SSLS101 - Plate circular posed subjected to a uniform pressure

Abstract:

One treats the case of a circular plate posed on edge in linear elasticity under 3 loadings (inertia loading, pressure, force distributed constant) which give the same deformed shape.

The first two modelizations make it possible to evaluate the influence of the mesh.

The test gathers 12 modelizations (model Coils-Kirchhoff, Mindlin-Reissner and COQUE_3D and SHB) more 2 modelizations of “connection” between shells or shells and 3D.

1 Problem of reference

1.1 Geometry

Coordinated of the points:

	O	A	B	C	D	E	F
x	0.	1.	$\sqrt{2}/2$	0.	0.5	0.	0.4
y	0.	0.	$\sqrt{2}/2$	1.	0.	0.5.0.4	
z	0.	0.	0.	0.	0.	0.	0.

1.2 Material properties

$E=1.Pa$
$\nu=0.3$
$\rho=1.kg/m^3$

1.3 Boundary conditions and loadings

simple Bearing on edge of the plate:
in all the points P such as $OP=R$: $u=v=w=0$

Uniform	FORCE_COQUE Pressure	$P=1\,N/m^2$
FORCE_COQUE	Charges distributed normal	$F3=-1\,N/m^2$
PESANTEUR	$g=10\,m/s^2$ according to Z from where	$FZ=\rho g t=-1\,N/m^2$

These three loadings lead to the same solution.

2 Reference solution

2.1 Method of calculating used for the reference solution

Two reference solutions are usable, for the computation of the deformed shape, according to the theory of plate used:

- the theory of Coils-Kirchhoff, usually used for the plates known as “thin”, that one will retain for the modelizations A, B and E,
- the theory of Reissner, including the effects of the shears for the plates known as “thick”, that one will retain for the modelizations F, G and H.

In any distant point of r center of the plate $r \leq R$, one has for the computation of the deflection:

$$w(r) = -P\,\frac{R^4}{64\,D}\left(1-\frac{r^2}{R^2}\right)\left(1+\frac{r^2}{R^2}-\frac{2(3+\nu)}{1+\nu}-\varphi\right) \text{ with } D=\frac{E\,t^3}{12(1-\nu^2)}$$

and $\varphi = 0\,(\text{Love-Kirchhoff})$ or $\varphi = \frac{16}{5}\left(\frac{t}{R}\right)^2 \frac{1}{1-\nu}\,(\text{Reissner})$

For the computation of the moments the two theories lead to the same statements:

$$M_{rr}(r) = \frac{PR^2}{16}(3+\nu)\left[\left(\frac{r}{R}\right)^2-1\right] \quad M_{\theta\theta}(r) = \frac{PR^2}{16}(3+\nu)\left[1-\frac{1+3\nu}{3+\nu}\left(\frac{r}{R}\right)^2\right]$$

In the center of the plate:

$$w(0) = -\frac{PR^4}{64\,D}\left(\frac{5+\nu}{1+\nu}\right)(\text{Love}-\text{Kirchhoff}) \text{ ou } w(0) = -\frac{PR^4}{64\,D}\left(\frac{5+\nu}{1+\nu}+\varphi\right)(\text{Reissner})$$

$$M_{rr}(0) = M_{\theta\theta}(0) = -\frac{PR^2}{16}(3+\nu)$$

Note:

Code_Aster and the calculates the moments with the nodes of each finite element in the reference of reference defined by the external norm reference axes defined on the shell (see AFFE_CARA_ELEM in the documentation of use).

The value of the moment M_{xx} (or M_{yy}), extracted field "EFGE_ELNO" , in a node pertaining to several finite elements can be regarded as being the average of the computed values on the elements which have this joint node. This average can be obtained by procedure POST_RELEVE [U4.74.03].

For each node, one a: $(M_{rr}+M_{\theta\theta}) = (M_{xx}+M_{yy}) = Sm$

for the point O	$M_{xx}=M_{yy}=M_{rr}=M_{\theta\theta}$
for the points $A et D$	$M_{xx}=M_{rr} et M_{yy}=M_{\theta\theta}$
for the points $C et E$	$M_{xx}=M_{\theta\theta} et M_{yy}=M_{rr}$
for the points $B et F$	$M_{xx}=M_{yy}=(M_{rr}+M_{\theta\theta})/2$

2.2 Results of reference

Marks with arrows and moments at the points: O, A, BC, DE, F.

