SSLS141 – Summarized shears and flexbeam

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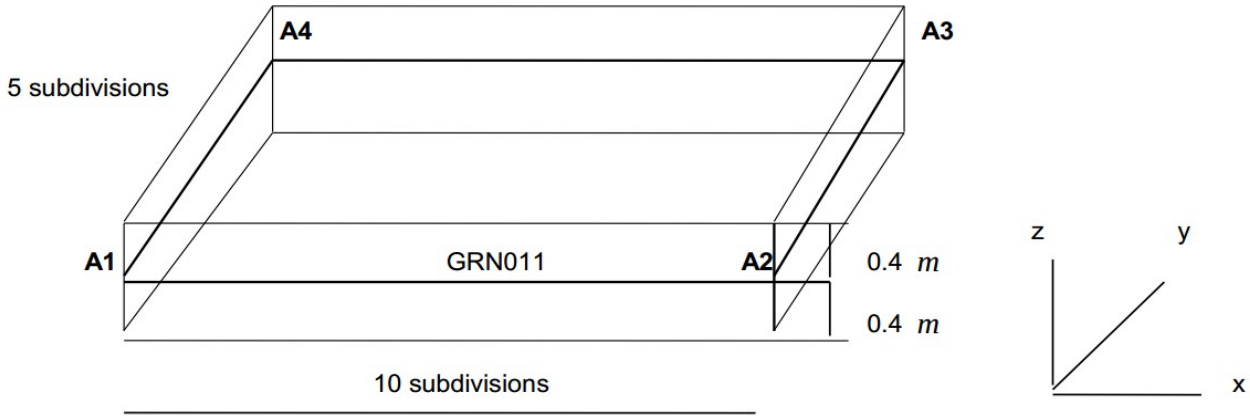
The purpose of this test is to validate the response in shears of the elements shells DST (QUAD4) and Q4G (QUAD4).

1 Problem of reference

1.1 Geometry



the coordinates of the points are given in meters (m):

$A1(0.0,0)$	$A3(10.5,0)$
$A2(10.0,0)$	$A4(0.5,0)$

Table 1.1-1

the thickness of the plate is $e=0,8\,m$

1.2 Properties of the material

the material has an isotropic elastic behavior:

Young modulus: $E=200000\,MPa$

Poisson's ratio: $\nu=0$

1.3 Boundary conditions and loadings

the edge $A1A4$ is clamped.
One applies a nodal force $Fz=-1000\,N$ to the edge $A2A3$.

1.4 Modelizations

the various modelizations are:

Standard	modelization of element	Meshes	Many elements of mesh
A	DST	QUAD4	50
B	DST	QUAD4	50

	(multi-layer, $ncou=2$)		
C	Q4G	QUAD4	800
D	Q4G (multi-layer, $ncou=2$)	QUAD4	800
E	DST	TRIA3 (regular mesh)	1600
F	DST	TRIA3 (irregular mesh)	1600

Table 1.4-1

2 Quantities and

2.1 result Reference solution of analytical

2.1.1 Model reference of the plate

One considers that the response of the plate corresponds to the response of a beam out of cantilever (because $\nu=0$). In this case, the displacement of the plate is given by displacements of each section of plate, according to the direction $x(u)$, $z(w)$ and their rotation compared to the axis $y(\theta_y)$.

$$U_x(x,z)=u(x)+z\,\theta_y(x) \quad U_z(x,z)=w(x)$$

In any point, the strains are:

$$\varepsilon_{xx}(x,z)=u'+z\,\theta_y' \text{ and } 2\,\varepsilon_{xz}(x,z)=\theta_y+w'=\gamma_z$$

the beam is embedded in $x=0$ and subjected to a vertical force F in $x=L$.

The resolution (principle of the virtual wors and equilibrium of the forces) give the statement of the forces generalized in the case of an elastic linear response:
N is the thrust load, M_y the moment along the axis y and T the shears.

$$\begin{array}{lll} N(x)=E\,A\,u'(x) & \text{and} & N(x)=0 \\ M_y(x)=E\,I\,\theta_y'(x) & & M_y(x)=-F(L-x) \\ T(x)=G\,k\,A\,\gamma_z & & T(x)=F \end{array}$$

where the parameter k is the factor of correction in shears

with the boundary conditions, one of deduced displacements $u,w\,et\,\theta_y$:

$$u(x)=0\ ,\ w(x)=\frac{F}{6\,E\,I}x^2(3\,L-x)+\frac{F\,x}{G\,k\,A}\ ,\ \theta_y(x)=\frac{F}{E\,I}\frac{x(x-2\,L)}{2}$$

the strains and stresses are:

$$\varepsilon_{xx}=z\frac{-F}{E\,I}(L-x) \text{ and } \sigma_{xx}=z\frac{-F}{I}(L-x)$$

$$2\,\varepsilon_{xz}=\frac{F}{G\,k\,A}=\gamma_z \text{ and } \sigma_{xz}=\frac{-F}{A}6(z^2/h^2-1/4)$$

where h is the height of the plate. The distribution of shear stresses is parabolic in the thickness of the plate to observe the conditions of free surface
($\sigma_{xz}(z=h/2)=\sigma_{xz}(z=-h/2)=0$).

