SSLS110 - Stability of a compressed square plate

Abstract:

A homogeneous isotropic linear plate square elastic simply leaned on its four sides is subjected to a linear compressive force acting on two on its sides.

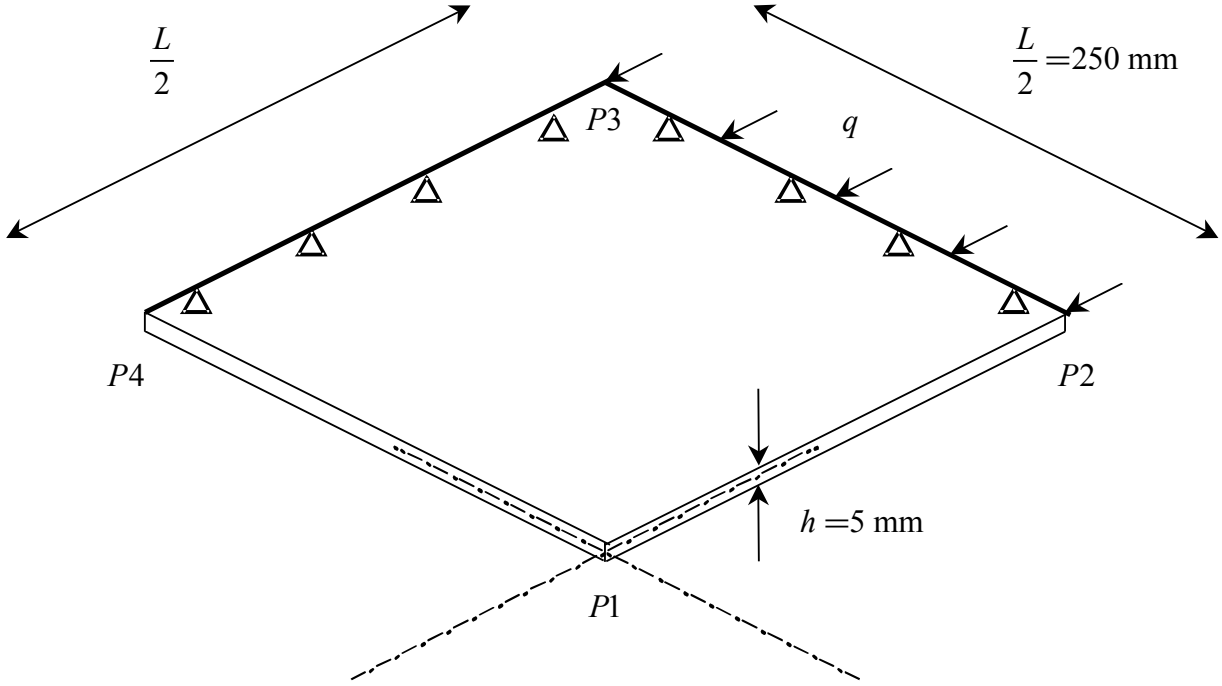
One calculates the critical loads leading to the elastic buckling of the plate. The geometrical stiffness matrix used in the resolution of the problem to the eigenvalues is that which is due to the initial stresses.

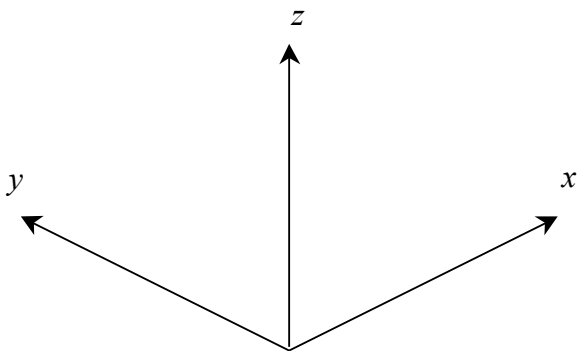
linear elastic mechanics,
buckling of a shell,
interest of the test: computation of the geometrical stiffness matrix of elements COQUE_3D,
2 modelizations.

1 Problem of reference

1.1 Geometry

Because of the geometrical and physical symmetry of the problem, only the quarter of the plate is modelled. By taking account the conditions of symmetry, one can collect only the buckling modes symmetric ones.





1.2 Material properties

$E=2.1\,E\,5\,Mpa$.

$\nu=0.3$

The transverse shear coefficient for the plate is worth $A_{CIS}=5/6$.

1.3 Boundary conditions and loadings

Boundary conditions: $P2P3$: $DZ=0.$

$P3P4$: $DZ=0.$

Symmetry $P1P2$: $DY=0.$ $DRX=0.$ $DRZ=0.$

$P4P1$: $DX=0.$ $DRY=0.$ $DRZ=0.$

Loading:

Linear compressive force q on $P2P3$

1.4 Remarks

It is not possible to solve the problem of strain of compression without introducing the conditions of symmetry. Indeed, to impose boundary conditions of symmetry for a quarter of plate amounts eliminating the modes from rigid body for the complete plate.

