SSLS125 - Buckling of a free cylinder under external pressure

Abstract:

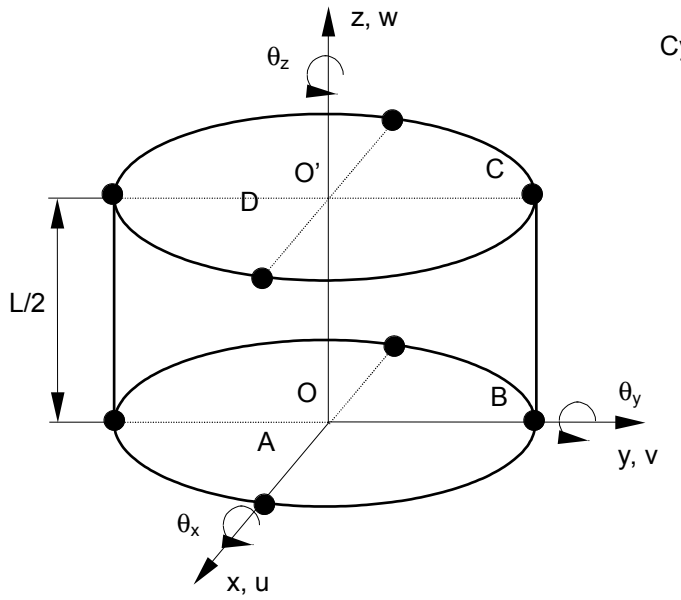
This test represents a computation of stability of a free thin cylindrical envelope at its ends subjected to an external pressure. One calculates the critical loads leading to the elastic buckling of Eulerian. The geometrical stiffness matrix used in the resolution of the problem to the eigenvalues is that which is due to the initial stresses.

It makes it possible to validate the modelization finite elements SHB (linear elements SHB8 and SHB6 and elements quadratic SHB20 and SHB15)

the critical load and the eigen mode obtained is compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry



Cylindre libre aux extrémités

the symmetry of the problem makes it possible to model a quarter of cylinder length L , with conditions of symmetry specific to lower edge.

$L=2\mathrm{m}$
Average radius $R=2\mathrm{m}$
Thickness $e=0.02\mathrm{m}$

1.2 Properties of the material

the properties of the material constituting the plate are:

$E=2.10^{11}\,Pa$	Modulus Young
$\nu=0.3$	Poisson's ratio

1.3 Boundary conditions and loadings

Loading:
pressure uniformly distributed of $p_{cr}=1.Pa$ on the cylindrical part.
Conditions of symmetry:
on AB : DZ = 0
on BC : DX=0
on DA : DY=0

1.4 Initial conditions

Without Reference solution

2 object

2.1 Method of calculating used for the reference solution

the critical pressure is given in [bib1] or [bib2] by the following statement:

$$P_{CR}=\frac{E}{12.(1-\nu^2)}.n^2.(\frac{e}{R})^3$$

with n number of the mode (here $n=2,4,6$)

2.2 Results of reference

the critical pressures (in Pa) are:

Mode (N)	Reference
2	73260
4	293040
6	659340

2.3 Uncertainties on the analytical

solution Solution

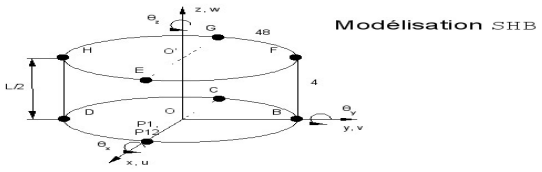
2.4 bibliographical References

1) S.P. TIMOSHENKO, J.M. MANAGES: Theory of elastic stability, page 500, second edition, DUNOD 1966.

2) BO O. ALMROTH, D.O. BRUSH: Buckling of bars, punts and shells, page 173, Mc Graw-Hill, New York, 1975.

3 Modelization A

3.1 Characteristic of the modelization



3.2 Characteristics of the mesh

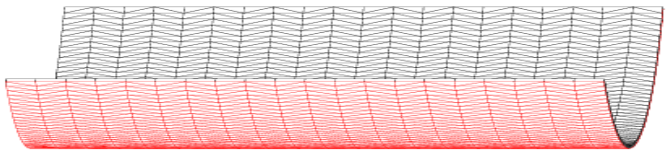
Many nodes: 882
Number of meshes and types: 400 HEXA8

3.3 Quantities tested and results

Identification	Mode (N)	Reference	*Aster*	% difference
Pressure criticizes (Pa)	2	73260	72492	B
	1.05	4	293040	293481
	0.15	6	659340	673600

4 2.2 Modelization

4.1 Characteristic of the modelization



4.2 Characteristics of the mesh

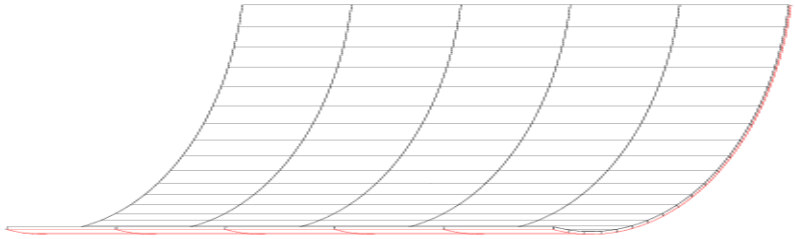
Many nodes: 2982 (20 elements in the height, 70 on the circumference)
Number of meshes and types: 2800 PENTA6

4.3 Quantities tested and results

Identification	Mode (N)	Reference	*Aster*	% difference
Pressure C criticizes Pa ( )		2	73260	75544
	3.1	4	293040	302291
	3.1	6	659340	680524

5 3.2 Modelization

5.1 Characteristic of the modelization



5.2 Characteristics of the mesh

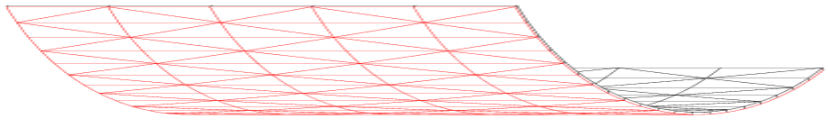
Many nodes: 828 (5 elements in the height, 20 on the circumference)
Number of meshes and types: 100 HEXA20

5.3 Quantities tested and results

Identification	Mode (N)	Reference	*Aster*	% difference
Pressure D criticizes Pa ( )		2	73260	71996
	-1.7	4	293040	287912
	-1.7	6	659340	647176

6 -1.8 Modelization

6.1 Characteristic of the modelization



6.2 Characteristics of the mesh

Many nodes: 1028 (5 elements in the height, 20 on the circumference)
Number of meshes and types: 200 PENTA15

6.3 Quantities tested and results

Identification	Mode (N)	Reference	*Aster*	% difference
Pressure criticizes (Pa)	2	73260	73235	-0.03
	4	293040	293111	0.02
	6	659340	659800	0.07

7 Summary of the results

the got results are satisfactory. Uncertainties on the critical pressure do not exceed 3% . It should be noted that to obtain this accuracy, element SHB6 needs a mesh finer than the other elements (3000 nodes instead of 1000).

The modal deformed shape obtained corresponds well to the expected circumferential mode: $n=2$ for the two modelizations.

This test made it possible to test modelization SHB in linear buckling of Eulerian of a thin structure subjected to an external pressure.