SSLS126 – Bending of a reinforced concrete slab (GLRC_DAMAGE `models`) leaned on two with dimensions: elastic mode of beam

Summarized:

This test represents the computation of a reinforced concrete slab, in bending, subjected to a pressure. It makes it possible to validate the model `modelization` DKTG with `GLRC_DAMAGE` for the linear elastic behavior and modelization `Q4GG` with the model `ELAS`. The slab is in configuration beam: simple bearings on two sides opposite of slab.

Four modelizations are carried out:

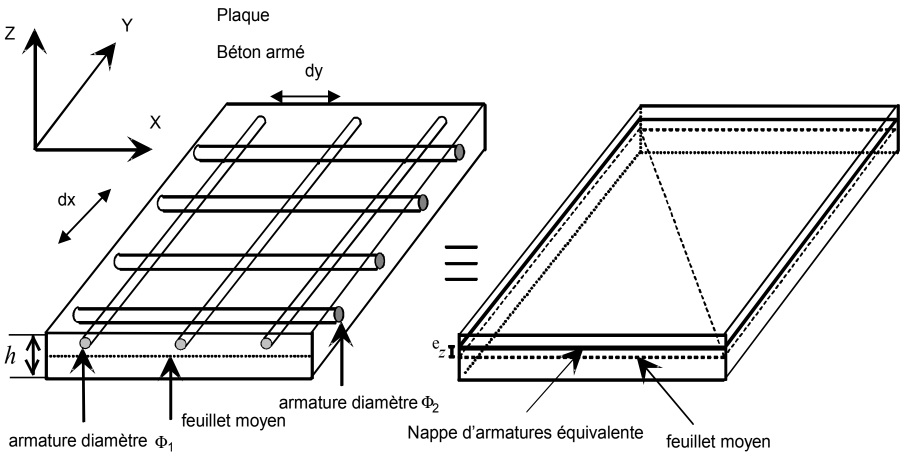
- Modelization A makes it possible to test the model `DKTG` with `TRIA3`,
- Modelization B makes it possible to test DKTG `with` QUAD4 `the model`.
- Modelization C makes it possible to test the model `Q4GG` with `TRIA3`,
- Modelization D makes it possible to test Q4GG `with` QUAD4 `the model`.

1 Problem of reference

1.1 Geometry

It is operated of a square slab, length $l=1.8\,m$, of thickness $h=0.12\,m$, bilateral simple bearings. The reinforcement of bending is parallel to edges; it is identical on each of the two sides and in each of the two meanings (dx, dy being spacings of irons in the directions x and y). The coating of the longitudinal irons closest to the sides is of $22\,mm$. The coating of irons compared to side edges of slab of $2\,cm$ is neglected. The table hereafter recapitulates the data of reinforcement. The geometrical percentage of steel μ is given for a face in a meaning.

Section of steels (mm)	Spacing (m) $dx \times dy$	Section steel/section of the concrete	outdistances grid/mean surface of slab
$HA10$	Nets $0{,}10 \times 0{,}10$	$\mu=0.65\%$	$e_z=\pm 38\,mm$



One notes $a_x=\frac{A_x}{d_x}$, $a_y=\frac{A_y}{d_y}$ rates of reinforcement (here: $a_x=a_y=7{,}854\,.\,10^{-4}\,m$), $A_x(A_y)$ being the section of an iron bar in the direction $x(y)$; e_z the distance from the three-dimensions functions at mean surface.

1.2 Material properties

the characteristics of steels are the following ones:

Modulus Young E_a	Poisson's ratio	Yield stress to 0.2% σ_y	Rupture limit σ_r	Slope of hardening	Lengthening to fracture
$210000\,MPa$	0,3	$500\,MPa$	$570\,MPa$	$473\,MPa$	15%

Those of the concrete are the following ones:

Modulus Young E_b	Poisson's ratio	Strength in compression, σ_c	Strength in tension, σ_t
$35700\,MPa$	0,22	$52{,}5\,MPa$	$4{,}4\,MPa$

1.3 Boundary conditions and loadings

the boundary conditions are summarized in simple bearings: vertical displacement blocked and free rotations on two edges in opposite, two other edges remaining free.

Uniform pressure: $p=0.01\,MPa$

1.4 Initial conditions

Without object.

