SSLS137 - Plate prestressed concrete with a excentré cable in bending

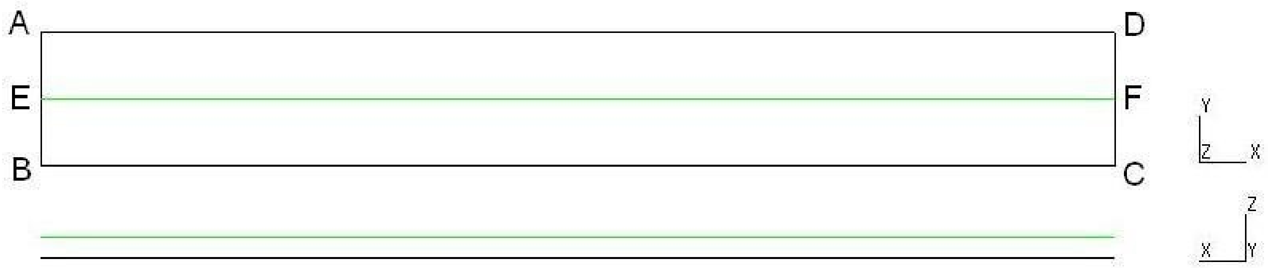
Summarized:

The purpose of this test is validating macro-command CALC_PRECONT for the elements shells. The reference solution is established from the theory of the beams. The first modelization is made with elements 3D and aim at validating the use of the theory of the beams like reference. Two modelizations of shells are then proposed (DKT and Q4GG) like two types of meshes different (TRIA3, QUAD4) for each modelization. Lastly, the modelization F takes again the modelization D by replacing the elastic constitutive law of the concrete by nonlinear constitutive law GLRC_DAMAGE.

1 Description

1.1 Geometry

The model of study is a rectangular plate length $L=4\mathrm{m}$, of width $l=0.5\mathrm{m}$ and of thickness $t=0.2\mathrm{m}$. A reinforcement of bending made up of only one reinforcement, crosses the plate to its half and is excentré of a distance $e_z=0.075\mathrm{m}$ median plane of the plate along the axis z . From a geometrical point of view, one knows only the area of the cross-section of the bar $A=0.00015\mathrm{m}^2$. The figure below illustrates the position of the cable for the model of study. The median plane of the plate is defined by the rectangle $ABCD$ and the cable by the segment EF .



1.2 Properties of the materials

the properties of the concrete for the plate and steel for the cable are indexed in the following table.

Concrete material		Steel
Modulus Young	$4\times10^{10}\,Pa$	$1.93\times10^{11}\,Pa$
Poisson's ratio	0.0 *	0.0
Density	$2500\,kg/m^3$	$7850\,kg/m^3$
Stress yield stress	n/a	$1.94\times10^{11}\,Pa$

*Il is justified thereafter that the model study can be comparable to a beam and that one will take a Poisson's ratio for the concrete equal to 0.0 .

1.3 Boundary conditions and loadings

the side defined by segment AB of the plate is embedded, all the degrees of freedom on this side are blocked and the other edges are left free to allow him a configuration of type beam. The order of application of the loadings is defined in the following way.

1.3.1 Loading 1:

It corresponds to the setting in tension of the cable. One imposes a tension in the cable equal to $3.75\times10^5\,N$, and only the node F of the cable located at free edge of the plate is active. The losses of tension by relaxation of steel and retreat of anchorage are neglected.

1.3.2 Loading 2:

The second loading applies a pressure $P_0=10^5\,Pa$, related by the axis $-z$, to all the plate.

1.4 Principal stages of the tests

One uses macro-command DEFI_CABLE_BP to obtain the kinematic relations between the plate and the cable as well as the loading related to the tension in the cable. Only the node F of the cable is considered active.

One launches then macro-command CALC_PRECONT to carry out the setting into prestressed plate from the tension of the cable given.

Command STAT_NON_LINE is also used to apply the loading of pressure to the plate. One extracts then the maximum value from displacement on free edge of the plate to the node D , this in order to compare them with the values of reference and thus to validate the setting in tension of the cables of prestressed for the elements shells.

2 Reference solution

For each modelization, the purpose is to find the good initial tension of prestressed cable after the application of CALC_PRECONT but also to determine the maximum displacement of the plate to the node D following the axis z after application of the pressure.