2.3 Uncertainty on the analytical

solution Solution

2.4 bibliographical References

1.TIMOSHENKO and WOINOWSKY-KRIEGER, Plates and shells, Béranger Edition - (1961).

3 Modelization A

3.1 Characteristic of the modelization

Shell element DKT (modelization of a quarter of plate)

3.1.1 limiting Conditions

in all the nodes of the arc ABC	: DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the segment $]OA[$	: DY: 0. , DRX: 0. , DRZ: 0.
in all the nodes of the segment $]OC[$	: DX: 0. , DRY: 0. , DRZ: 0.
with the node is outside the field of definition with a right profile of the EXCLU type node: O	DX: 0. , DY: 0. , DRX: 0. , DRY: 0. , DRZ: 0.

Not O meshes: $M30\ M33$
Not A meshes: $M76$
Not B meshes: $M39\ M40\ M51$
Not C meshes: $M1$
Not D meshes: $M55\ M56\ M65$
Not E meshes: $M8\ M17\ M18$
Not F meshes: $M34\ M35\ M37\ M41\ M46\ M47\ M48$

3.2 Characteristics of the mesh

Many nodes: 50
Number of meshes and types: 76 TRIA3

3.3 Quantities tested and results

One tests the parameters of the data structure results:

Identification	Standard Value of reference	of Reference
INST for NUME_ORDRE= 3	ANALYTIQUE	0.6
ITER_GLOB for NUME_ORDRE=3	NON_REGRESSION	1
INST for NUME_ORDRE= 4	ANALYTIQUE	1
ITER_GLOB for Standard	NUME_ORDRE=4	1

1	Identification of reference	Values of reference
O $w(r)$	ANALYTIQUE	– 695.6256
D $w(r)$	ANALYTIQUE	– 489.727
E $w(r)$	ANALYTIQUE	– 489.727
F $w(r)$	ANALYTIQUE	– 435.8974

Standard		Identification of reference	Values of reference	Tolerance
O	M_{rr}	"ANALYTIQUE"	– 0.20625	1.5%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.20625	1.5%
A	M_{rr}	"ANALYTIQUE"	0.	0.01
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.0875	6%
B	M_{rr}	"ANALYTIQUE"	– 0.04375	13%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.04375	14%
	M_{rr}	C'ANALYTIQUE"	– 0.0875	6%
	$M_{\theta\theta}$	"ANALYTIQUE"	0.	0.01
	M_{rr}	D'ANALYTIQUE"	– 0.15469	0.5%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	0.5%
E	M_{rr}	"ANALYTIQUE"	– 0.15469	0.3%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	0.3%
F	M_{rr}	"ANALYTIQUE"	– 0.14025	10%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.16825	10%

Elastic strain energy

Standard				Identification of reference	Values of reference	Tolerance %
Quantity		Node	Nets			
ENEL_ELNO	*TOTALE*	N2	M1	"NON_DEFINI"	94.237	0.1
	MEMBRANE			"NON_DEFINI"	0.0.0.1	
	FLEXION			formulates "NON_DEFINI	„	93.112
	CISAILLE			formula "NON_DEFINI	„	0.1
	COUPL_MF			"NON_DEFINI"	0.0.0.1	

Identification				Standard of reference	Values of reference	Tolerance %
Quantity		Not	Mesh			
ENEL_ELGA	*TOTALE*	1	M1	"NON_DEFINI"	77.080	0.1
	MEMBRANE			"NON_DEFINI"	0.0.0.1	
	FLEXION			"NON_DEFINI"	75.955	0.1
	CISAILLE			"NON_DEFINI"	1.1248	0.1
	COUPL_MF			"NON_DEFINI"	0.0.0.1	

Identification			Standard of reference	Values of reference	Tolerance %
Quantity		ENEL_ELEM			
formulates	*TOTALE*	"	NON_DEFINI" 0.781	0.1	formula
	MEMBRANE		NON_DEFINI" 0.0.0.1	Nets	
	FLEXION		"NON_DEFINI"	0.767	0.1
	CISAILLE		"NON_DEFINI"	0.015	0.1
	COUPL_MF		"NON_DEFINI"	0.0.0.1	

One also tests into non regression the various components of elastic strain energy in the case of a computation with MECA_STATIQUE.