2 Reference solution

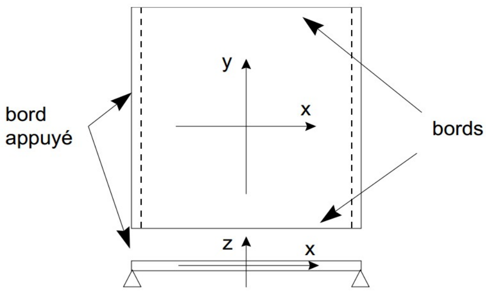
2.1 Method of calculating used for the reference solution

the elastic relations, connecting the membrane forces $\boldsymbol{N}$ and of bending $\boldsymbol{M}$ to the membrane strains ε and the curvatures κ and taking account of two symmetric grids, are written:

$$\boldsymbol{N}=\left(\frac{E_b h}{1-\nu_b^2}\begin{bmatrix}1 & \nu_b & 0\\ \nu_b & 1 & 0\\ 0 & 0 & \frac{1-\nu_b}{2}\end{bmatrix}+2E_a\begin{bmatrix}a_x & 0 & 0\\ 0 & a_y & 0\\ 0 & 0 & 0\end{bmatrix}\right)\boldsymbol{\varepsilon}$$

$$\boldsymbol{M}=\left(\frac{E_b h^3}{12(1-\nu_b^2)}\begin{bmatrix}1 & \nu_b & 0\\ \nu_b & 1 & 0\\ 0 & 0 & \frac{1-\nu_b}{2}\end{bmatrix}+2E_a e_z^2\begin{bmatrix}a_x & 0 & 0\\ 0 & a_y & 0\\ 0 & 0 & 0\end{bmatrix}\right)\boldsymbol{\kappa}$$

As regards a configuration beam, one assigns to the concrete a Poisson's ratio equal to 0 to cancel any bending in the perpendicular direction. Two opposite edges of slab are simply supported, two others remaining free:



The equivalent flexural stiffness which takes account of steels is:

$$(EI)_{éq}=\frac{E_b l h^3}{12}+2E_a a_x l e_z^2,$$

that is to say here: $(EI)_{éq}=10.111\ MNm^2$.

The elastic solution is calculated in beam theory for a value of equivalent pressure $p'=pl$. One obtains the values of the moments in configuration “plates” by division by the width of slab l.

Quantity in the center	Statement
Marks with arrows in the center under surface pressure	$w(l/2)=\frac{5p'l^4}{384(EI)_{éq}}$
Curvature	$\kappa_{xx}(l/2)=\frac{p'l^2}{8(EI)_{éq}}$
Strain	$\varepsilon_{xx}=\kappa_{xx}\frac{h}{2}$
total Moment (out of beam)	$M(l/2)=p'l^2/8$
total Moment (out of plate)	$M(l/2)=pl^2/8$

2.2 Results of reference

Marks with arrows in the center under surface pressure: $w=2{,}433.10^{-4}m$
Curvature: $k=7{,}210.10^{-4}m^{-1}$
Strain: $\varepsilon_{xx}=-0.4326.10^{-4}$ on the lower skin
total Moment (out of beam): $M=7290\,Nm$
Total moment (out of plate): $M=4050\,Nm/ml$

