SSLS120 - Cylindrical thin shell under hydrostatic pressure

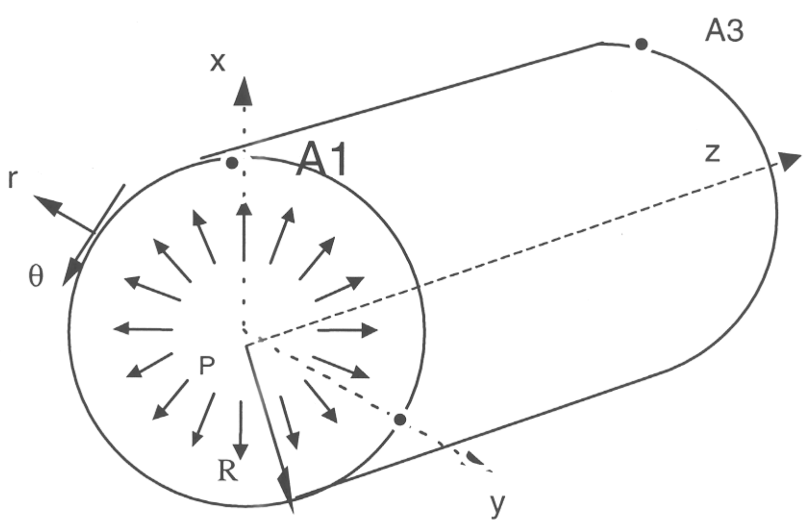
Summarized:

This test represents a static computation of thin cylindrical tank filled with water. It makes it possible to validate the good taking into account of the pressures function of the geometry, as well as the orthotropic elastic materials. 5 modelizations finite elements are used: `AXIS`, `COQUE_3D` with meshes `QUAD9`, `COQUE_3D` with meshes `TRIA7`, and `DKT` with meshes `QUAD4` and `3D` with meshes `HEXA20`. Displacements and the forced obtained are compared with an analytical reference solution.

1 Problem of reference

1.1 Geometry

Tank cylindrical of average radius $R=5.7\,m$, thickness $e=0.04\,m$ and height $L=16\,m$, simply supported at its base (rotation is free), and subjected to an internal hydrostatic pressure.



Average radius: $R=5.7\,m$
Thickness: $e=0.04\,m$
Height: $L=16\,m$

1.2 Material properties

the properties of the materials constituting the plate are:

Material 1 : isotropic elastic: Young's modulus $E=2.1\,10^{11}\,Pa$

Poisson's ratio $\nu=0.3$

Material 2 : orthotropic elastic: In order to free itself from the dependence of the notations to orthotropic reference, one gives the characteristic materials in the cylindrical coordinate system

$E_r=2.1\,E\,11\,Pa$
$E_\theta=2.1\,E\,11\,Pa$
$E_z=4\,E\,11\,Pa$
$\nu_{r\theta}=0.075$
$\nu_{rz}=0.075$
$\nu_{\theta z}=0.075$
$G_{r\theta}=0.35\,E\,10\,Pa$
$G_{rz}=0.45\,E\,10\,Pa$
$G_{\theta z}=0.45\,E\,10\,Pa$

1.3 Boundary conditions and loadings

Bases $z=0$ simply supported,

internal Pressure varying linearly according to z : $p(z)=P0.(L-z)/L$
with $P0=15000\,Pa$.

1.4 Initial conditions

Without object.

2 Reference solution

2.1 Method of calculating used for the isotropic reference solution

- Material : Analytical solution [bib1], obtained with the mean assumption of shell:

$$\sigma_{zz}=0$$

$$\sigma_{\theta\theta}=P0\,R\frac{(L-z)}{L\,e}$$

$$u_r=\frac{P0\,R^2}{E\,e}\left[1-\frac{z}{L}\right]$$

$$u_z=\frac{-P0\,R\,\nu\,z}{E\,e}\left[1-\frac{z}{2L}\right]$$

Radial displacement at the base of the cylinder: $u_r(z=0)=\frac{P0\,R^2}{E\,e}$

Vertical displacement at the top of cylinder: $u_z(z=L)=\frac{-P0\,R\,L\,\nu}{2\,E\,e}$

Circumferential stress in bottom of the orthotropic $\sigma_{\theta\theta}(z=0)=\frac{P0\,R}{e}$

- cylinder Material : The solution can be deduced from the preceding one: the stresses being statically determinate, it is enough to amend the constitutive law, and to integrate the strains. So that the solution is independent of the various notations (the value E_T does not have the same significance according to orthotropic reference), one places oneself out of cylindrical coordinate system (r,θ,z) .

