SSLS127 – Bending of a reinforced concrete slab (GLRC_DAMAGE `models`) leaned on 4 with dimensions: elastic mode of plate

Summarized:

This test represents the computation of a reinforced concrete slab, in bending, subjected to a pressure. It makes it possible to validate the model `modelization` DKTG with `GLRC_DAMAGE` for the linear elastic behavior and modelization `Q4GG` with the model `ELAS`. The slab is in configuration paves: simple bearings on the four with dimensions ones.

Four modelizations are carried out:

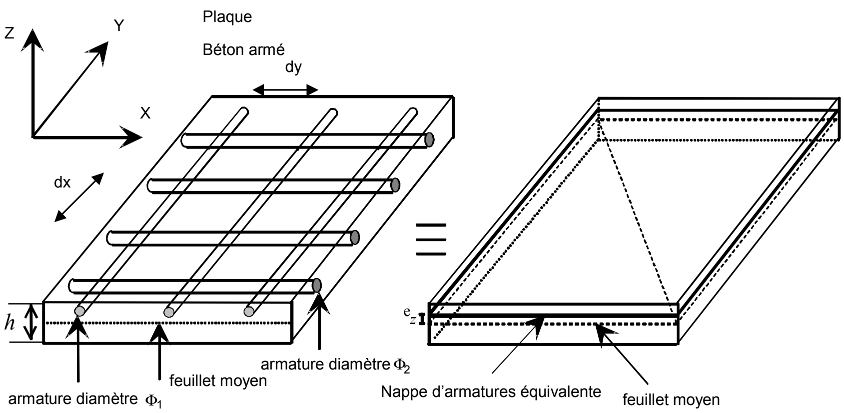
1) Modelization A makes it possible to test the model `DKTG` with `TRIA3`,
2) Modelization B makes it possible to test DKTG `with` QUAD4 `the model`.
3) Modelization C makes it possible to test the model `Q4GG` with `TRIA3`,
4) Modelization D makes it possible to test Q4GG `with` QUAD4 `the model`.

1 Problem of reference

1.1 Geometry

Paves square, length $l=1.8m$, of thickness $h=0.12m$, out of simple bearing on four edges. The reinforcement of bending is parallel to edges; it is identical on each of the two sides and in each of the two meanings (dx , dy being spacings of irons in the directions x and y). The coating of the longitudinal irons closest to the sides is of $22\,mm$. The coating of irons compared to side edges of slab of $2\,cm$ is neglected. The table hereafter recapitulates the data of reinforcement. The geometrical percentage of steel μ is given for a face in a meaning.

Diameter of reinforcements	Spacing	Section steel/section of the concrete	outdistances grid/mean surface of slab
$\Phi=0{,}01\,\mathrm{m}$	$dx=dy=0{,}1\,\mathrm{m}$	$\mu=0{,}65$	$e_s=\pm 0{,}038\,\mathrm{m}$



One notes $a_x=\frac{A_x}{d_x}$ rate $a_y=\frac{A_y}{d_y}$ of reinforcement and the (here: $a_x=a_y=7{,}854.10^{-4}\,\mathrm{m}$), A_x (A_y) being the area of the section of an iron bar in the direction $x(y)$; e_s is the distance from the three-dimensions functions at mean surface.

1.2 Material properties

the mechanical properties of steels are the following ones:

Modulus Young E_a	Poisson's ratio	Yield stress to 0.2% σ_y	Rupture limit σ_r	Slope of hardening	Lengthening with fracture
$210000\,MPa$	0,3	$500\,MPa$	$570\,MPa$	$473\,MPa$	15%

Those of the concrete are the following ones:

Modulus Young E_b	Poisson's ratio	Strength in compression σ_c	Strength in tension σ_t
$35700\,MPa$	0,22	$52{,}5\,MPa$	$4{,}4\,MPa$

1.3 Boundary conditions and loadings

- the boundary conditions are summarized in simple bearings: vertical displacement blocked and free rotations on four edges of slab.
- Uniform pressure $p=0{,}01\,MPa$

1.4 Initial conditions

Without object.