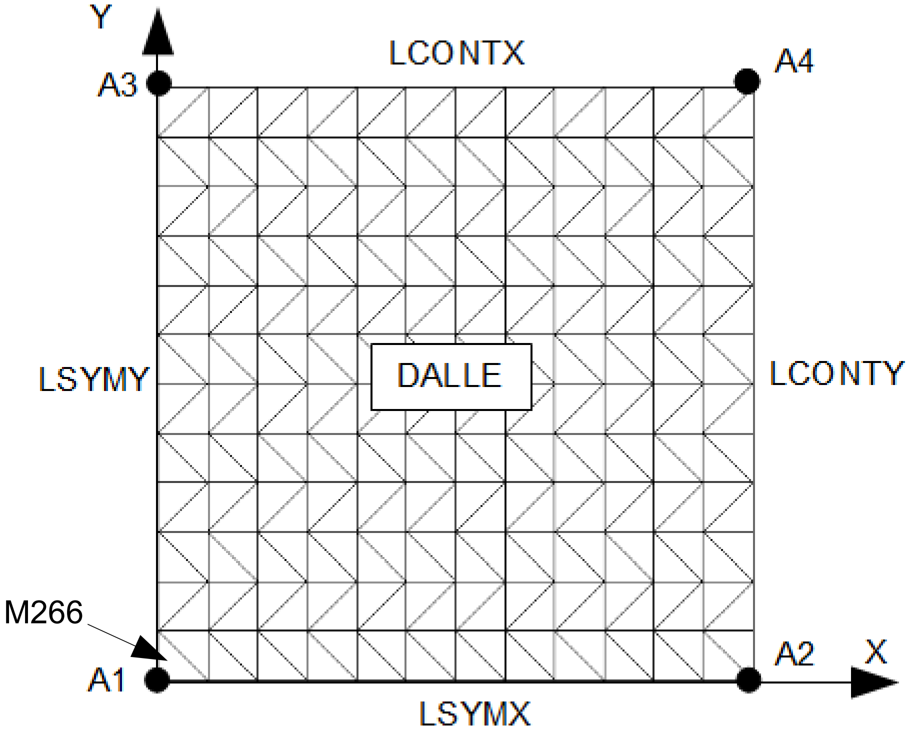
2.3 Uncertainty on the analytical

solution Solution.

2.4 Bibliographical reference

[1] KOECHLIN P., MILL S., “Model total behavior of the reinforced concrete plates under dynamic loading of bending: Model GLRC”, EDF Notes/R & D /AMA HT-62/01/028A.

3 Modelization A



3.1 Characteristics of the mesh

Many nodes: 169
Number of meshes and type: 288 TRIA3

3.2 Quantities tested and results

Value	Identification	Reference	% Tolerance
$w(x=0, y=0)$	$DZ(A1)$	2.433 10-4	0.90
$M_{xx}(x=0, y=0)$	$MXX(A1)$	4050.	2.0
$\kappa_{xx}(x=0, y=0)$	$KXX(A1)$	7.21 10-4	3.0

Value	Standard			Identification of reference	Tolérance %	Reference
M_{xx}	MXX	$M266$	$Point\,3$	"NON_REGRESSION"	3976.763	1.e-10
κ_{xx}	KXX	$M266$	$Point\,3$	"NON_REGRESSION"	7.2348 10-4	1.e-10

•the quantities are expressed in the reference defined by the angles nautiquesformuleet $\alpha=33°$ $\beta=12°$.

Standard	value	Identification of reference	Reference	% Tolerance
$M_{xx}(x=0, y=0)$	$MXX(A1)$	"NON_REGRESSION"	2991.193	1.e-10
$M_{yy}(x=0, y=0)$	$MYY(A1)$	"NON_REGRESSION"	1631.459	1.e-10
$M_{xy}(x=0, y=0)$	$MXY(A1)$	"NON_REGRESSION"	-1527.005	1.e-10
$\kappa_{xx}(x=0, y=0)$	$KXX(A1)$	"NON_REGRESSION"	4.9785 10-4	1.e-10
$\kappa_{yy}(x=0, y=0)$	$KYY(A1)$	"NON_REGRESSION"	1.8694 10-4	1.e-10
$\kappa_{xy}(x=0, y=0)$	$KXY(A1)$	"NON_REGRESSION"	-3.4916 10-4	1.e-10

Standard	Value			Identification of reference	Tolérance%	Reference
M_{xx}	MXX	$M266$	$Point\,3$	"NON_REGRESSION"	2986.069	1.e-10
M_{yy}	MYY	$M266$	$Point\,3$	"NON_REGRESSION"	1629.876	1.e-10
M_{xy}	MXY	$M266$	$Point\,3$	"NON_REGRESSION"	-1524.810	1.e-10
κ_{xx}	KXX	$M266$	$Point\,3$	"NON_REGRESSION"	4.9695 10-4	1.e-10
κ_{yy}	KYY	$M266$	$Point\,3$	"NON_REGRESSION"	1.8685 10-4	1.e-10
κ_{xy}	KXY	$M266$	$Point\,3$	"NON_REGRESSION"	-3.4865 10-4	1.e-10

3.3 Remarks

the coefficients of the following matrixes of elasticity, used during computations, were calculated with $\nu_b=0$:

1) Stamp elasticity out of membrane: $\begin{Bmatrix} 4614. & 0 & 0 \\ 0 & 4614. & 0 \\ 0 & 0 & 2142. \end{Bmatrix} 10^6 N/m$