4 Modelization B

4.1 Characteristic of the modelization

Shell element DKT (modelization of a quarter of plate)

4.1.1 limiting Conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the segment $]OA[$: DY: 0. , DRX: 0. , DRZ: 0.
in all the nodes of the segment $]OC[$: DX: 0. , DRY: 0. , DRZ: 0.
with the node is outside the field of definition with a right profile of the EXCLU type node: O DX: 0. , DY: 0. , DRX: 0. , DRY: 0. , DRZ: 0.

Not O meshes: $M1\ M2$
Not A meshes: $M248\ M255$
Not B meshes: $M292\ M293\ M296$
Not C meshes: $M74\ M75$
Not D meshes: $M76\ M108\ M109$
Not E meshes: $M34\ M40\ M41$
Not F meshes: $M122\ M123\ M124\ M148\ M152\ M153$

4.2 Characteristics of the mesh

Many nodes: 170
Number of meshes and types: 296 TRIA3

4.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
O $w(r)$	"ANALYTIQUE"	– 695.6256	0.2%
D $w(r)$	"ANALYTIQUE"	– 489.727	0.2%
E $w(r)$	"ANALYTIQUE"	– 489.727	0.2%
F $w(r)$	"ANALYTIQUE"	– 435.8974	0.2%

Standard		Identification of reference	Values of reference	Tolerance
O	M_{rr}	"ANALYTIQUE"	– 0.20625	0.1%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.20625	0.2%
A	M_{rr}	"ANALYTIQUE"	0.	0.0025
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.0875	3.5%
B	M_{rr}	"ANALYTIQUE"	– 0.04375	6%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.04375	6%
	M_{rr}	C'ANALYTIQUE"	– 0.0875	3.5%
	$M_{\theta\theta}$	"ANALYTIQUE"	0.	0.0025
	M_{rr}	D'ANALYTIQUE"	– 0.15469	0.5%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	0.5%
E	M_{rr}	"ANALYTIQUE"	– 0.15469	0.5%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	0.5%
F	M_{rr}	"ANALYTIQUE"	– 0.14025	0.3%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.16825	0.3%

5 Modelization C

5.1 Characteristic of the modelization

Shell element SHB8 (modelization of a quarter of plate)

5.1.1 limiting Conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the face $]OA[$: DY: 0. ,
in all the nodes of the face $]OC[$: DX: 0. ,
with the node is outside the field of definition with a right profile of the EXCLU type node: O DX: 0. , DY: 0. ,

The mesh is built starting from the surface mesh of the modelization E, by thickening, using CREA_MAILLAGE/COQU_VOLU. 2 layers of HEXA8 meshes are built.

5.2 Characteristics of the mesh

Many nodes: 338
Number of meshes and types: 147 HEXA8

5.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
O $w(r)$	"ANALYTIQUE"	– 695.6256	0.5%
D $w(r)$	"ANALYTIQUE"	– 489.727	0.5%
E $w(r)$	"ANALYTIQUE"	– 489.727	0.5%
F $w(r)$	"ANALYTIQUE"	– 435.8974	0.5%

6 Modelization D

6.1 Characteristic of the modelization

Shell element SHB20 (modelization of a quarter of plate)

6.1.1 limiting Conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the face $]OA[$: DY: 0. ,
in all the nodes of the face $]OC[$: DX: 0. ,
with the node is outside the field of definition with a right profile of the EXCLU type node: O DX: 0. , DY: 0. ,

6.2 Characteristic of the mesh

Many nodes: 1137
Number of meshes and types: 147 HEXA20

6.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
O $w(r)$	"ANALYTIQUE"	– 695.6256	1.2%
D $w(r)$	"ANALYTIQUE"	– 489.727	1.3%
E $w(r)$	"ANALYTIQUE"	– 489.727	1.3%
F $w(r)$	"ANALYTIQUE"	– 435.8974	1.5%

7 Modelization E

7.1 Characteristic of the modelization

Shell element DKQ (modelization of a quarter of plate)

7.1.1 limiting Conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the segment $]OA[$: DY: 0. , DRX: 0. , DRZ: 0.
in all the nodes of the segment $]OC[$: DX: 0. , DRY: 0. , DRZ: 0.
with the node is outside the field of definition with a right profile of the EXCLU type node: O DX: 0. , DY: 0. , DRX: 0. , DRY: 0. , DRZ: 0.