2 Reference solution

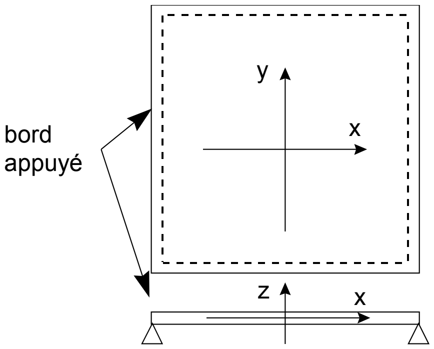
2.1 Method of calculating used for the reference solution

the elastic relations, connecting the membrane forces N and of bending M to the membrane strains ε and the curvatures κ and taking account of two symmetric grids, are written:

$$\boldsymbol{N}=\left(\frac{E_b h}{1-\nu_b^2}\begin{bmatrix}1 & \nu_b & 0\\ \nu_b & 1 & 0\\ 0 & 0 & \frac{1-\nu_b}{2}\end{bmatrix}+2E_a\begin{bmatrix}a_x & 0 & 0\\ 0 & a_y & 0\\ 0 & 0 & 0\end{bmatrix}\right)\boldsymbol{\epsilon}$$

$$\boldsymbol{M}=\left(\frac{E_b h^3}{12(1-\nu_b^2)}\begin{bmatrix}1 & \nu_b & 0\\ \nu_b & 1 & 0\\ 0 & 0 & \frac{1-\nu_b}{2}\end{bmatrix}+2E_a e_s^2\begin{bmatrix}a_x & 0 & 0\\ 0 & a_y & 0\\ 0 & 0 & 0\end{bmatrix}\right)\boldsymbol{\kappa}$$

As regards a configuration paves, one assigns to the concrete the usual Poisson's ratio $\nu_b=0{,}22$. The slab is simply leaned on the four edges:



The equivalent flexural stiffness is:

$D_{éq}=\frac{E_b h^3}{12(1-\nu_b^2)}+2E_a e_s^2 a_x$,

that is to say here: $D_{éq}=5{,}8786\,\mathrm{MNm}$;

Moreover: $\nu_{éq}=\frac{\nu_b E_b h^3}{12(1-\nu_b^2)D_{éq}}$,

that is to say: $\nu_{éq}=0{,}2022$

Quantity	Statement
Marks with arrows in the center under pressure [2]	$w=\frac{0{,}0464\,pl^4}{12(1-\nu_{éq}^2)D_{éq}}$
Curvature in the center [2]	$\kappa_{xx}=\kappa_{yy}=\frac{0{,}04784\,pl^2}{(1+\nu_{éq})D_{éq}}$
total Moment in the center [2]	$M_{xx}=M_{yy}=0{,}04784\,pl^2$

2.2 Results of reference

- Marks with arrows in the center under pressure: $w=6{,}926.10^{-5}\,\mathrm{m}$
- Curvature in the center: $\kappa_{xx}=\kappa_{yy}=2{,}193.10^{-4}\,\mathrm{m}^{-1}$
- Total moment in the center: $M_{xx}=M_{yy}=1550\,\mathrm{Nm/ml}$

2.3 Uncertainty on the analytical

solution Solution.

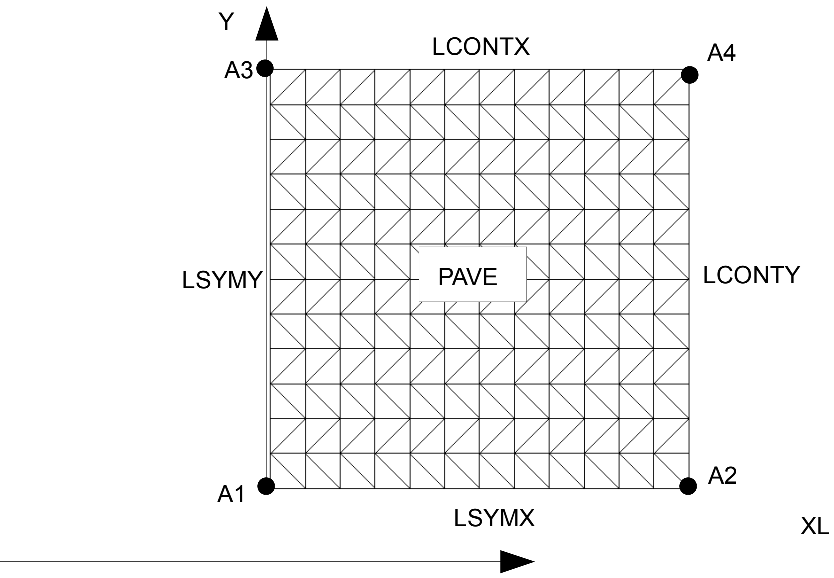
2.4 Bibliographical references

[1] KOECHLIN P., MILL S., “Models total behavior of the reinforced concrete plates under dynamic loading of bending: Model GLRC”, EDF Notes/R & D /AMA HT-62/01/028A.