2) Stamp elasticity in bending: $\begin{Bmatrix} 5.617 & 0 & 0 \\ 0 & 5.617 & 0 \\ 0 & 0 & 2.57 \end{Bmatrix} 10^6 N/m$

To be certain to remain in the elastic domain, the yield stresses expressed in orthotropic reference, are built-in arbitrarily with a very high value:

Yield stresses in positive bending:

Direction x : $1.10^{10} MNm/ml$

Direction y : $1.10^{10} MNm/ml$

Yield stresses in negative bending:

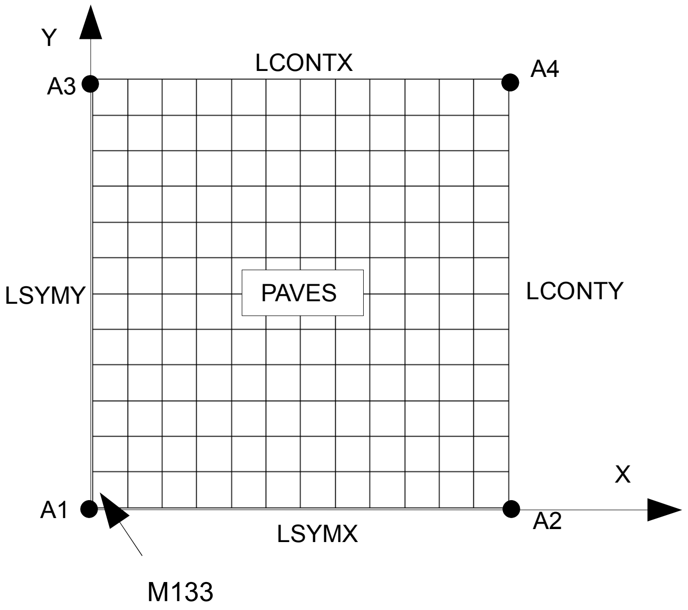
Direction x : $-1.10^{10} MNm/ml$

Direction y : $-1.10^{10} MNm/ml$

As the structure remains in the elastic domain, the kinematical coefficient of recall (constant of Prager) can take an unspecified value.

4 Modelization B

4.1 Characteristic of modelization



Modelization DKTG (QUAD4)

- Boundary conditions:
  . Dimensioned $A2A4$: $DZ=0$

- Conditions of symmetry
  . Dimensioned $A1A2$: $DY=DRX=0$
  . Dimensioned $A1A3$: $DX=DRY=0$

The slab is symmetric compared to the planes $(X=0)$ and $(Y=0)$, computations are carried out on a quarter of slab.

4.2 Characteristics of the mesh

Many nodes: 169
Number of meshes and type: 144 QUAD4

4.3 Features tested

macro-command POST_COQUE make it possible and to extract the forces strains in an unspecified point from the shell.

4.4 Values tested

Value	Identification	Reference	% Tolerance
$w(x=0, y=0)$	$DZ(A1)$	2.433 10-4	0.5
$M_{xx}(x=0, y=0)$	$MXX(A1)$	4050.	2.0
$\kappa_{xx}(x=0, y=0)$	$KXX(A1)$	7.21 10-4	1.0

Value	Standard			Identification of reference	Tolérance %	Reference
M_{xx}	MXX	$M133$	$Point\ 4$	"NON_REGRESSION"	3978.021	1.e-10
κ_{xx}	KXX	$M133$	$Point\ 4$	"NON_REGRESSION"	7.2355 10-4	1.e-10

•the quantities are expressed in the reference defined by the angles nautiquesformuleet $\alpha=33°$ $\beta=12°$.

Standard	value	Identification of reference	Reference	% Tolerance
$M_{xx}(x=0, y=0)$	$MXX(A1)$	"NON_REGRESSION"	2993.222	1.e-10
$M_{yy}(x=0, y=0)$	$MYY(A1)$	"NON_REGRESSION"	1635.193	1.e-10
$M_{xy}(x=0, y=0)$	$MXY(A1)$	"NON_REGRESSION"	-1525.093	1.e-10
$\kappa_{xx}(x=0, y=0)$	$KXX(A1)$	"NON_REGRESSION"	4.9808 10-4	1.e-10
$\kappa_{yy}(x=0, y=0)$	$KYY(A1)$	"NON_REGRESSION"	1.8756 10-4	1.e-10
$\kappa_{xy}(x=0, y=0)$	$KXY(A1)$	"NON_REGRESSION"	-3.4872 10-4	1.e-10