Not O meshes: $M1$
Not A meshes: $M147$
Not B meshes: $M98\ M111$
Not C meshes: $M14$
Not D meshes: $M85\ M99$
Not E meshes: $M7\ M8$
Not F meshes: $M91\ M92\ M105$

7.2 Characteristics of the mesh

Many nodes: 169
Number of meshes and types: 147 QUAD4

7.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
O $w(r)$	"ANALYTIQUE"	– 695.6256	0.1%
D $w(r)$	"ANALYTIQUE"	– 489.727	0.15%
E $w(r)$	"ANALYTIQUE"	– 489.727	0.15%
F $w(r)$	"ANALYTIQUE"	– 435.8974	0.15%

Standard		Identification of reference	Values of reference	Tolerance
O	M_{rr}	"ANALYTIQUE"	– 0.20625	0.1%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.20625	0.1%
A	M_{rr}	"ANALYTIQUE"	0.	0.0004
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.0875	0.5%
B	M_{rr}	"ANALYTIQUE"	– 0.04375	1.1%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.04375	1.1%
	M_{rr}	C'ANALYTIQUE"	0.	0.0004
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.0875	0.5%
	M_{rr}	D'ANALYTIQUE"	– 0.15469	0.1%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	0.1%
E	M_{rr}	"ANALYTIQUE"	– 0.15469	0.5%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	0.1%

F	M_{rr}	"ANALYTIQUE"	– 0.14025	0.35%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.16825	0.35%

8 Modelization F

8.1 Characteristic of the modelization

Shell element DST (modelization of a quarter of plate)

8.1.1 limiting Conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the segment $]OA[$: DY: 0. , DRX: 0. , DRZ: 0.
in all the nodes of the segment $]OC[$: DX: 0. , DRY: 0. , DRZ: 0.
with the node is outside the field of definition with a right profile of the EXCLU type node: O DX: 0. , DY: 0. , DRX: 0. , DRY: 0. , DRZ: 0.

Not O meshes: $M1\ M2$
Not A meshes: $M248\ M255$
Not B meshes: $M3292\ M293\ M296$
Not C meshes: $M74\ M75$
Not D meshes: $M76\ M108\ M109$
Not E meshes: $M34\ M40\ M41$
Not F meshes: $M122\ M123\ M124\ M148\ M152\ M153$

8.2 Characteristics of the mesh

Many nodes: 170
Number of meshes and types: 296 TRIA3

8.3 Quantities tested and Standard

Identification	results of reference	Values of reference Reissner	Tolerance
O $w(r)$	"ANALYTIQUE"	– 703.40	0.2%
$w(r)$	D'ANALYTIQUE"	– 495.56	0.1%
E $w(r)$	"ANALYTIQUE"	– 495.56	0.1%
F $w(r)$	"ANALYTIQUE"	– 441.18	0.1%

Standard		Identification of reference	Values of reference	Tolerance
O	M_{rr}	"ANALYTIQUE"	– 0.20625	0.5%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.20625	0.55%
A	M_{rr}	"ANALYTIQUE"	0.	0.02
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.0875	9%
B	M_{rr}	"ANALYTIQUE"	-0.04375	9.5%
	$M_{\theta\theta}$	"ANALYTIQUE"	-0.04375	9.5%
	M_{rr}	C'ANALYTIQUE"	– 0.0875	9%
	$M_{\theta\theta}$	"ANALYTIQUE"	0.	0.02
	M_{rr}	D'ANALYTIQUE"	– 0.15469	0.9%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	0.9%

E	M_{rr}	"ANALYTIQUE"	– 0.15469	0.9%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	0.9%
F	M_{rr}	"ANALYTIQUE"	– 0.14025	0.2%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.16825	0.2%

9 Modelization G

9.1 Characteristic of the modelization

Shell element DSQ (modelization of a quarter of plate)

9.1.1 limiting Conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the segment $]OA[$: DY: 0. , DRX: 0. , DRZ: 0.
in all the nodes of the segment $]OC[$: DX: 0. , DRY: 0. , DRZ: 0.
with the node is outside the field of definition with a right profile of the EXCLU type node: O : DX: 0. , DY: 0. , DRX: 0. , DRY: 0. , DRZ: 0.