[2] J.Dulac, “Behavior dynamic elastoplastic of reinforced concrete slabs. Tests CEMETE – December 1979 – Slabs 8 to 12”: Note EDF: ESE/GC/82/13/A

3 Modelization A

3.1 Characteristic of modelization



Modelization Q4GG (TRIA3)

- Boundary conditions:
  . Dimensioned $A2A4$: $DZ=0$
- Conditions of symmetry
  . Dimensioned $A1A2$: $DY=DRX=0$
  . Dimensioned $A1A3$: $DX=DRY=0$

slab is symmetric compared to the planes $(X=0)$ and $(Y=0)$, computations are carried out on a quarter of slab.

3.2 Characteristics of the mesh

Many nodes: 169
Number of meshes and type: 288 TRIA3

3.3 Quantities tested and results

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Standard	***value***	**Identification of reference**	***Reference***	***Tolerance (%)***
$w(x=0,y=0)$	$DZ(A1)$	"ANALYTIQUE"	6,926.10-5	12.00%
$M_{xx}(x=0,y=0)$	$MXX(A1)$	"ANALYTIQUE"	1550	8.00%
$M_{yy}(x=0,y=0)$	$MYY(A1)$	"ANALYTIQUE"	1550	8.00%
$\kappa_{xx}(x=0,y=0)$	$KXX(A1)$	"ANALYTIQUE"	2,193.10-4	8.00%
$\kappa_{yy}(x=0,y=0)$	$KYY(A1)$	"ANALYTIQUE"	2,193.10-4	8.00%

the quantities are expressed in the reference defined by the angles nautiquesformuleet $\alpha=33°$ $\beta=12°$

Standard	**Value**	**Identification of reference**	**Reference**	**Tolerance**
$M_{xx}(x=0,y=0)$	$MXX(A1)$	"ANALYTIQUE"	1550.0	8.%
$M_{yy}(x=0,y=0)$	$MYY(A1)$	"ANALYTIQUE"	1550.0	8.%
$M_{xy}(x=0,y=0)$	$MXY(A1)$	"ANALYTIQUE"	0.	2.
$\kappa_{xx}(x=0,y=0)$	$KXX(A1)$	"ANALYTIQUE"	2.193 10-4	3.%
$\kappa_{yy}(x=0,y=0)$	$KYY(A1)$	"ANALYTIQUE"	2.193 10-4	3.%

$\kappa_{xy}(x=0, y=0)$	$KXY(A1)$	"ANALYTIQUE"	0.	0.001

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Standard	value			Identification of reference	Tolérance%	Reference
M_{xx}	MXX	$M266$	$Point\,3$	"NON_REGRESSION"	1445.794	1.e-6
M_{yy}	MYY	$M266$	$Point\,3$	"NON_REGRESSION"	1447.847	1.e-6
M_{xy}	MXY	$M266$	$Point\,3$	"NON_REGRESSION"	0.526	1.e-6
κ_{xx}	KXX	$M266$	$Point\,3$	"NON_REGRESSION"	2.14096 10-4	1.e-6
κ_{yy}	KYY	$M266$	$Point\,3$	"NON_REGRESSION"	2.14565 10-4	1.e-6
κ_{xy}	KXY	$M266$	$Point\,3$	"NON_REGRESSION"	1.2018 10-7	1.e-6

3.4 Remarks

the coefficients of the following matrixes of elasticity, used during computations, were calculated with $\nu_b=0,22$:

1) Stamp elasticity out of membrane: $\begin{bmatrix} 4832 & 990,4 & 0 \\ 990,4 & 4832 & 0 \\ 0 & 0 & 1756 \end{bmatrix} 10^6\,\mathrm{N/m}$

2) Stamp elasticity in bending: $\begin{bmatrix} 5,879 & 1,188 & 0 \\ 1,188 & 5,879 & 0 \\ 0 & 0 & 2,107 \end{bmatrix} 10^6\,\mathrm{N/m}$

To be certain to remain in the elastic domain, the yield stresses expressed in orthotropic reference, are built-in arbitrarily with a very high value:

1. Yield stresses in positive bending:

   Directorate X: $1.10^{10}\,\mathrm{MNm/ml}$
   Direction there: $1.10^{10}\,\mathrm{MNm/ml}$

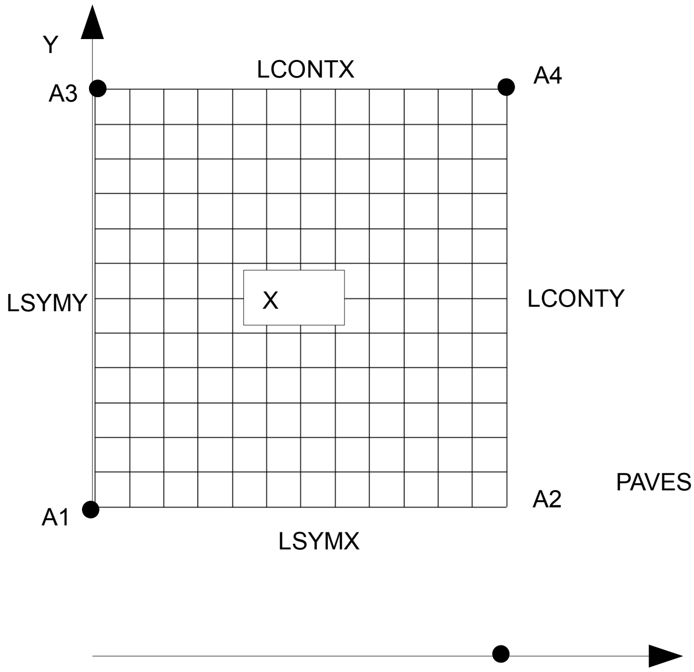
2. Yield stresses in negative bending:

   Directorate X: $-1.10^{10}\,\mathrm{MNm/ml}$
   Direction there: $-1.10^{10}\,\mathrm{MNm/ml}$

As the structure remains in the elastic domain, the kinematical coefficient of recall (constant of Prager) can take an unspecified value.

4 Modelization B

4.1 Characteristic of modelization



Modelization Q4GG (QUAD4)

- Boundary conditions:
  . Dimensioned $A2A4$: $DZ=0$
  . Dimensioned $A3A4$: $DZ=0$

- Conditions of symmetry
  . Dimensioned $A1A2$: $DY=DRX=0$
  . Dimensioned $A1A3$: $DX=DRY=0$

slab is symmetric compared to the planes $(X=0)$ and $(Y=0)$, computations are carried out on a quarter of slab.

4.2 Characteristics of the mesh

Many nodes: 169

Number of meshes and type: 144 QUAD4

4.3 Quantities tested and results

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Standard	*value*	**Identification of reference**	*Reference*	*Tolerance (%)*
$w(x=0, y=0)$	$DZ(A1)$	"ANALYTIQUE"	6,926.10-5	12.00%
$M_{xx}(x=0, y=0)$	$MXX(A1)$	"ANALYTIQUE"	1550	8.00%
$M_{yy}(x=0, y=0)$	$MYY(A1)$	"ANALYTIQUE"	1550	8.00%
$\kappa_{xx}(x=0, y=0)$	$KXX(A1)$	"ANALYTIQUE"	2,193.10-4	8.00%
$\kappa_{yy}(x=0, y=0)$	$KYY(A1)$	"ANALYTIQUE"	2,193.10-4	8.00%

the quantities are expressed in the reference defined by the angles nautiqueset $\alpha=33°$ $\beta=12°$.

Standard	value	Identification of reference	Reference	Tolerance
$M_{xx}(x=0, y=0)$	$MXX(A1)$	"ANALYTIQUE"	1550.0	8.%
$M_{yy}(x=0, y=0)$	$MYY(A1)$	"ANALYTIQUE"	1550.0	8.%
$M_{xy}(x=0, y=0)$	$MXY(A1)$	"ANALYTIQUE"	0.	0.01
$\kappa_{xx}(x=0, y=0)$	$KXX(A1)$	"ANALYTIQUE"	2.193 10-4	3.%

$\kappa_{yy}(x=0, y=0)$	$KYY(A1)$	"ANALYTIQUE"	2.193 10-4	3.%
$\kappa_{xy}(x=0, y=0)$	$KXY(A1)$	"ANALYTIQUE"	0.	0.001