Radial displacement at the base of the cylinder: $u_r(z=0)=\frac{P0\,R^2}{E_r e}$

Vertical displacement at the top of cylinder: $u_z(z=L)=\frac{-P0\,R\,L\,\nu_{\theta z}}{2\,E_\theta e}$

Circumferential stress in bottom of the cylinder $\sigma_{\theta\theta}(z=0)=\frac{P0\,R}{e}$

2.2 Results of isotropic

reference Material :
Radial displacement at the base of the cylinder: $Ur(A1)=5.8017857E-05\,m$
Vertical displacement at the top of cylinder: $Uz(A3)=-2.442857E-05\,m$
Circumferential stress in bottom of the cylinder: $Stt(A1)=2.1375E+06\,Pa$

Orthotropic material :
Radial displacement at the base of the cylinder: $Ur(A1)=5.8017857E-05\,m$
Vertical displacement at the top of cylinder: $Uz(A3)=-6.107143E-06\,m$
Circumferential stress in bottom of the cylinder: $Stt(A1)=2.1375E+06\,Pa$

2.3 Uncertainty on the analytical

solution Solution.

2.4 Bibliographical references

1) PILKEY W.D.: “Formulated for stress, Strain and Structural Matrixes”. Wiley & Idiots, New York, 1994.

3 Modelization A

3.1 Characteristic of the modelization

Modelization `AXIS`. One nets only one generator of the cylinder. 2 meshes QUAD8 in the thickness and 400 on the height.

3.2 Characteristics of the mesh

Many nodes: 3206

Number of meshes and types: 800 QUAD8

3.3 Values tested

<u>isotropic Material</u>

Value	Identification	Reference
$Ur(z=0)$	$DX(PM)$	5.8018E – 05
$Uz(z=L)$	$DY(A3)$	– 2.4429E – 05
$Uz(z=L)$	$DY(A4)$	– 2.4429E – 05
$SigmaTT(z=0)$	$SIZZ(PM)$	2.1375E+06

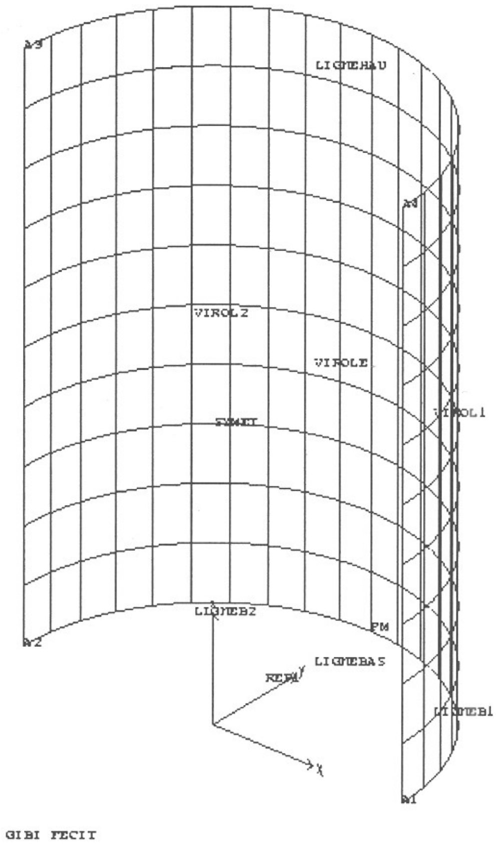
<u>orthotropic Material</u>

Value	Identification	Reference
$Ur(z=0)$	$DX(A1)$	5.8018E – 05
$Uz(z=L)$	$DY(A3)$	– 6.10714E – 06
$Uz(z=L)$	$DY(A4)$	– 2.4429E – 05
$SigmaTT(z=0)$	$SIZZ(PM)$	2.1375E+06

4 Modelization B

4.1 Characteristic of the modelization

Modelization COQUE_3D. One nets only half of the cylinder (symmetry compared to the plane $y=0$) 10 meshes QUAD9 in the height and 20 on the semicircumference.



