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## SSLS133 - Flexbeam with variable thickness

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### Abstract:

This test represents a quasi-static computation of a flexbeam with variable thickness. It is embedded at an end, and is subjected to a vertical force at the other end. This test makes it possible to test shell elements voluminal SHB8 and SHB20 to manage the variations of thickness. Four modelizations are tested:

- Finite elements SHB8 for a linear variation of the thickness of the plate (modelization *A* ).
- Finite elements SHB20 for a linear variation of the thickness of the plate (modelization *B* ).
- Finite elements SHB8 for a quadratic variation of the thickness of the plate (modelization *C* ).
- Finite elements SHB20 for a quadratic variation of the thickness of the plate (modelization *D* ).

Displacements obtained are compared with the elastic analytical solution of a beam in bending. This test makes it possible to show the capacities and the limiting of elements SHB8 and SHB20 to manage the variations of thickness.

## 1 Problem of reference

### 1.1 Geometry

#### 1.1.1 Plates with thickness varying linearly

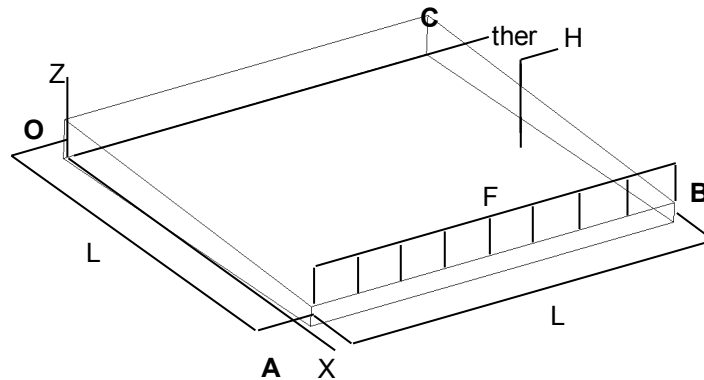


Figure 1.1.1-1 :

Length  $L=100\text{ m}$  , width  $l=100\text{ m}$  .

The thickness  $h$  varies linearly:

$$h(x) = ax + b$$

We pose  $h(x=0)=h_1=10\text{ m}$  and  $h(x=L)=h_2=5\text{ m}$  what gives us:

$$a = (h_2 - h_1)/L \text{ and } b = h_1$$

#### 1.1.2 Plates with thickness varying quadratically

the thickness  $h$  varies in a quadratic way:

$$h(x) = ax^2 + bx + c$$

We pose  $h(x=0)=h_1=10\text{ m}$  ,  $h(x=L)=h_2=5\text{ m}$  and  $h(x=L/2)=h_{12}=6,25\text{ m}$  what gives us

$$a = (2(h_1 + h_2) - 4h_{12})/L^2, b = (4h_{12} - h_2 - 3h_1)/L \text{ and } c = h_1$$

## 1.2 Material properties

Modulus Young:  $E=2.10^{11}\text{ Pa}$

Poisson's ratio:  $\nu=0.0$

## 1.3 Boundary conditions and loadings

Boundary conditions:

Embedded on the side  $OC$  :  $u=v=w=0$  ,  $\theta_x=\theta_y=\theta_z=0$

Loading:

At the end  $AB$  , a load uniformly distributed of resultant:

Force parallel with the axis  $Z$  ;  $F_z=1N$

## 2 Reference solution

### 2.1 Method of calculating used for the reference solution

the results of reference are got by the theory of the elastic beams.

In the case of a linear variation of the thickness, vertical displacement at the end  $AB$  is given by [1]:

$$w(x) = -\frac{FL^2}{2EI_{y_1}c^3} \frac{\left(2Lcx + c^2x^2 - c^3x^2 + 2L(L+cx)\ln\left(\frac{L}{L+cx}\right)\right)}{(L+cx)}$$

With

$$c = \left(\frac{I_{y_2}}{I_{y_1}}\right)^{\frac{1}{3}} - 1 \quad \text{and} \quad I_{y_i} = \frac{bh_i^3}{12}$$

In the case of a quadratic variation of the thickness, it is possible to find a formula exact of displacement. However its general statement is sufficiently complex not to be able to be written here. We formulated the approximate function of vertical displacement according to  $x$  our case:

$$w(x) = 3 \cdot 10^{-8} \frac{2x-200}{x^2-200x+20000} + 6 \cdot 10^{-10} \arctan(0.01x-1) - 3 \cdot 10^{-12}x + 7.71238 \cdot 10^{-10} \text{ m}$$

### 2.2 Quantities and results of reference

Displacement of the points  $A$  and  $B$  following  $Z$ .