Value	Standard			Identification of reference	Tolérance%	Reference
M_{xx}	MXX	$M133$	$Point\,4$	"NON_REGRESSION"	1444.999	1.e-10
M_{yy}	MYY	$M133$	$Point\,4$	"NON_REGRESSION"	1447.976	1.e-10
M_{xy}	MXY	$M133$	$Point\,4$	"NON_REGRESSION"	-0.6626	1.e-10
κ_{xx}	KXX	$M133$	$Point\,4$	"NON_REGRESSION"	2.1394 10-4	1.e-10
κ_{yy}	KYY	$M133$	$Point\,4$	"NON_REGRESSION"	2.1462 10-4	1.e-10
κ_{xy}	KXY	$M133$	$Point\,4$	"NON_REGRESSION"	-1.5151 10-7	1.e-10

4.4 Remarks

the coefficients of the following matrixes of elasticity, used during computations, were calculated with $\nu_b=0{,}22$:

- Stamp elasticity out of membrane: $\begin{bmatrix} 4832 & 990{,}4 & 0 \\ 990{,}4 & 4832 & 0 \\ 0 & 0 & 1756 \end{bmatrix} 10^6\,\mathrm{N/m}$
- Stamp elasticity in bending: $\begin{bmatrix} 5{,}879 & 1{,}188 & 0 \\ 1{,}188 & 5{,}879 & 0 \\ 0 & 0 & 2{,}107 \end{bmatrix} 10^6\,\mathrm{N/m}$

To be certain to remain in the elastic domain, the yield stresses expressed in orthotropic reference, are built-in arbitrarily with a very high value:

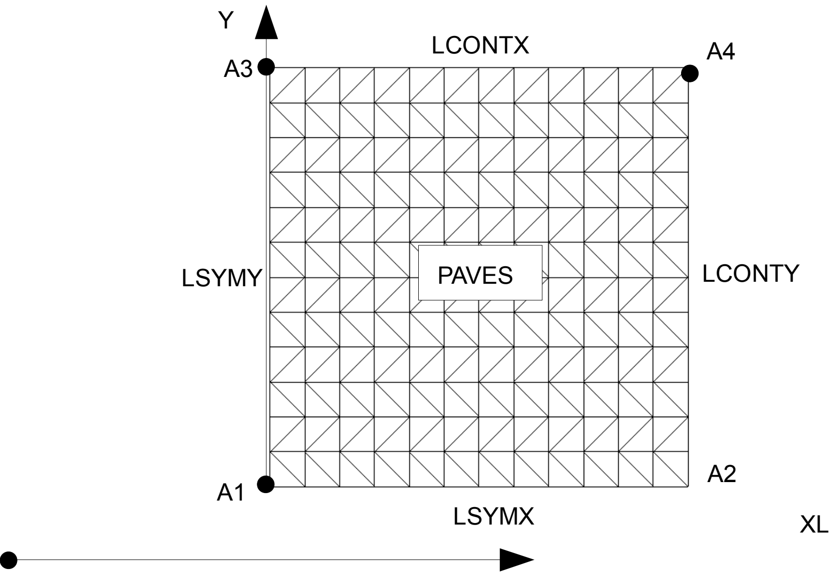
1. Yield stresses in positive bending:
   Direction x : $1.10^{10}\,\mathrm{MNm/ml}$
   Direction y : $1.10^{10}\,\mathrm{MNm/ml}$
2. Yield stresses in negative bending:

   Direction x : $-1.10^{10}\,\mathrm{MNm/ml}$
   Direction y : $-1.10^{10}\,\mathrm{MNm/ml}$

As the structure remains in the elastic domain, the kinematical coefficient of recall (constant of Prager) can take an unspecified value.

5 Modelization C

5.1 Characteristic of modelization



Modelization `Q4GG (TRIA3)`

- Boundary conditions:
  . Dimensioned $A2A4$: $DZ=0$
- Conditions of symmetry
  . Dimensioned $A1A2$: $DY=DRX=0$
  . Dimensioned $A1A3$: $DX=DRY=0$

slab is symmetric compared to the planes $(X=0)$ and $(Y=0)$, computations are carried out on a quarter of slab.