2.1 Results of reference

to determine maximum displacement, one uses the theory of the beams working in bending. For a beam fixed at an end and free of the other under distributed loading, maximum displacement at the loose lead called deflection, is given by:

$$f=-\frac{qL^4}{8\text{EI}}$$

q : the distributed force in N/m .

L : the length of the beam in m .

E : the Young's modulus of the plate, therefore concrete in Pa .

$I=\frac{t^3 l}{12}$: the quadratic moment of the beam compared to the axis formulates m^4 .

For the compressive force:

Thanks to its eccentring, the cable takes part in the stiffness of the model:

$$(EI)_{eq}=E_b\frac{t^3 l}{12}+E_a a_x l\times e_z^2$$

$a_x=\frac{A}{dx}$: is the rate of reinforcement.

$$(EI)_{eq}=13.50\, MN.m^2$$

the deflection under the loading of pressure is calculated as follows:

$$f_p=\frac{-P_0 l\times L^4}{8(EI)_{eq}}$$

Then:

$$f_p=-0.118552\text{m}$$

For the setting in tension of the cable of prestressed:

The cable put in tension applies a compressive force then $-F_0$ and a bending moment $-e_z F_0$ on the loose lead of the plate.

With the principle of superposition, the statement of the normal stress is:

$$\sigma_x=\frac{-F_0}{tl}(1+\frac{12 e_z z}{t^2})$$

If one neglects the effects of the Poisson's ratio the field of displacement is given by:

$$\begin{cases} u(x,y,z)=\frac{-F_0}{E_b tl}(1+\frac{12 e_z z}{t^2})x \\ v(x,y,z)=0 \\ w(x,y,z)=\frac{F_0}{E_b tl}(\frac{6 e_z}{t^2} x^2) \end{cases}$$

Displacements are given at the loose lead of the beam to the node D , that is to say in $(4, 0.5, 0)$:

$$\begin{cases} u(x,y,z)=-0.375\text{mm} \\ v(x,y,z)=0 \\ w(x,y,z)=16.875\text{mm} \end{cases}$$

By superposition, the theoretical maximum displacement of the plate prestressed under the compressive force is of:

$$f_{tot}=f_p+w$$

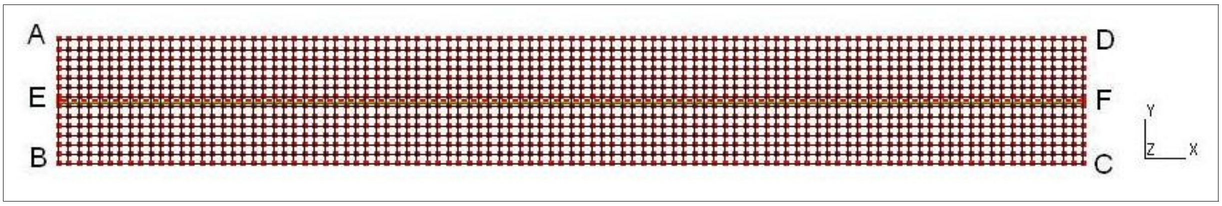
$$f_{tot}=-0.101677\text{m}$$

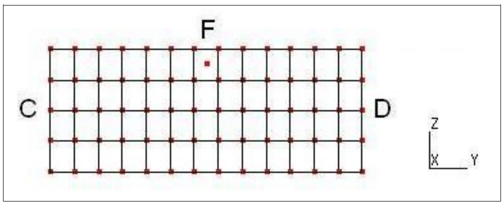
2.2 Uncertainty on the analytical

solution Solution.

3 Modelization A

3.1 Characteristic of the modelization





- Modelization: 3D
- Standard of finite elements: Hexahedron (HEXA8), dimension of 0.05m for the height and 0.04m for the length and 0.04m to the maximum for the width.

The plate is cut out in 13 elements over its width, 100 over its length and 4 on its thickness.
Steel reinforcement is modelled by elements BARS. The cable is discretized in 101 elements.

3.2 Quantities tested and results

One tests the value of displacement to the node D following the axis z.

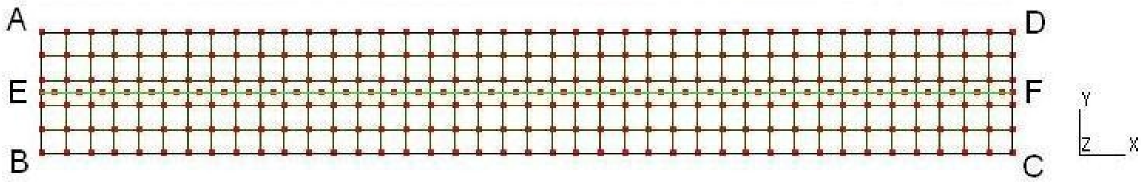
Node	Component	Value of reference (m)	Accuracy
D	DZ	0.000000 -0.101677	$2.0\text{E}-2$

One tests also the tension in several elements of the cable.

