SSLS124 - Beam in bending with various slenderness

Summarized:

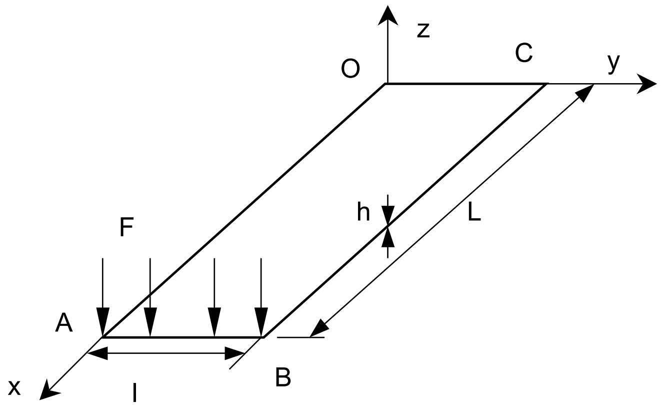
This test represents a computation quasi-static of a beam in bending, embedded at an end, and subjected to a vertical force at the other end. This test makes it possible to validate, for a linear elastic design, up to five values of slenderness (variable thickness), in each following modelization:

- Finite elements SHB8 for a regular mesh (modelization A)
- Finite elements SHB8 for a nonregular mesh (modelization B)
- Finite elements SHB6 for a regular mesh (modelization C)
- Finite elements SHB20 for a regular mesh (modelization D)
- Finite elements SHB20 for a nonregular mesh (modelization E)
- Finite elements SHB15 for a regular mesh (modelization F)
- Finite elements SHB15 for a nonregular mesh (modelization G)

displacements obtained are compared with the elastic analytical solution of a beam in bending. This test makes it possible to show the limits of the elements in term of slenderness, on the one hand, and to show their good convergence for a very irregular mesh, on the other hand.

1 Problem of reference

1.1 Geometry



Length $L=100\,m$, width $l=10\,m$.
Thickness: **case 1** $h=10\,m$, **case 2** $h=1\,m$, **case 3** $h=0.1\,m$, **case 4** $h=0.05\,m$, **case 5** $h=0.02\,m$

1.2 Properties of the materials

One considers an elastic material:
$E=2.10^{11}\,Pa$
$\nu=0.3$

1.3 Boundary conditions and loadings

Fixed support on the side OC : $u=v=w=0$, $\theta_x=\theta_y=\theta_z=0$
With the end AB , a load uniformly distributed of resultant:
$F_z=1\,N$

2 Reference solution

2.1 Method of calculating used for the reference solution

the results of reference are got by the theory of the elastic beams.
Vertical displacement at the end AB is given by:

$$U_y=\frac{F L^3}{3 E I_z}$$

With:

$$I_z=\frac{l h^3}{12}$$

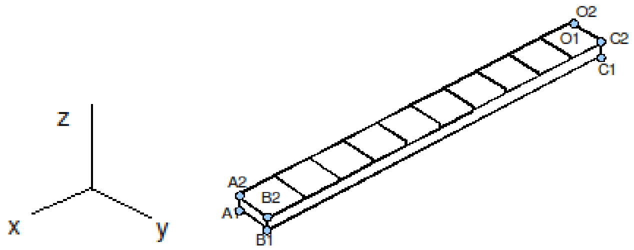
2.2 Results of reference

Displacement of the points A and B following Z.

3 Modelization A

3.1 Characteristic of the modelization

Element SHB8



Cutting: a regular mesh is considered in this modelization.