5.2 Characteristics of the mesh

Many nodes: 169
Number of meshes and type: 288 `TRIA3`

5.3 Quantities tested and results

Standard	Value	Identification of Reference	Reference	% Tolerance
$w(x=0,y=0)$	$DZ(A1)$	"ANALYTIQUE"	6,926.10-5	20.00%
$M_{xx}(x=0,y=0)$	$MXX(A1)$	"ANALYTIQUE"	1550	8%
$M_{yy}(x=0,y=0)$	$MYY(A1)$	"ANALYTIQUE"	1550	8%
$\kappa_{xx}(x=0,y=0)$	$KXX(A1)$	"ANALYTIQUE"	2,193.10-4	8%
$\kappa_{yy}(x=0,y=0)$	$KYY(A1)$	"ANALYTIQUE"	2,193.10-4	8%

the quantities are expressed in the reference defined by the angles nautiquesformuleet $\alpha=33°$ $\beta=12°$

Standard	Value	Identification of reference	Reference	Tolerance
$M_{xx}(x=0,y=0)$	$MXX(A1)$	"ANALYTIQUE"	1550.0	4.5%
$M_{yy}(x=0,y=0)$	$MYY(A1)$	"ANALYTIQUE"	1550.0	4.5%
$M_{xy}(x=0,y=0)$	$MXY(A1)$	"ANALYTIQUE"	0.	0.2
$\kappa_{xx}(x=0,y=0)$	$KXX(A1)$	"ANALYTIQUE"	2.193 10-4	2.%
$\kappa_{yy}(x=0,y=0)$	$KYY(A1)$	"ANALYTIQUE"	2.193 10-4	2.%

$\kappa_{xy}(x=0, y=0)$	$KXY(A1)$	"ANALYTIQUE"	0.	0.001

Value	Standard			Identification of reference	Reference	Tolérance %
M_{xx}	MXX	$M266$	$Point\,1$	"NON_REGRESSION"	1499.292	1.e-6
M_{yy}	MYY	$M266$	$Point\,1$	"NON_REGRESSION"	1499.390	1.e-6
M_{xy}	MXY	$M266$	$Point\,1$	"NON_REGRESSION"	0.11	1.e-6
κ_{xx}	KXX	$M266$	$Point\,1$	"NON_REGRESSION"	2.221 10-4	1.e-6
κ_{yy}	KYY	$M266$	$Point\,1$	"NON_REGRESSION"	2.221 10-4	1.e-6
κ_{xy}	KXY	$M266$	$Point\,1$	"NON_REGRESSION"	2.5 10-8	1.e-6

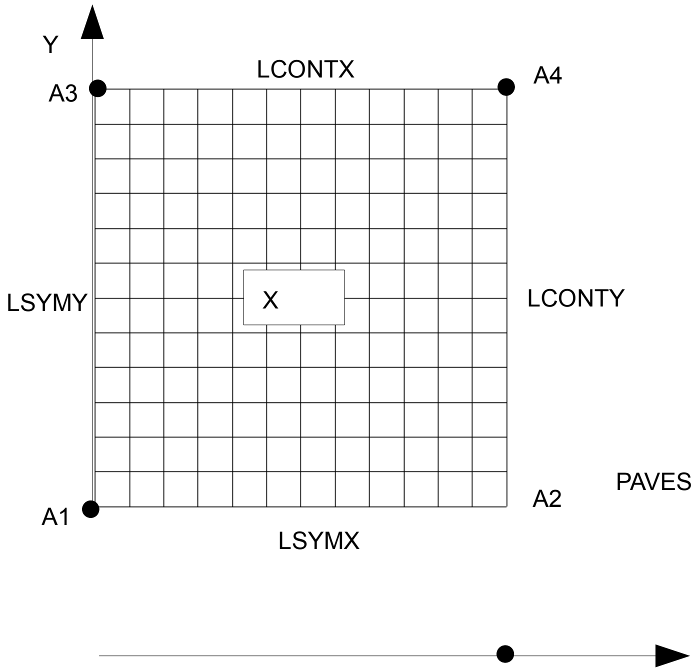
5.4 Remarks

the coefficients of the following matrixes of elasticity, used during computations, were calculated with $\nu_b=0{,}22$:

- Stamp elasticity out of membrane: $\begin{bmatrix} 4832 & 990{,}4 & 0 \\ 990{,}4 & 4832 & 0 \\ 0 & 0 & 1756 \end{bmatrix} 10^6 \mathrm{N/m}$
- Matrix of elasticity in bending: $\begin{bmatrix} 5{,}879 & 1{,}188 & 0 \\ 1{,}188 & 5{,}879 & 0 \\ 0 & 0 & 2{,}107 \end{bmatrix} 10^6 \mathrm{N/m}$

6 Modelization D

6.1 Characteristic of modelization



Modelization `Q4GG` (`QUAD4`)

- Boundary conditions:
  . Dimensioned $A2A4$: $DZ=0$
  . Dimensioned $A3A4$: $DZ=0$

- Conditions of symmetry
  . Dimensioned $A1A2$: $DY=DRX=0$
  . Dimensioned $A1A3$: $DX=DRY=0$

slab is symmetric compared to the planes $(X=0)$ and $(Y=0)$, computations are carried out on a quarter of slab.