Standar d	Value			Identification of reference	Tolérance%	Reference
M_{xx}	MXX	$M133$	$Point\,4$	"NON_REGRESSION"	2989.086	1.e-10
M_{yy}	MYY	$M133$	$Point\,4$	"NON_REGRESSION"	1632.114	1.e-10
M_{xy}	MXY	$M133$	$Point\,4$	"NON_REGRESSION"	-1523.138	1.e-10
κ_{xx}	KXX	$M133$	$Point\,4$	"NON_REGRESSION"	4.9743 10-4	1.e-10
κ_{yy}	KYY	$M133$	$Point\,4$	"NON_REGRESSION"	1.8715 10-4	1.e-10
κ_{xy}	KXY	$M133$	$Point\,4$	"NON_REGRESSION"	-3.4827 10-4	1.e-10

4.5 Remarks

the coefficients matrixes of following elasticities, used during computations, were calculated with $\nu_b=0$:

1) Stamp elasticity out of membrane: $\begin{Bmatrix} 4614. & 0 & 0 \\ 0 & 4614. & 0 \\ 0 & 0 & 2142. \end{Bmatrix} 10^6\,N/m$

2) Stamp elasticity in bending: $\begin{Bmatrix} 5.617 & 0 & 0 \\ 0 & 5.617 & 0 \\ 0 & 0 & 2.57 \end{Bmatrix} 10^6\,N/m$

To be certain to remain in the elastic domain, the yield stresses expressed in orthotropic reference, are built-in arbitrarily with a very high value:

Yield stresses in positive bending:

Direction x : $1.10^{10}\,MNm/ml$

Direction y : $1.10^{10}\,MNm/ml$

Yield stresses in negative bending:

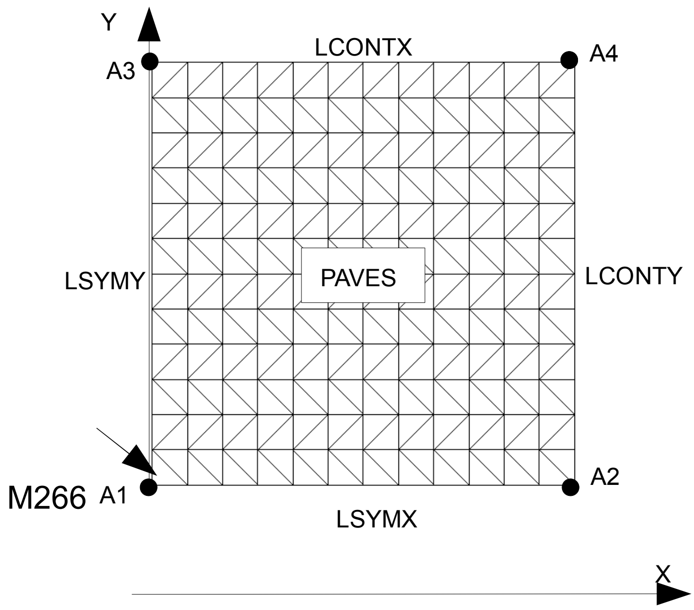
Direction x : $-1.10^{10}\,MNm/ml$

Direction y : $-1.10^{10}\,MNm/ml$

As the structure remains in the elastic domain, the kinematical coefficient of recall (constant of Prager) can take an unspecified value.

5 Modelization C

5.1 Characteristic of modelization



Modelization `Q4GG (TRIA3)`

- Boundary conditions:
  . Dimensioned $A2A4$: $DZ=0$
- Conditions of symmetry
  . Dimensioned $A1A2$: $DY=DRX=0$
  . Dimensioned $A1A3$: $DX=DRY=0$

the slab is symmetric compared to the planes $(X=0)$ and $(Y=0)$, computations are carried out on a quarter of slab.