### 2.3 Bibliographical references

[1] [V3.01.400] SSSL400 – non-prismatic Beam, subjected to forces specific or distributed.

## 3 Modelization A

### 3.1 Characteristic of the modelization

Element SHB8 and thickness varying linearly

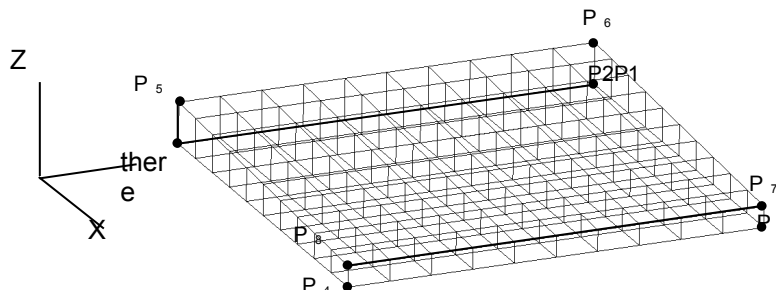


Figure 3.1-1 : Mesh of the modelization A

Cutting: a regular mesh is considered in this modelization.

Regular mesh:

100 meshes SHB8 : 10 according to the width, 10 according to the length, 1 according to the thickness

Boundary conditions:

All nodes inside the side  $P_1P_2P_6P_5$  : blocked displacement following  $x$

All the nodes on the edge  $P_1P_5$  : blocked displacement following  $y$

All the nodes on the edge  $P_2P_1$  : blocked displacement following  $z$

Loading:

Force linearly distributed on the edge  $P_8P_7$  :  $F = 1$

Name of the nodes:

Not P1	N022	Not P5	N020
Not P2	N002	Not P6	N001
Not P3	N102	Not P7	N100
Not P4	N172	Not P8	N171

### 3.2 Characteristics of the mesh

Many nodes: 242

Number of meshes and types: 100 SHB8

### 3.3 Quantities tested and regular

Mesh results:

Not	Quantity in unit	Reference	Aster	% difference
P7	displacement W ( m )	3.2710 10-10	3.2711 10-10	+0.004
P8	displacement W ( m )	3.2710 10-10	3.2711 10-10	+0.004

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

## 4 Modelization B

### 4.1 Characteristic of the modelization

Element SHB20 and thickness varying linearly

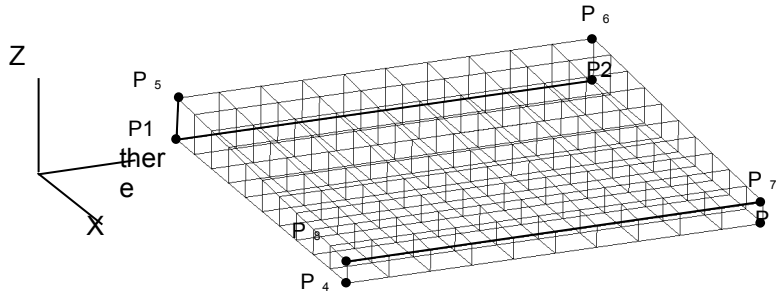


Figure 4.1-1 : Mesh of the modelization B

Cutting: a regular mesh is considered in this modelization.

Regular mesh:

100 meshes SHB20 : 10 according to the width, 10 according to the length, 1 according to the thickness

Boundary conditions:

All nodes inside the side  $P_1P_2P_6P_5$  : blocked displacement following  $x$

All the nodes on the edge  $P_1P_5$  : blocked displacement following  $y$

All the nodes on the edge  $P_2P_1$  : blocked displacement following  $z$

Loading:

Force linearly distributed on the edge  $P_8P_7$  :  $F = 1$

Name of the nodes:

Not P1	N347	Not P5	N340
Not P2	N579	Not P6	N572
Not P3	N006	Not P7	N002
Not P4	N074	Not P8	N067