Regular mesh:
10 meshes SHB8 : 1 according to the width, 10 according to the length, 1 according to the thickness

Five values of thickness are considered in this modelization: **case 1** $h=10\,m$, **case 2** $h=1\,m$, **case 3** $h=0.1\,m$, **case 4** $h=0.05\,m$, **case 5** $h=0.02\,m$

Boundary conditions:
- In all the nodes on the side OC : following blocked displacement X
- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: blocked displacement following Z

Loading:
- in $A2$: nodal force according to X : $FX=0{,}5$
- in $B2$: nodal force according to Y : $FY=0{,}5$

Name of the nodes:

Not $O1$	$N40$	Not $O2$	$N44$
Not $A1$	$N03$	Not $A2$	$N01$
Not $B1$	$N04$	Not $B2$	$N02$
Not $C1$	$N43$	Not $C2$	$N39$

3.2 Characteristic of the mesh

Many nodes: 44 Number of meshes and types: 11 SHB8
In the case of the regular mesh, each element are a perfect square on side of length $10\mathrm{m}$

3.3 Quantities tested and regular

Mesh results:

Thickness	Not	Quantity in unit	Reference	% tolerances
Case 1	$A2$	displacement $W(m)$	2.0E-9	1
$h=10\text{m}$	$B2$	displacement $W(m)$	2.0E-9	1
Cases 2	$A2$	displacement $W(m)$	2.0E-6	1
$h=1\text{m}$	$B2$	displacement $W(m)$	2.0E-6	1
Cases 3	$A2$	displacement $W(m)$	2.0E-3	1
$h=0.1\text{m}$	$B2$	displacement $W(m)$	2.0E-3	1
Cases 4	$A2$	displacement $W(m)$	1.60E-2	0.1
$h=0.05\text{m}$	$B2$	displacement $W(m)$	1.60E-2	0.1
Cases 5	$A2$	displacement $W(m)$	0.25	0.1
$h=0.02\text{m}$	$B2$	displacement $W(m)$	0.25	0.1

One also tests the NON-regression of the computation of fields SIEQ_ELGA and SIEQ_ELNO.

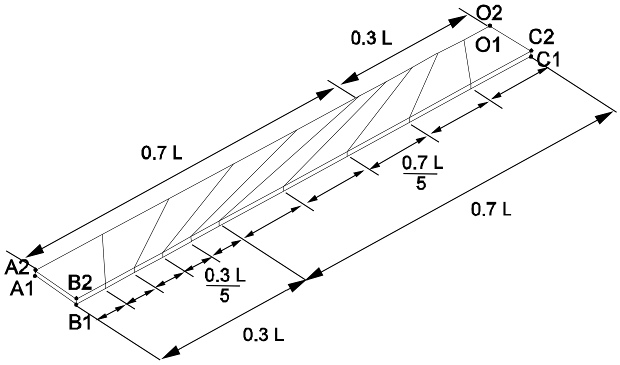
3.4 Remarks

the use of operator STAT_NON_LINE for cases 3,4 and 5 makes it possible to better approach the reference solution. Indeed, for strong slenderness, the stiffness matrix becomes quasi-singular (it is necessary to increase the number of decimals lost with the resolution, using key word NPREC) and the accuracy of resolution of the linear system decreases. The nonlinear solver, by carrying out iterations, allows to converge towards the analytical solution.

4 Modelization B

4.1 Characteristic of the modelization

Element SHB8



Cutting: an irregular mesh is considered in this modelization.
NON-regular mesh:
10 meshes SHB8 : 1 according to the width, 10 according to the length, 1 according to the thickness

Three values of thickness are considered in this modelization: **case 1** $h=10\,m$, **case 2** $h=1\,m$, **case 3** $h=0.1\,m$

Boundary conditions:

- In all the nodes on the side OC : following blocked displacement X
- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: blocked displacement following Z

Loading:

- in $A2$: nodal force according to X : $FX=0{,}5$
- in $B2$: nodal force according to Y : $FY=0{,}5$

Names of the nodes:

Not $O1$	$N40$	Not $O2$	$N44$
Not $A1$	$N03$	Not $A2$	$N01$
Not $B1$	$N04$	Not $B2$	$N02$
Not $C1$	$N43$	Not $C2$	$N39$

4.2 Characteristic of the mesh

Many nodes: 44 Number of meshes and types: 11 SHB8

4.3 Quantities tested and NON-regular

Mesh results:

Thickness	Not	Quantity in unit	Reference	% tolerance
Case 1	$A2$	displacement $W(m)$	2.0E-09	4
$h=10\mathrm{m}$	$B2$	displacement $W(m)$	2.0E-09	4
Cases 2	$A2$	displacement $W(m)$	2.0E-06	5
$h=1\mathrm{m}$	$B2$	displacement $W(m)$	2.0E-06	5
Cases 3	$A2$	displacement $W(m)$	2.0E-03	5
$h=0.1\mathrm{m}$	$B2$	displacement $W(m)$	2.0E-03	5

One also tests the NON-regression of the computation of fields `SIEQ_ELGA` and `SIEQ_ELNO`.