5.2 Characteristic of the mesh

Many nodes: 169
Number of meshes and type: 288 `TRIA3`

5.3 Quantities tested and results

Standard	Value	Identification of Reference	Reference	% Tolerance
$w(x=0, y=0)$	$DZ(A1)$	"ANALYTIQUE"	2.433 10-4	2.0
$M_{xx}(x=0, y=0)$	$MXX(A1)$	"ANALYTIQUE"	4050.	2.5
$\kappa_{xx}(x=0, y=0)$	$KXX(A1)$	"ANALYTIQUE"	7.21 10-4	3.0

Value	Standard			Identification of reference	Tolérance %	Reference
M_{xx}	MXX	$M266$	$Point\,1$	"NON_REGRESSION"	3963.039	1.e-10
κ_{xx}	KXX	$M266$	$Point\,1$	"NON_REGRESSION"	7.2205 10-4	1.e-10

•the quantities are expressed in the reference defined by the angles nautiquesformuleet $\alpha=33°$ $\beta=12°$.

Standard	value	Identification of reference	Reference	% Tolerance
$M_{xx}(x=0, y=0)$	$MXX(A1)$	"NON_REGRESSION"	2968.546	1.e-10
$M_{yy}(x=0, y=0)$	$MYY(A1)$	"NON_REGRESSION"	1604.914	1.e-10
$M_{xy}(x=0, y=0)$	$MXY(A1)$	"NON_REGRESSION"	-1531.384	1.e-10
$\kappa_{xx}(x=0, y=0)$	$KXX(A1)$	"NON_REGRESSION"	4.9465 10-4	1.e-10
$\kappa_{yy}(x=0, y=0)$	$KYY(A1)$	"NON_REGRESSION"	1.8285 10-4	1.e-10
$\kappa_{xy}(x=0, y=0)$	$KXY(A1)$	"NON_REGRESSION"	-3.5016 10-4	1.e-10

Standard	Value			Identification of reference	Tolérance%	Reference
M_{xx}	MXX	$M266$	$Point\,1$	"NON_REGRESSION"	2968.546	1.e-10
M_{yy}	MYY	$M266$	$Point\,1$	"NON_REGRESSION"	1604.914	1.e-10
M_{xy}	MXY	$M266$	$Point\,1$	"NON_REGRESSION"	-1531.384	1.e-10
κ_{xx}	KXX	$M266$	$Point\,1$	"NON_REGRESSION"	4.9465 10-4	1.e-10
κ_{yy}	KYY	$M266$	$Point\,1$	"NON_REGRESSION"	1.8285 10-4	1.e-10
κ_{xy}	KXY	$M266$	$Point\,1$	"NON_REGRESSION"	-3.5016 10-4	1.e-10

5.4 Remarks

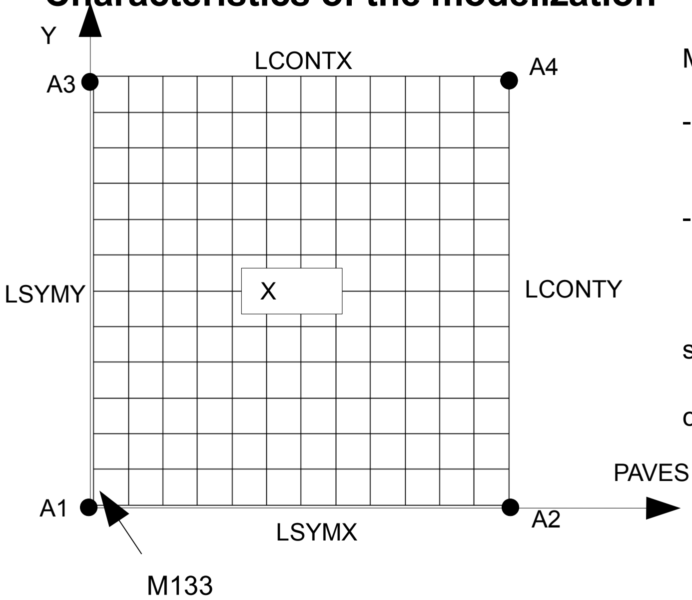
the coefficients of the following matrixes of elasticity, used during computations, were calculated with $\nu_b=0$:

1) Stamp elasticity out of membrane: $\begin{Bmatrix} 4614. & 0 & 0 \\ 0 & 4614. & 0 \\ 0 & 0 & 2142. \end{Bmatrix} 10^6 N/m$

2) Matrix of elasticity in bending: $\begin{Bmatrix} 5.617 & 0 & 0 \\ 0 & 5.617 & 0 \\ 0 & 0 & 2.57 \end{Bmatrix} 10^6 N/m$

6 D Modelization

6.1 Characteristics of the modelization



Modelization Q4GG (QUAD4)

- Boundary conditions:
  . Dimensioned $A2A4$: $DZ=0$

- Conditions of symmetry
  . Dimensioned $A1A2$: $DY=DRX=0$
  . Dimensioned $A1A3$: $DX=DRY=0$

slab is symmetric compared to the planes $(X=0)$ and $(Y=0)$, computations are carried out on a quarter of slab.