### 4.2 Characteristics of the mesh

Many nodes: 803

Number of meshes and types: 100 SHB20

### 4.3 Quantities tested and regular

Mesh results:

Not	Quantity in unit	Reference	Aster	% difference
P7	displacement $W$ ( $m$ )	3.2710 10 <sup>-10</sup>	3.2866 10 <sup>-10</sup>	+0.476
P8	displacement $W$ ( $m$ )	3.2710 10 <sup>-10</sup>	3.2866 10 <sup>-10</sup>	+0.476

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## 5 Modelization C

### 5.1 Characteristic of the modelization

Element SHB8 and thickness varying quadratically

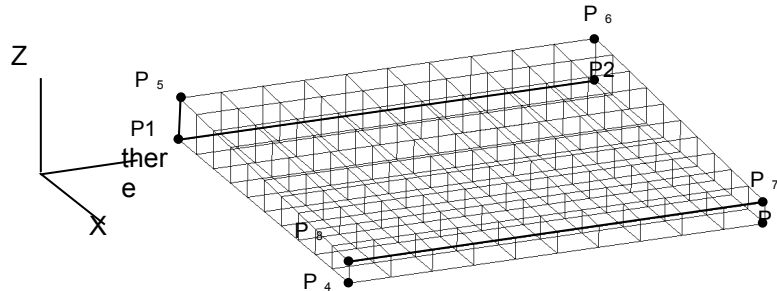


Figure 5.1-1 : Mesh of the modelization C

the characteristics are the same ones as for the modelization *A*

Name of the nodes:

Not P1	N005	Not P5	N003
Not P2	N006	Not P6	N004
Not P3	N008	Not P7	N002
Not P4	N007	Not P8	N001

### 5.2 Characteristics of the mesh

The mesh is the same one as the modelization *A* except for the thickness which varies here in a quadratic way.

### 5.3 Quantities tested and regular

Mesh results:

Not	Quantity in unit	Reference	Aster	% difference
P7	displacement W ( m )	4.7124 10 <sup>-10</sup>	5.1212 10 <sup>-10</sup>	+8.67
P8	displacement W ( m )	4.7124 10 <sup>-10</sup>	5.1212 10 <sup>-10</sup>	+8.67

## 6 Modelization D

### 6.1 Characteristic of the modelization

Element SHB20 and thickness varying quadratically

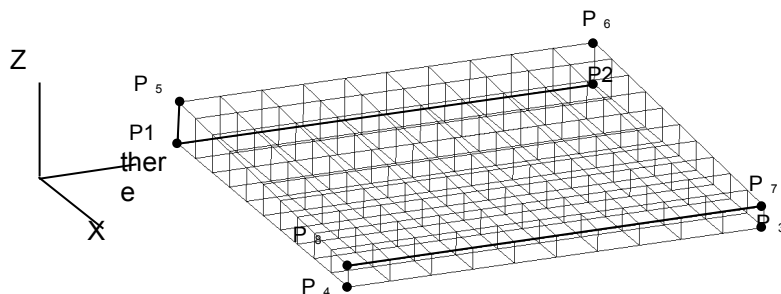


Figure 6.1-1 : Mesh of the modelization D

the characteristics are the same ones as for the modelization *B*

Name of the nodes:

Not P1	N005	Not P5	N003
Not P2	N006	Not P6	N004
Not P3	N008	Not P7	N002
Not P4	N007	Not P8	N001

### 6.2 Characteristics of the mesh

The mesh is the same one as the modelization *B* except for the thickness which varies here in a quadratic way.

### 6.3 Quantities tested and regular

Mesh results:

Not	Quantity in unit	Reference	Aster	% difference
P7	displacement W ( <i>m</i> )	4.7124 10 <sup>-10</sup>	4.6754 10 <sup>-10</sup>	-0.784
P8	displacement W ( <i>m</i> )	4.7124 10 <sup>-10</sup>	4.6754 10 <sup>-10</sup>	-0.784



## 7 Summary of the results

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In the case in a variation lineaire of the thickness of the plate, good solutions are obtained some is the finite element used (SHB8 or SHB20).

When the geometrical variation is of a quadratic nature, elements SHB20 provide more precise results (error  $<1\%$  ) that elements SHB8 (error  $<9\%$  ).