2 Reference solution

2.1 Reference solution

- Critical loads

the analytical solution obtained with a theory of thin plate in isotropic homogeneous linear elasticity [bib1] without taking into account of the transverse energy of shears determines i the ème critical load:

$$q_{cr\,I}=\frac{D\pi^2}{L^2}\left(i+\frac{1}{i}\right)^2$$

with:

$D=\frac{E\,h^3}{12\,(1-\nu^2)}$: the coefficient of flexural rigidity of the shell

h : the thickness

L : the length on the side of the square plate.

- Strain of membrane

the analytical statement of the strain of membrane along the axis X is the following one:

$$e_{xx}=\frac{q\times L}{(h\times L\times E)}$$

Result of reference was calculated with $q=1.\,N/mm$

2.2 Results of reference

Certain modes corresponding to the critical loads of the analytical solution are not symmetric and cannot be collected with the conditions of symmetry for a quarter of plate. The Values of the critical loads obtained thus correspond to the first 3 symmetric modes of buckling:

Mode 1 of the quarter of the plate = Mode 1 of all the plate
Mode 2 of the quarter of the plate = Mode 3 of all the plate
Mode 3 of the quarter of the plate = Mode 5 of all the plate

2.3 Uncertainty on the solution

exact Solution for a theory of plate without transverse shears.

2.4 Bibliographical references

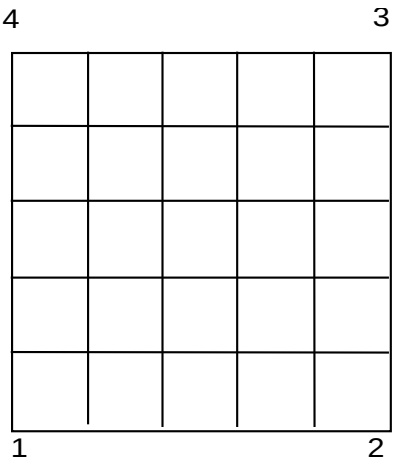
1) J.G. EISLEY "Mechanics of Elastic Structures: Classical years Finite Methods Element". Prentice Hall, Englewood Cliffs N.J. 07632 (19XX).

2) "Stability of Square Punt Biaxial Under Loading". The SAMCEF software User' S Manuals V7.1. (1998).

3 Modelization A

3.1 Characteristic of modelization

MEC3QU9H (shell 3D)



modelization COQUE_3D

3.2 Characteristics of the mesh

Many nodes: 121

Number of meshes and types: 25 QUAD9

3.3 Values tested

- Critical loads

Standard	Identification of reference	Reference	Tolerance %
mode 1	"ANALYTIQUE"	– 3.79600E+02	1.0
mode 2	"ANALYTIQUE"	– 1.05444E+03	0.6
mode 3	"ANALYTIQUE"	– 2.56609E+03	0.4

- Strain of Standard

membrane			Identification of reference	Reference	Tolérance %
Value	Nets	Node			
EXX	*MA000045*	*NC000003*	"ANALYTIQUE"	– 9.5238095E-07	1.e-4

Standard			Identification of reference	Reference	Tolérance %
Value	Do not net				
EXX	*MA000045*	1	"ANALYTIQUE"	– 9.5238095E-07	1.e-4

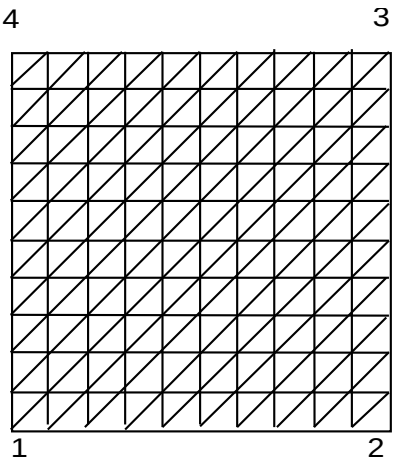
3.4 Remarks

energy due to the transverse shears is not neglected.

4 Modelization B

4.1 Characteristic of modelization

MEC3TR7H (COQUE_3D)



modelization COQUE_3D

4.2 Characteristics of the mesh

Many nodes: 641

Number of meshes and types: 200 TRIA7

4.3 Values tested

- Critical loads

Standard	Identification of reference	Reference	Tolerance %
mode 1	"ANALYTIQUE"	– 3.79600E+02	0.6
mode 2	"ANALYTIQUE"	– 1.05444E+03	0.3
mode 3	"ANALYTIQUE"	– 2.56609E+03	0.7

- Strain of Standard

membrane			Identification of reference	Reference	Tolérance %
Value	Nets	Node			
EXX	*MA000240*	*NC000003*	"ANALYTIQUE"	– 9.5238095E-07	1.e-4

Standard			Identification of reference	Reference	Tolérance %
Value	Do not net				
EXX	*MA000240*	1	"ANALYTIQUE"	– 9.5238095E-07	1.e-4

4.4 Remarks

energy due to the transverse shears is not neglected.

5 Summary of the results

the got results are very satisfactory for both element types, QUAD9 and TRIA7, even if it is necessary to employ one plus a large number of elements triangles.