Net	Value of reference (N)	Accuracy
$M28$	3.75×10^{5}	$1.0\text{E}-8$
$M326$	3.75×10^{5}	$1.0\text{E}-8$
$M547$	3.75×10^{5}	$1.0\text{E}-8$

4 Modelization B

4.1 Characteristic of the modelization



- Modelization: DKT
- Type of finite elements: Quadrangle (QUAD4), dimension of 0.1m.

The plate is then cut out in 5 elements over its width and 40 over its length.
Steel reinforcement is modelled by elements BARS . The cable is discretized in 41 elements since each node of the cable is in the middle of an element quadrangle.

4.2 Quantities tested and results

One D tests the value of displacement to the node along the axis z.

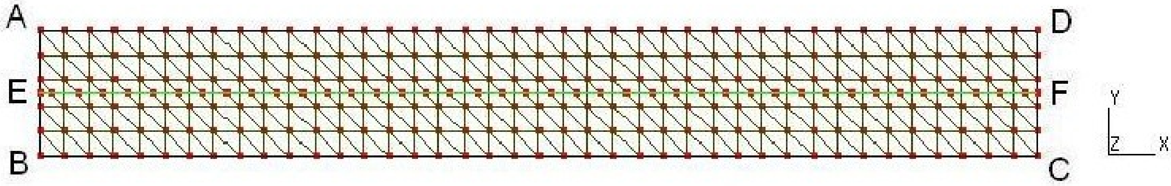
Node	Component	Value of reference (m)	Accuracy
D	DZ	-0.101677 0.000000	$1.0\text{E}-2$

One tests also the tension in several elements of the cable.

Net	Value of reference (N)	Accuracy
$M10$	3.75×10^{5}	$1.0\text{E}-8$
$M70$	3.75×10^{5}	$1.0\text{E}-8$
$M130$	3.75×10^{5}	$1.0\text{E}-8$

5 Modelization C

5.1 Characteristic of the modelization



- Modelization: DKT
- Type of finite elements: Triangle (TRIA3), dimension of $0.1\mathrm{m}$.

Two elements triangles are created from an element quadrangle of the model of the preceding modelization.
The plate is then cut out in 5 elements over its width and 40 over its length.
Steel reinforcement is modelled by elements BARS. The cable is discretized in 41 elements.

5.2 Quantities tested and results

One D tests the value of displacement to the node along the axis z .

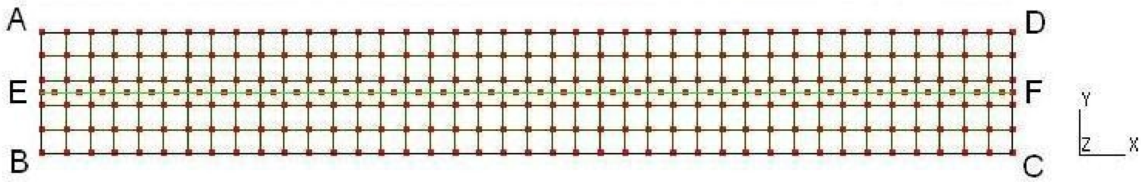
Node	Component	Value of reference (m)	Accuracy
D	DZ	-0.101677 0.000000	$1.0\mathrm{E}-2$

One tests also the tension in several elements of the cable.

Net	Value of reference (N)	Accuracy
$M20$	3.75×10^5	$1.0\mathrm{E}-8$
$M72$	3.75×10^5	$1.0\mathrm{E}-8$
$M127$	3.75×10^5	$1.0\mathrm{E}-8$

6 Modelization D

6.1 Characteristic of the modelization



- Modelization: `Standard`
- Q4GG of finite elements: Quadrangle (QUAD4), dimension of $0.1\mathrm{m}$.

The plate is then cut out in 5 elements over its width and 40 over its length.
Steel reinforcement is modelled by elements `BARS`. The cable is discretized in 41 elements.

6.2 Quantities tested and results

One tests the value of displacement to the node D following the axis z .