The vertical response in displacement of the point $x=L$ is:

$$U_z(L)=\frac{F}{3\,E\,I}L^3+\frac{F\,L}{G\,k\,A}$$

2.2 Uncertainties on the solution

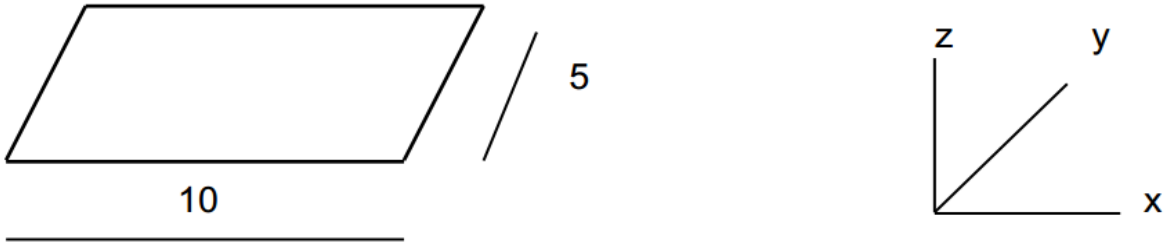
the reference solution is analytical. There is thus no uncertainty.

3 Modelization A

3.1 Characteristic of the modelization

The model consists of 1 plate with a layer.
The elements used are shell elements DST.

3.2 Characteristics of the mesh



The mesh is regular. There are 10 subdivisions according to x and 5 subdivisions according to y; that is to say on the whole 50 meshes DSQ (QUAD4) and 66 nodes.

3.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
DZ ($A3$)	"ANALYTIQUE"	-3.92125E-05	0.001
MXX ($A1$)	"ANALYTIQUE"	10000	0.001
QX ($A1$)	"ANALYTIQUE"	-1000	0.001
SIXX ($M1, PT1, SSPT2$)	"ANALYTIQUE"		0.0.1.0E-8
SIXX ($M1, PT1, SSPT3$)	"ANALYTIQUE"	1981.17	0.001
SIXZ ($M1, PT1, SSPT2$)	"ANALYTIQUE"	-1875.0	0.001
SIXZ ($M1, PT1, SSPT3$)	"ANALYTIQUE"		0.0.1.0E-8
SIXX ($M50, PT2, SSPT2$)	"ANALYTIQUE"		0.0.1.0E-8
SIXX ($M50, PT2, SSPT3$)	"ANALYTIQUE"	91768.83	0.001
SIXZ ($M50, PT2, SSPT2$)	"ANALYTIQUE"	-1875.0	0.001
SIXZ ($M50, PT2, SSPT3$)	"ANALYTIQUE"		0.0.1.0E-8
EPXX ($M50, PT2, SSPT2$)	"ANALYTIQUE"		0.0.1.0E-8
EPXX ($M50, PT2, SSPT3$)	"ANALYTIQUE"	4.58844E-07	0.001

EPXZ (*M50 , PT2 , SSPT2*)	"ANALYTIQUE"	-7.5E-09	0.001
EPXZ(*M50 , PT2 , SSPT3*)	"ANALYTIQUE"	-7.5E-09	0.001

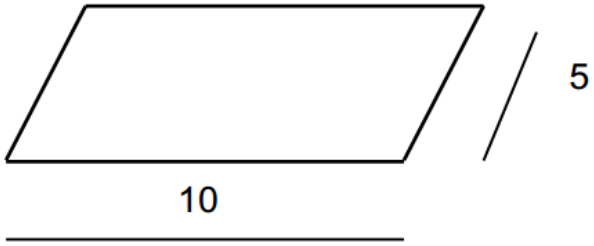
Table 3.3-1

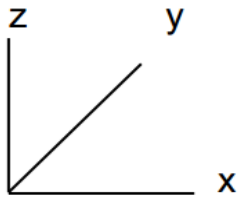
4 Modelization B

4.1 Characteristic of the modelization

The model consist of 1 plate with 2 layers.
The elements used are shell elements DST.

4.2 Characteristics of the mesh





The mesh is regular. There are 10 subdivisions according to x and 5 subdivisions according to y; i.e. a total of 50 meshes DST (QUAD4) and 66 nodes.