4.4 Remarks

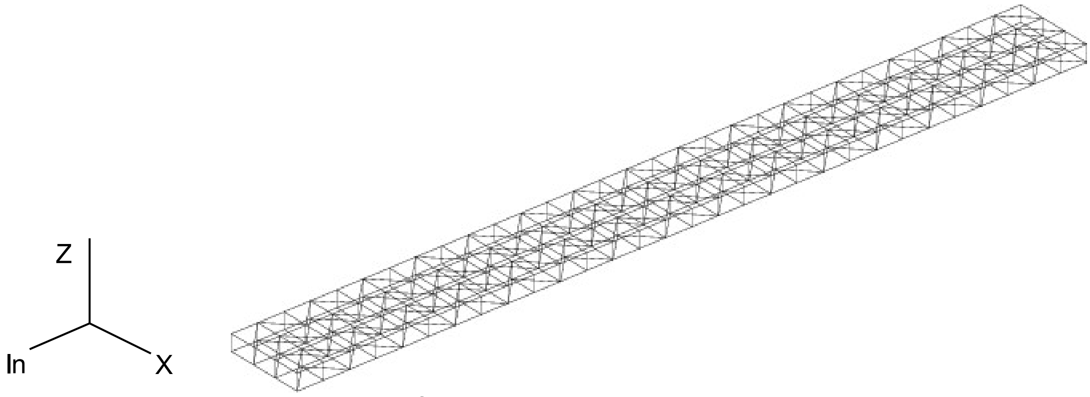
One can pass the same remark as for modelization a: when slenderness increases until becoming very important, the stiffness matrix becomes very badly conditioned and quasi-singular. It is all the more marked in this modelization that the mesh is, voluntarily, of poor quality (irregular with distorted elements).
The conditioning evaluated with the MUMPS solver in case 3 exceeds 10^{12} and the solution of the linear system obtained, even after iterative refinement, has a variability between platforms of computation. For this reason, one does not calculate higher slenderness in this modelization.

In spite of poor quality of the mesh and with slenderness all the same important, one notes the good behavior of the element with a good approximation of the solution in bending.

5 Modelization C

5.1 Characteristic of the modelization

Element SHB6



this modelization one adapted the surface mesh there to the thickness considered:
(the mesh above corresponds to the case 2)

- nbl = many elements according to the width,
- nbL = many elements according to the length,
- 1 element according to the thickness

	h	nbL	nbl
cases 1	10	10	1
case 2	5	30	3
cases 3	2	100	10
cases 4	1	100	10

Boundary conditions:

- In all the nodes on the side OC : following blocked displacement X
- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: blocked displacement following Z

Loading:

- in $A2$: nodal force according to X : $FX=0,5$
- in $B2$: nodal force according to Y : $FY=0,5$

5.2 Characteristics of the mesh

case 1Nombre of nodes: 44 Number of meshes and types: 20 SHB6
case 2Nombre of nodes: 248 Number of meshes and types: 180 SHB6
case 3 and 4Nombre of nodes: 2222 Number of meshes and types: 500 SHB6

5.3 Quantities tested and regular

Mesh results:

Thickness	Not	Quantity in unit	Reference	% tolerance
Case 1	$A2$	displacement $W(m)$	2.00E-009	1
$h=10\text{m}$	$B2$	2,00E-009 $W(m)$	2.00E-009	1.5
Cases 2	$A2$	displacement $W(m)$	1.60E-006	1
$h=5\text{m}$	$B2$	displacement $W(m)$	1.60E-006	1
Cases 3	$A2$	displacement $W(m)$	2.50E-007	1
$h=2\text{m}$	$B2$	displacement $W(m)$	2.50E-007	1
Cases 4	$A2$	displacement $W(m)$	2.00E-006	4
$h=1\text{m}$	$B2$	displacement $W(m)$	2.00E-006	4

One also tests the NON-regression of the computation of fields SIEQ_ELGA and SIEQ_ELNO.