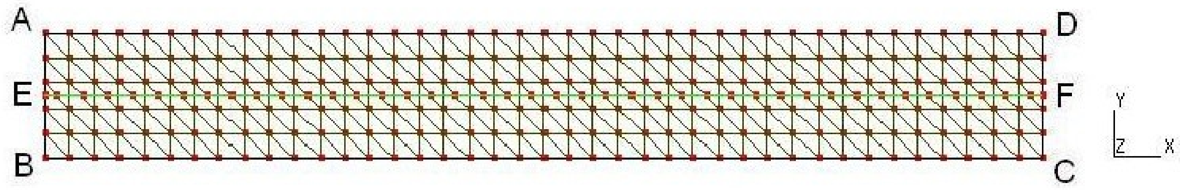
Node	Component	Value of reference (m)	Accuracy
D	DZ	0.000000 -0.101677	$1.0\mathrm{E}-2$

One tests also the tension in several elements of the cable.

Net	Value of reference (N)	Accuracy
$M10$	3.75×10^5	$1.0\mathrm{E}-8$
$M70$	3.75×10^5	$1.0\mathrm{E}-8$
$M130$	3.75×10^5	$1.0\mathrm{E}-8$

7 Modelization E

7.1 Characteristic of the modelization



- Modelization: `Standard`
- Q4GG of finite elements: Triangle (TRIA3), dimension of $0.1\mathrm{m}$.

Two elements triangles are created from an element quadrangle of the model of the preceding modelization.
The plate is then cut out in 5 elements over its width and 40 over its length.
Steel reinforcement is modelled by elements `BARS`. The cable is discretized in 41 elements.

7.2 Quantities tested and results

One tests the value of displacement to the node D following the axis z .

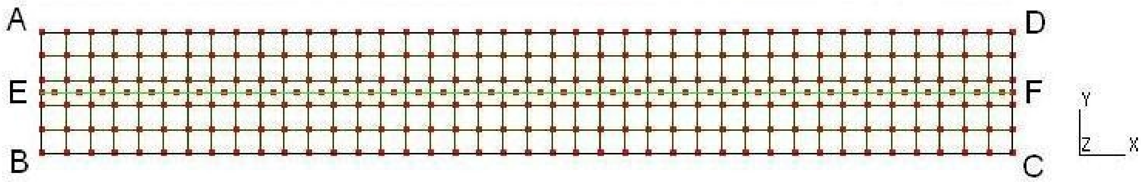
Node	Component	Value of reference (m)	Accuracy
D	`DZ`	0.000000 -0.101677	$1.0\mathrm{E}-2$

One tests also the tension in several elements of the cable.

Net	Value of reference (N)	Accuracy
$M20$	3.75×10^5	$1.0\mathrm{E}-8$
$M72$	3.75×10^5	$1.0\mathrm{E}-8$
$M127$	3.75×10^5	$1.0\mathrm{E}-8$

8 Modelization F

8.1 Characteristic of the modelization



- Modelization: `Standard`
- Q4GG of finite elements: Quadrangle (QUAD4), dimension of $0.1\mathrm{m}$.

The plate is then cut out in 5 elements over its width and 40 over its length. It is modelled by nonlinear model `GLRC_DAMAGE`.
The steel wire rope is modelled by elements `BARS`. The cable is discretized in 41 elements.

8.2 Quantities tested and results

One tests the value of displacement to the node D along the axis z.

Node	Component	Value of reference (m)	Accuracy
D	`DZ`	0.000000 -0.101677	$1.0\mathrm{E}-2$

One tests also the tension in several elements of the cable.

Net	Value of reference (N)	Accuracy
$M10$	3.75×10^5	$1.0\mathrm{E}-8$
$M70$	3.75×10^5	$1.0\mathrm{E}-8$
$M130$	3.75×10^5	$1.0\mathrm{E}-8$

9 Synthesis

A the results got with the modelization correspond well to the analytical solution suggested from the theory of the beams. That thus validates the choice of the model of beam like reference.

In the modelizations B with E (DKT and Q4GG), the values of the stresses in the cable in output of CALC_PRECONT are well those expected. In addition, the values of deflection obtained are almost identical to the analytical solution. It is noticed that the percentage of error is less important with modelization DKT for this case of study, but especially that the error on the deflection is quite less than with the modelization 3D .

The modelization F allows to validate that the computation of prestressing is correct when the concrete is modelled by model GLRC_DAMAGE .

This validates the use of macro-command CALC_PRECONT for the shells.