4.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
DZ ($A3$)	"ANALYTIQUE"	-3.92125E-05	0.001
MXX ($A1$)	"ANALYTIQUE"	10000	0.001
QX ($A1$)	"ANALYTIQUE"	-1000	0.001
SIXX ($M1, PT1, SSPT3$)	"ANALYTIQUE"	0.	1.0E-8
SIXX ($M1, PT1, SSPT6$)	"ANALYTIQUE"	1981.17	0.001
SIXZ ($M1, PT1, SSPT3$)	"ANALYTIQUE"	-1875.0	0.001
SIXZ ($M1, PT1, SSPT6$)	"ANALYTIQUE"		0.0.1.0E-8
SIXX ($M50, PT2, SSPT3$)	"ANALYTIQUE"		0.0.1.0E-8
SIXX ($M50, PT2, SSPT6$)	"ANALYTIQUE"	91768.83	0.001
SIXZ ($M50, PT2, SSPT3$)	"ANALYTIQUE"	-1875.0	0.001
SIXZ ($M50, PT2, SSPT6$)	"ANALYTIQUE"		0.0.1.0E-8
EPXX ($M50, PT2, SSPT3$)	"ANALYTIQUE"		0.0.1.0E-8
EPXX ($M50, PT2, SSPT6$)	"ANALYTIQUE"	4.58844E-07	0.001
EPXZ ($M50, PT2, SSPT3$)	"ANALYTIQUE"	-7.5E-09	0.001
EPXZ ($M50, PT2, SSPT6$)	"ANALYTIQUE"	-7.5E-09	0.001

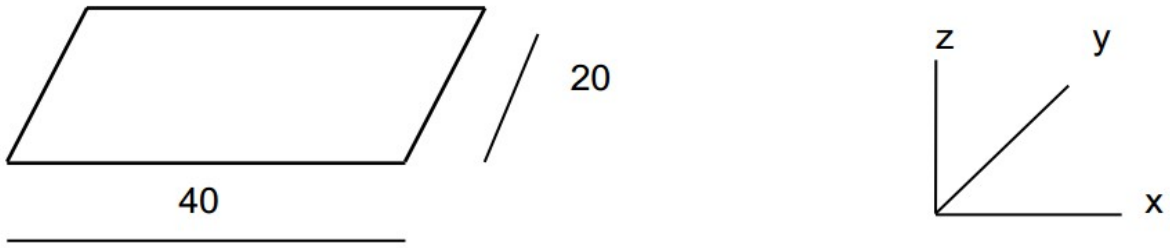
Table 4.3-1

5 Modelization C

5.1 Characteristic of the modelization

The model consists of 1 plate with a layer.
The elements used are shell elements Q4G.

5.2 Characteristics of the mesh



The mesh is regular. There are 40 subdivisions according to x and 20 subdivisions according to y; i.e. a total of 800 meshes Q4G (QUAD4) and 861 nodes.

5.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
DZ ($A3$)	"ANALYTIQUE"	-3.92125E-05	0.001
MXX ($A1$)	"ANALYTIQUE"	10000	0.02
QX ($A1$)	"ANALYTIQUE"	-1000	0.001
SIXX ($M141, PT1, SSPT2$)	"ANALYTIQUE"		0.0.1.0E-8
SIXX ($M141, PT1, SSPT3$)	"ANALYTIQUE"	2839.04	0.3
SIXZ ($M141, PT1, SSPT2$)	"ANALYTIQUE"	-1875.0	0.001
SIXZ ($M141, PT1, SSPT3$)	"ANALYTIQUE"		0.0.1.0E-8
SIXX ($M901, PT2, SSPT2$)	"ANALYTIQUE"		0.0.1.0E-8
SIXX ($M901, PT2, SSPT3$)	"ANALYTIQUE"	93254.70	0.01
SIXZ ($M901, PT2, SSPT2$)	"ANALYTIQUE"	-1875.0	0.001
SIXZ ($M901, PT2, SSPT3$)	"ANALYTIQUE"		0.0.1.0E-8
EPXX ($M901, PT2, SSPT2$)	"ANALYTIQUE"		0.0.1.0E-8
EPXX ($M901, PT2, SSPT3$)	"ANALYTIQUE"	4.66273E-07	0.01
EPXZ ($M901, PT2, SSPT2$)	"ANALYTIQUE"	-7.5E-09	0.001
EPXZ($M901, PT2, SSPT3$)	"ANALYTIQUE"	-7.5E-09	0.001

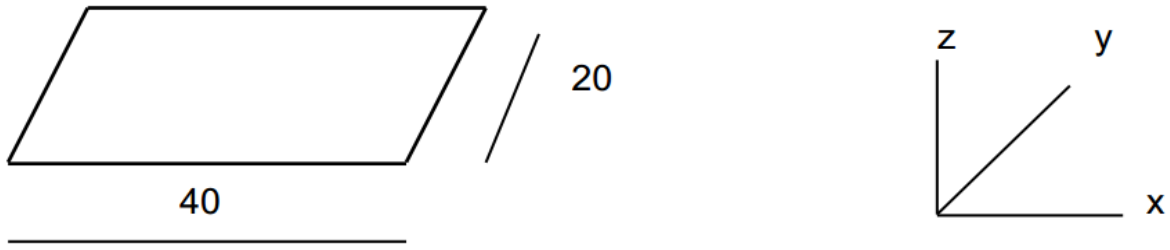
Table 5.3-1

6 Modelization D

6.1 Characteristic of the modelization

The model consist of 1 plate with 2 layers.
The elements used are shell elements Q4G.

