an uncertainty as for the graphic taking away.

A contrario, the solutions suggested by Batoz in theory “deep shells” are well checked by the four modelizations, with relative errors of less than 5% for the long cylinder.

It appears that:

- modelizations TRIA3 and QUAD4 are appreciably equivalent for this problem,
- the relative errors are much weaker for the cylinder long (modelizations C and D) that for the cylinder short (modelizations A and B): at the point F, the error is reduced of a factor 10 compared to the reference solution of Batoz,
- the refinement of the mesh does not minimize in a decisive way the relative variations, as well with the TRIA3 as with the QUAD4.

It is thus noted that the results indeed degrade when the ratio length on the diameter decreases, the geometrical effects become important with this kind of modelization. It would be desirable to be able to carry out a computation in finite elements of shells in orthotropic medium, in order to better take into account the curvature, the plates constituting a borderline case.