4.2 Characteristics of the mesh

Many nodes: 664

Number of meshes and type: 200 QUAD9

4.3 Values tested

isotropic Material

Value	Identification	Reference
$Ur(z=0)$	$DX(PM)$	5.8018E – 05
$Ur(z=0)$	$DX(A1)$	5.8018E – 05
$Ur(z=0)$	$DX(A2)$	– 5.8018E – 05
$Uz(z=L)$	$DZ(A3)$	– 2.4429E-05
$Uz(z=L)$	$DZ(A4)$	– 2.4429E-05
$SigmaTT(z=0)$	$SIZZ(PM)$	2.1375E+06

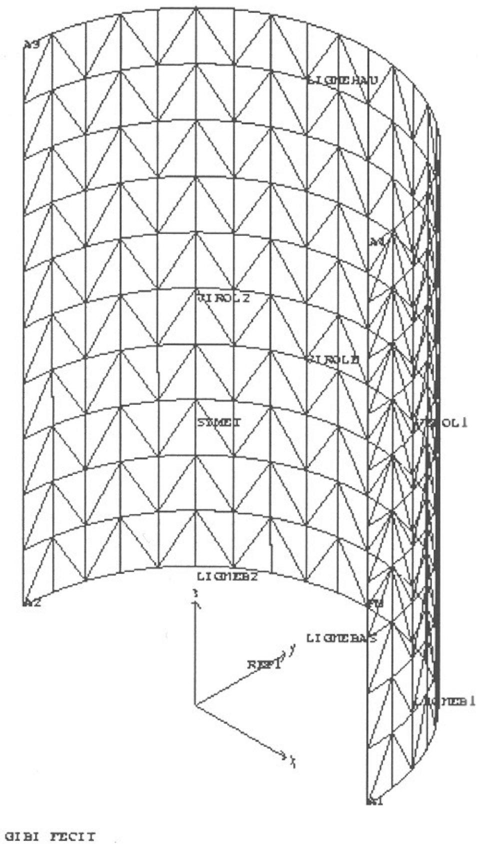
orthotropic Material

Value	Identification	Reference
$Ur(z=0)$	$DX(PM)$	5.8018E-05
$Ur(z=0)$	$DX(A1)$	5.8018E-05
$Ur(z=0)$	$DX(A2)$	– 5.8018E-05
$Uz(z=L)$	$DZ(A3)$	– 6.10714E-06
$Uz(z=L)$	$DZ(A4)$	– 6.10714E-06
$SigmaTT(z=0)$	$SIZZ(PM)$	2.1375E+06

5 Modelization C

5.1 Characteristic of the modelization

Modelization COQUE_3D. One nets only half of the cylinder (symmetry compared to the plane $y=0$) 10 meshes TRIA7 in the height and 20 on the semicircumference.



5.2 Characteristics of the mesh

Many nodes: 864

Number of meshes and types: 400 TRIA7

5.3 Values tested

isotropic Material

Value	Identification	Reference
$Ur(z=0)$	$DY(PM)$	5.8018E-05
$Ur(z=0)$	$DX(A1)$	5.8018E-05
$Ur(z=0)$	$DX(A2)$	– 5.8018E-05
$Uz(z=L)$	$DZ(A3)$	– 2.4429E-05
$Uz(z=L)$	$DZ(A4)$	– 2.4429E-05
$SigmaTT(z=0)$	$SIZZ(PM)$	2.1375E+06