6.2 Characteristics of the mesh



The mesh is regular. There are 40 subdivisions according to x and 20 subdivisions according to y; i.e. a total of 800 meshes Q4G (QUAD4) and 861 nodes.

6.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
DZ ($A3$)	"ANALYTIQUE"	-3.92125E-05	0.001
MXX ($A1$)	"ANALYTIQUE"	10000	0.02
QX ($A1$)	"ANALYTIQUE"	-1000	0.001
SIXX ($M141, PT1, SSPT3$)	"ANALYTIQUE"		0.0.1.0E-8
SIXX ($M141, PT1, SSPT6$)	"ANALYTIQUE"	2839.04	0.3
SIXZ ($M141, PT1, SSPT3$)	"ANALYTIQUE"	-1875.0	0.001
SIXZ ($M141, PT1, SSPT6$)	"ANALYTIQUE"		0.0.1.0E-8
SIXX ($M901, PT2, SSPT3$)	"ANALYTIQUE"		0.0.1.0E-8
SIXX ($M901, PT2, SSPT6$)	"ANALYTIQUE"	93254.70	0.01
SIXZ ($M901, PT2, SSPT3$)	"ANALYTIQUE"	-1875.0	0.001
SIXZ ($M901, PT2, SSPT6$)	"ANALYTIQUE"		0.0.1.0E-8
EPXX ($M901, PT2, SSPT3$)	"ANALYTIQUE"		0.0.1.0E-8
EPXX ($M901, PT2, SSPT6$)	"ANALYTIQUE"	4.66273E-07	0.01
EPXZ ($M901, PT2, SSPT3$)	"ANALYTIQUE"	-7.5E-09	0.001
EPXZ($M901, PT2, SSPT6$)	"ANALYTIQUE"	-7.5E-09	0.001

Table 6.3-1

7 Modelization E

7.1 Characteristic of the modelization

The model consist of 1 plate with 1 layer.
The elements used are shell elements DST.

7.2 Characteristics of the mesh

The mesh is regular. There are 40 subdivisions according to x and 20 subdivisions according to y; i.e. a total of 1600 meshes DST (TRIA3) and 861 nodes.

7.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
DZ ($A3$)	"ANALYTIQUE"	-3.92125E-05	0.13
DZ ($A2$)	"ANALYTIQUE"	-3.92125E-05	0.13
MXX ($A4$)	"ANALYTIQUE"	10000	0.33
MXX ($A1$)	"ANALYTIQUE"	10000	0.02

Table 7.3-1

8 Modelization F

8.1 Characteristic of the modelization

The model consists of 1 plate with 1 layer.
The elements used are shell elements DST.

8.2 Characteristics of the mesh

The mesh is free. There are 1600 meshes DST (TRIA3) and 853 nodes.

8.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
DZ (*A3*)	"ANALYTIQUE"	-3.92125E-05	0.011
DZ (*A2*)	"ANALYTIQUE"	-3.92125E-05	0.009
MXX (*A4*)	"ANALYTIQUE"	10000	0.053
MXX (*A1*)	"ANALYTIQUE"	10000	0.022

Table 8.3-1

9 Summary of the results

- For a plate with one or more layers, elements DST (QUAD4) give good estimates of displacements, generalized forces, strains and forced (Modelization A and B).
- For a finer mesh, elements Q4G (QUAD4) lead to errors a little more important, in particular for axial stresses SIXX (Modelization C and D).
- Two element types DST (QUAD4) and Q4G (QUAD4) respectively give the same results with one (Modelization A and C) or two layers (Modelization B and D).
- Elements DST (TRIA3) give results much less good (Modelizations E and F). In particular one highlights important effects of mesh: the modelizations E and F have the same number of elements but the mesh E is regulated whereas the mesh F is free. The error on the deflection is of 13 % for E and 1 % for F. One notes similar variations at the time. There is also a strong dissymmetry at the time between the points $A1$ and $A4$ for the 2 meshes. This is with the fact that the meshes are not symmetric compared to the loading. On a symmetric mesh the problem does not appear any more. This element thus has a behavior in bending very depend on the mesh, which undoubtedly deserves a thorough study.