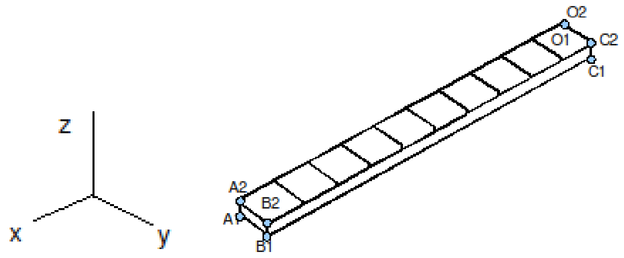
5.4 Remarks

slenderness are much weaker than for the other modelizations. Element SHB6 tolerates slenderness less large indeed than other modelizations SHB and is more sensitive to locking in shears.

6 Modelization D

6.1 Characteristic of the modelization

Element SHB20



Cutting: a regular mesh is considered in this modelization.

Regular mesh:
10 meshes SHB20 : 1 according to the width, 10 according to the length, 1 according to the thickness

Three values of thickness are considered in this modelization: **case 1** $h=10m$, **case 2** $h=1m$, **case 3** $h=0.1m$

Boundary conditions:

- In all the nodes on the side OC : following blocked displacement X
- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: blocked displacement following Z

Loading:

- in $A2$: nodal force according to X : $FX=0{,}5$
- in $B2$: nodal force according to Y : $FY=0{,}5$

6.2 Characteristics of the mesh

Many nodes: 128 Number of meshes and types: 10 SHB20
In the case of the regular mesh, each element are a perfect square on side of length 10m

6.3 Quantities tested and regular

Mesh results:

Thickness	**Not**	**Quantity in unit**	**Reference**	**% tolerance**
Case 1	$A2$	displacement $W(m)$	2.0E-9	1
$h=10\text{m}$	$B2$	displacement $W(m)$	2.0E-9	1
Cases 2	$A2$	displacement $W(m)$	2.0E-6	1
$h=1\text{m}$	$B2$	displacement $W(m)$	2.0E-6	1
Cases 3	$A2$	displacement $W(m)$	2.0E-3	1
$h=0.1\text{m}$	$B2$	displacement $W(m)$	2.0E-3	1

One also tests the NON-regression of the computation of fields SIEQ_ELGA and SIEQ_ELNO.

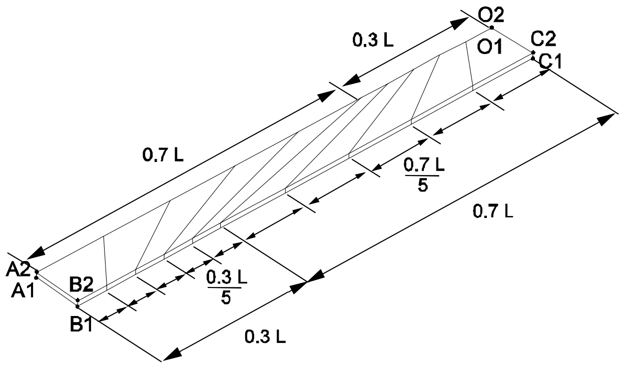
6.4 Remarks

the same remarks as for the modelizations A and B apply. Here in case 3, conditioning reaches already 10^{13}.

7 Modelization E

7.1 Characteristic of the modelization

Element SHB20



Cutting: an irregular mesh is considered in this modelization.