6.2 Characteristics of the mesh

Many nodes: 169

Number of meshes and type: 144 `QUAD4`

6.3 Quantities tested and results

Standard	Value	Identification of Reference	Reference	% Tolerance
$w(x=0, y=0)$	$DZ(A1)$	"ANALYTIQUE"	6,926.10-5	20.00%
$M_{xx}(x=0, y=0)$	$MXX(A1)$	"ANALYTIQUE"	1550	8%
$M_{yy}(x=0, y=0)$	$MYY(A1)$	"ANALYTIQUE"	1550	8%
$\kappa_{xx}(x=0, y=0)$	$KXX(A1)$	"ANALYTIQUE"	2,193.10-4	8%
$\kappa_{yy}(x=0, y=0)$	$KYY(A1)$	"ANALYTIQUE"	2,193.10-4	8%

the quantities are expressed in the reference defined by the angles nautiquesformuleet $\alpha=33°$ $\beta=12°$.

Standard	value	Identification of reference	Reference	Tolerance
$M_{xx}(x=0, y=0)$	$MXX(A1)$	"ANALYTIQUE"	1550.0	4.5%
$M_{yy}(x=0, y=0)$	$MYY(A1)$	"ANALYTIQUE"	1550.0	4.5%
$M_{xy}(x=0, y=0)$	$MXY(A1)$	"ANALYTIQUE"	0.	0.001

$\kappa_{xx}(x=0, y=0)$	$KXX(A1)$	"ANALYTIQUE"	2.193 10-4	1.e-10
$\kappa_{yy}(x=0, y=0)$	$KYY(A1)$	"ANALYTIQUE"	2.193 10-4	1.e-10
$\kappa_{xy}(x=0, y=0)$	$KXY(A1)$	"ANALYTIQUE"	0.	0.001

Value	Standard			Identification of reference	Tolérance%	Reference
M_{xx}	MXX	$M133$	$Point\,4$	"NON_REGRESSION"	1505.472	1.e-10
M_{yy}	MYY	$M133$	$Point\,4$	"NON_REGRESSION"	1508.529	1.e-10
M_{xy}	MXY	$M133$	$Point\,4$	"NON_REGRESSION"	-0.68	1.e-10
κ_{xx}	KXX	$M133$	$Point\,4$	"NON_REGRESSION"	2.2290 10-4	1.e-10
κ_{yy}	KYY	$M133$	$Point\,4$	"NON_REGRESSION"	2.2359 10-4	1.e-10
κ_{xy}	KXY	$M133$	$Point\,4$	"NON_REGRESSION"	-1.56 10-7	1.e-10

6.4 Remarks

the coefficients of the following matrixes of elasticity, used during computations, were calculated with $\nu_b=0,22$:

- Stamp elasticity out of membrane: $\begin{bmatrix} 4832 & 990,4 & 0 \\ 990,4 & 4832 & 0 \\ 0 & 0 & 1756 \end{bmatrix} 10^6 \mathrm{N/m}$
- Matrix of elasticity in bending: $\begin{bmatrix} 5,879 & 1,188 & 0 \\ 1,188 & 5,879 & 0 \\ 0 & 0 & 2,107 \end{bmatrix} 10^6 \mathrm{N/m}$

7 Summary of the results

By comparing the results of the four modelizations with the analytical solution, one observes:

- DKTG : to the maximum $10,7\%$ of variation for displacements, and 6.5% for the moment and the curvature.
- Q4GG : to the maximum 19.1% of variation for displacements, and 3.5% the curvature.

One of the reasons is the assumption of homogeneity of the material in the formulas which are used to calculate the values of reference [1], whereas the matrixes of elasticity used take account of the NON-homogeneous reality of the reinforced concrete. One can thus estimate that these modelizations validate modelization DKTG and the model GLRC in elastic behavior, modelization Q4GG and the model ELAS.