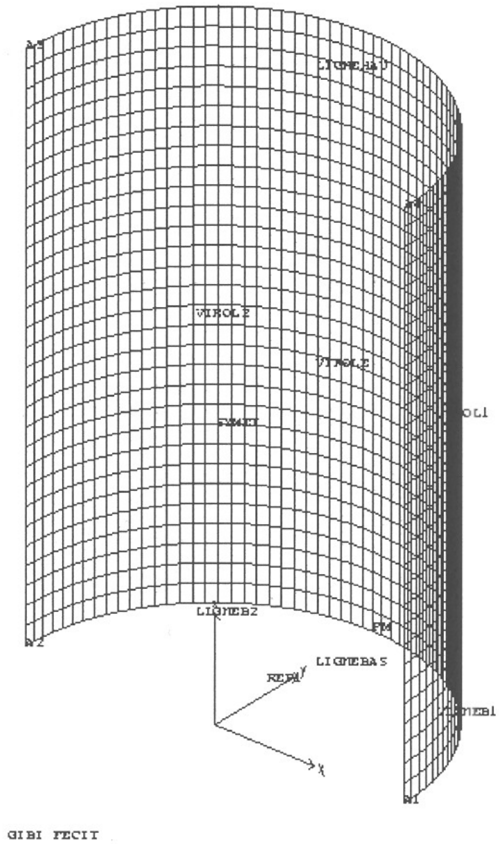
orthotropic Material

Value	Identification	Reference
$Ur(z=0)$	$DX(PM)$	5.8018E-05
$Ur(z=0)$	$DX(A1)$	5.8018E-05
$Ur(z=0)$	$DX(A2)$	– 5.8018E-05
$Uz(z=L)$	$DZ(A3)$	– 6.10714E-06
$Uz(z=L)$	$DZ(A4)$	– 6.10714E-06
$SigmaTT(z=0)$	$SIZZ(PM)$	2.1375E+06

6 Modelization D

6.1 Characteristic of the modelization

Modelization DKT. One nets only half of the cylinder (symmetry compared to the plane $y=0$) 30 meshes QUAD4 in the height and 60 on the semicircumference.



6.2 Characteristics of the mesh

Many nodes: 1894

Number of meshes and types: 1800 QUAD4

6.3 Values tested

isotropic Material

Value	Identification	Reference
$Ur(z=0)$	$DY(PM)$	5.8018E-05
$Ur(z=0)$	$DX(A1)$	5.8018E-05
$Ur(z=0)$	$DX(A2)$	– 5.8018E-05
$Uz(z=L)$	$DZ(A3)$	– 2.4429E-05
$Uz(z=L)$	$DZ(A4)$	– 2.4429E-05
$SigmaTT(z=0)$	$SIZZ(PM)$	2.1375E+06

orthotropic Material

Value	Identification	Reference
$Ur(z=0)$	$DY(PM)$	5.8018E-05
$Ur(z=0)$	$DX(A1)$	5.8018E-05
$Ur(z=0)$	$DX(A2)$	– 5.8018E-05
$Uz(z=L)$	$DZ(A3)$	– 6.10714E-06
$Uz(z=L)$	$DZ(A4)$	– 6.10714E-06
$SigmaTT(z=0)$	$SIZZ(PM)$	2.1375E+06

7 Modelization E

7.1 Characteristic of the modelization

Modelization 3D. One nets only half of the cylinder (symmetry compared to the plane $y=0$) 10 meshes HEXA20 in the height, 40 on the semicircumference and 2 in the thickness.

7.2 Characteristics of the mesh

Many nodes: 4725

Number of meshes and types: 800 HEXA20

7.3 Values tested

orthotropic Material by `MECA_STATIQUE`

Value	Identification	Reference
$Ur(z=0)$	$DY(PM)$	5.8018E-05
$Ur(z=0)$	$DX(A1)$	5.8018E-05
$Uz(z=L)$	$DZ(A3)$	– 6.10714E-06
$Uz(z=L)$	$DZ(A4)$	– 6.10714E-06

orthotropic Material by `STAT_NON_LINE`

Value	Identification	Reference
$Ur(z=0)$	$DY(PM)$	5.8018E-05
$Ur(z=0)$	$DX(A1)$	5.8018E-05
$Uz(z=L)$	$DZ(A3)$	– 6.10714E-06
$Uz(z=L)$	$DZ(A4)$	– 6.10714E-06

8 Summary of the results

the results of the five modelizations are very close to the analytical solution: to the maximum 0.4% variation for modelizations COQUE_3D and DKT, and less than 2% of variation for the axisymmetric modelization and 3D, which is explained by the fact why the analytical solution is a mean solution shell.

This test thus validates on the one hand the forces of pressure varying linearly with the geometry, for thin shells, and on the other hand the taking into account of orthotropic elasticity.