NON-regular mesh:

10 meshes SHB20 : 1 according to the width, 10 according to the length, 1 according to the thickness

Three values of thickness are considered in this modelization: **case 1** $h=10m$, **case 2** $h=1m$, **case 3** $h=0.1m$

Boundary conditions:

- In all the nodes on the side OC : following blocked displacement X
- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: blocked displacement following Z

Loading:

- in $A2$: nodal force according to X : $FX=0,5$
- in $B2$: nodal force according to Y : $FY=0,5$

7.2 Characteristics of the mesh

Many nodes: 128 Number of meshes and types: 10 SHB20

7.3 Quantities tested and NON-regular

Mesh results:

Thickness	Not	Quantity in unit	Reference	% tolerance
Case 1	$A2$	displacement $W(m)$	2.0E-9	1
$h=10\text{m}$	$B2$	displacement $W(m)$	2.0E-9	1
Cases 2	$A2$	displacement $W(m)$	2.0E-6	7
$h=1\text{m}$	$B2$	displacement $W(m)$	2.0E-6	7
Cases 3	$A2$	2.0E-3 $W(m)$	displacement	2.5
$h=0.1\text{m}$	$B2$	2.0E-3 $W(m)$	displacement	2.5

One also tests the NON-regression of the computation of fields SIEQ_ELGA and SIEQ_ELNO.

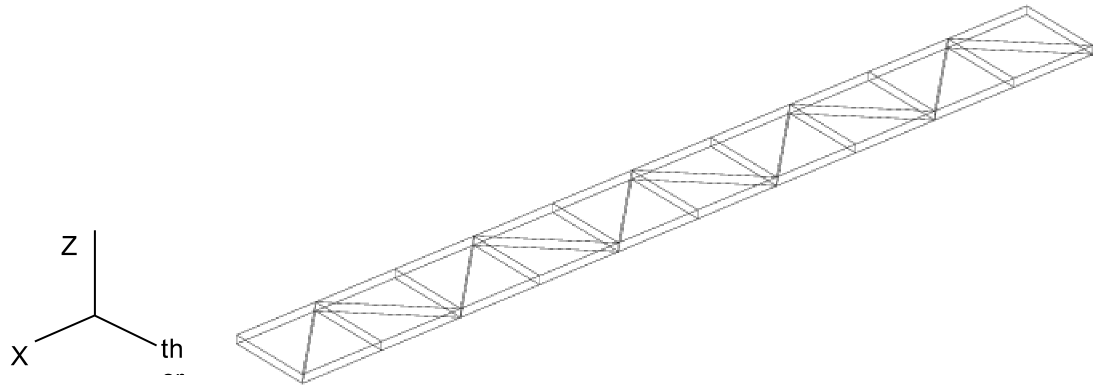
7.4 Remarks

As one can note it, the deterioration of the quality of the mesh affects the quality of result for strong slenderness but remains neutral with respect to standard slenderness.
In case 3, conditioning reaches already 10^{13} .

8 Modelization F

8.1 Characteristic of the modelization

Element `SHB15`



Cutting: a regular mesh is considered in this modelization.

Regular mesh:
20 meshes `SHB15` : 1 according to the width, 10 according to the length, 1 according to the thickness

Three values of thickness are considered in this modelization: **case 1** $h=10\,m$, **case 2** $h=1\,m$, **case 3** $h=0.1\,m$

Boundary conditions:
- In all the nodes on the side OC : following blocked displacement X
- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: blocked displacement following Z

Loading:
- in $A2$: nodal force according to X : $FX=0{,}5$
- in $B2$: nodal force according to Y : $FY=0{,}5$

8.2 Characteristics of the mesh

Many nodes: 148

Number of meshes and types: 20 `SHB15`

8.3 Quantities tested and regular

Mesh results:

Thickness	Not	Quantity in unit	Reference	% tolerance
Case 1	$A2$	displacement $W(m)$	2.0E-9	1
$h=10\text{m}$	$B2$	displacement $W(m)$	2.0E-9	1
Cases 2	$A2$	2.0E-6 $W(m)$	displacement	1.5
$h=1\text{m}$	$B2$	2.0E-6 $W(m)$	displacement	1.5
Cases 3	$A2$	displacement $W(m)$	2.0E-3	2
$h=0.1\text{m}$	$B2$	displacement $W(m)$	2.0E-3	2

One also tests the NON-regression of the computation of fields `SIEQ_ELGA` and `SIEQ_ELNO`.