6.2 Characteristics of the mesh

Many nodes: 169
Number of meshes and type: 144 QUAD4

6.3 Features tested

macro-command POST_COQUE make it possible and to extract the forces strains in an unspecified point from the shell.

6.4 Values tested

Standard	Value	Identification of Reference	Reference	% Tolerance
$w(x=0, y=0)$	$DZ(A1)$	"ANALYTIQUE"	2.433 10-4	1.5
$M_{xx}(x=0, y=0)$	$MXX(A1)$	"ANALYTIQUE"	4050.	2.5
$\kappa_{xx}(x=0, y=0)$	$KXX(A1)$	"ANALYTIQUE"	7.21 10-4	1.0

Value	Standard			Identification of reference	Tolérance %	Reference
M_{xx}	MXX	$M133$	$Point\ 4$	"NON_REGRESSION"	3966.605	1.e-10
κ_{xx}	KXX	$M133$	$Point\ 4$	"NON_REGRESSION"	7.22745 10-4	1.e-10

•the quantities are expressed in the reference defined by the angles nautiquesformuleet $\alpha=33°$ $\beta=12°$.

Standard	value	Identification of reference	Reference	% Tolerance
$M_{xx}(x=0, y=0)$	$MXX(A1)$	"NON_REGRESSION"	2970.339	1.e-10
$M_{yy}(x=0, y=0)$	$MYY(A1)$	"NON_REGRESSION"	1605.001	1.e-10
$M_{xy}(x=0, y=0)$	$MXY(A1)$	"NON_REGRESSION"	-1533.299	1.e-10
$\kappa_{xx}(x=0, y=0)$	$KXX(A1)$	"NON_REGRESSION"	4.9499 10-4	1.e-10
$\kappa_{yy}(x=0, y=0)$	$KYY(A1)$	"NON_REGRESSION"	1.8280 10-4	1.e-10
$\kappa_{xy}(x=0, y=0)$	$KXY(A1)$	"NON_REGRESSION"	-3.5059 10-4	1.e-10

Standard	Value			Identification of reference	Tolérance%	Reference
M_{xx}	MXX	$M133$	$Point\,4$	"NON_REGRESSION"	2971.183	1.e-10
M_{yy}	MYY	$M133$	$Point\,4$	"NON_REGRESSION"	1605.217	1.e-10
M_{xy}	MXY	$M133$	$Point\,4$	"NON_REGRESSION"	-1533.160	1.e-10
κ_{xx}	KXX	$M133$	$Point\,4$	"NON_REGRESSION"	4.9514 10-4	1.e-10
κ_{yy}	KYY	$M133$	$Point\,4$	"NON_REGRESSION"	1.8280 10-4	1.e-10
κ_{xy}	KXY	$M133$	$Point\,4$	"NON_REGRESSION"	-3.5056 10-4	1.e-10

6.5 Remarks

the coefficients matrixes of following elasticities, used during computations, were calculated with $\nu_b=0$:

1) Stamp elasticity out of membrane: $\begin{Bmatrix} 4614. & 0 & 0 \\ 0 & 4614. & 0 \\ 0 & 0 & 2142. \end{Bmatrix} 10^6 N/m$

2) Matrix of elasticity in bending: $\begin{Bmatrix} 5.617 & 0 & 0 \\ 0 & 5.617 & 0 \\ 0 & 0 & 2.57 \end{Bmatrix} 10^6 N/m$

7 Summary of the results

the results of the four modelizations are close to the analytical solution:

- DKTG : to the maximum 0.9% of variation for displacements, and 3.0% the curvature.
- Q4GG : to the maximum 2.0% variation for displacements formulates, and 3.0% for the moment and the curvature.

These modelizations validate:

- modelization DKTG with the model GLRC_DAMAGE in elastic behavior.
- modelization Q4GG with the model ELAS.