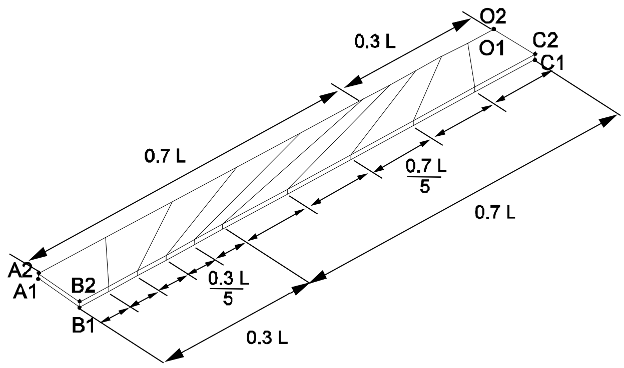
8.4 Remarks

the same remarks as for the modelizations A and B apply. Here in case 3, conditioning reaches already 10^{13}.

9 Modelization G

9.1 Characteristic of the modelization

Element SHB15



Cutting: an irregular mesh is considered in this modelization.

NON-regular mesh:

20 meshes SHB15 : 1 according to the width, 10 according to the length, 1 according to the thickness

Three values of thickness are considered in this modelization: **case 1** $h=10\,m$, **case 2** $h=1\,m$, **case 3** $h=0.1\,m$

Boundary conditions:
In all the nodes on the side OC : following blocked displacement X

- in $C1$: following blocked displacement Y and Z
- in $C2$: following blocked displacement Y
- in $O1$: blocked displacement following Z

Loading:

- in $A2$: nodal force according to X : $FX=0{,}5$
- in $B2$: nodal force according to Y : $FY=0{,}5$

9.2 Characteristics of the mesh

Many nodes: 148

Number of meshes and types: 20 SHB15

9.3 Quantities tested and NON-regular

Mesh results:

Thickness	**Not**	**Quantity in unit**	**Reference**	**% tolerance**
Case 1	$A2$	2.0E-9 $W(m)$	displacement	2.5
$h=10$m	$B2$	displacement $W(m)$	2.0E-9	2
Cases 2	$A2$	displacement $W(m)$	2.0E-6	7
$h=1$m	$B2$	displacement $W(m)$	2.0E-6	7
Cases 3	$A2$	displacement $W(m)$	2.0E-3	7.5
$h=0.1$m	$B2$	displacement $W(m)$	2.0E-3	7.5

One also tests the NON-regression of the computation of fields SIEQ_ELGA and SIEQ_ELNO.

9.4 Remarks

As one can note it, the deterioration of the quality of the mesh affects the quality of result including for weak slenderness.
In case 3, conditioning reaches already 10^{13} .

10 Summary of the results

In the case of regular mesh:

- for the SHB8 (modelization A), good performances are obtained, even when the slenderness of the element (side ratio/thickness) reached 500.
- for the SHB6 (modelization C), one obtains good performances on condition that refining the surface mesh as slenderness increases. Nevertheless, for slenderness beyond 50, the results are degraded. Element SHB6 presents a locking (rather weak) in bending.
- for the quadratic elements (modelizations D and F), the results are very good until a slenderness of 200.
- In the case of the nonregular mesh (modelizations B , E and G), some is the slenderness of the element, for the SHB8 one tends to underestimate the stiffness of the beam of approximately 4% , but the results remain good until a slenderness of 500. The quadratic elements give good performances until a slenderness of 200.

For the range of slenderness usually met in modelizations plates and shells (from 10 to 100), elements SHB get good results whatever the quality of the mesh.
When one makes tighten the thickness by zero, of the phenomena of locking can make the stiffness matrix singular and thus to prevent the resolution or slow down convergence into nonlinear.