Not O meshes: $M1$
Not A meshes: $M147$
Not B meshes: $M98\ M111$
Not C meshes: $M14$
Not D meshes: $M85\ M99$
Not E meshes: $M7\ M8$
Not F meshes: $M91\ M92\ M105$

9.2 Characteristics of the mesh

Many nodes: 169
Number of meshes and types: 147 QUAD4

9.3 Quantities tested and Standard

Identification	results of reference	Values of reference Reissner	Tolerance
O $w(r)$	"ANALYTIQUE"	– 703.40	0.15%
D $w(r)$	"ANALYTIQUE"	– 495.56	0.15%
E $w(r)$	"ANALYTIQUE"	– 495.56	0.15%
F $w(r)$	"ANALYTIQUE"	– 441.18	0.2%

Standard		Identification of reference	Values of reference	Tolerance
O	M_{rr}	"ANALYTIQUE"	– 0.20625	1%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.20625	1%
A	M_{rr}	"ANALYTIQUE"	0.	0.01
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.0875	2.5%
B	M_{rr}	"ANALYTIQUE"	-0.04375	7.5%
	$M_{\theta\theta}$	"ANALYTIQUE"	-0.04375	7.5%
	M_{rr}	C'ANALYTIQUE"	– 0.0875	2.5%

	$M_{\theta\theta}$	"ANALYTIQUE"	0.	0.01
E	M_{rr}	D'ANALYTIQUE"	– 0.15469	0.6%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	0.6%
	M_{rr}	"ANALYTIQUE"	– 0.15469	0.7%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	0.7%
F	M_{rr}	"ANALYTIQUE"	– 0.14025	19%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.16825	19%

Standard				Identification of reference	Values of reference	Tolerance
Quantity		Not	Mesh			
ENEL_ELNO	*TOTALE*	formulates	M14	"NON_REGRESSION"	45.79374	0.1%
	MEMBRANE			"NON_REGRESSION"	0.0	0.1%
	FLEXION			"NON_REGRESSION"	44.78267	0.1%
	CISAILLE			"NON_REGRESSION"	1.01107	0.1%
	COUPL_MF			"NON_REGRESSION"	0.0	0.1%

Identification				Standard of reference	Values of reference	Tolerance
Quantity		Not	Mesh			
ENEL_ELGA	*TOTALE*	1	M14	"NON_REGRESSION"	57.36959	0.1%
	MEMBRANE			"NON_REGRESSION"	0.0	0.1%
	FLEXION			"NON_REGRESSION"	56.47085	0.1%
	CISAILLE			"NON_REGRESSION"	0.89874	0.1%
	COUPL_MF			"NON_REGRESSION"	0.0	0.1%

Identification			Standard of reference	Values of reference	Tolerance
Quantity		Nets			
ENEL_ELEM	*TOTALE*	M14	"NON_REGRESSION"	0.42940	0.1%
	MEMBRANE		"NON_REGRESSION"	0.0	0.1%
	FLEXION		"NON_REGRESSION"	0.42229	0.1%
	CISAILLE		"NON_REGRESSION"	7.11225E-03	0.1%
	COUPL_MF		"NON_REGRESSION"	0.0	0.1%

10 Modelization H

10.1 Characteristic of the modelization

Shell element Q4G (modelization of a quarter of plate)

10.1.1 limiting Conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the segment $]OA[$: DY: 0. , DRX: 0. , DRZ: 0.
in all the nodes of the segment $]OC[$: DX: 0. , DRY: 0. , DRZ: 0.
with the node is outside the field of definition with a right profile of the EXCLU type node: O DX: 0. , DY: 0. , DRX: 0. , DRY: 0. , DRZ: 0.

Not O meshes: $M1$
Not A meshes: $M147$
Not B meshes: $M98\ M111$
Not C meshes: $M14$
Not D meshes: $M85\ M99$
Not E meshes: $M7\ M8$
Not F meshes: $M91\ M92\ M105$

10.2 Characteristics of the mesh

Many nodes: 169
Number of meshes and types: 147 QUAD4

10.3 Quantities tested and Standard

Identification		results of reference	Values of reference Reissner	Tolerance
O	$w(r)$	"ANALYTIQUE"	– 703.40	0.2%
D	$w(r)$	"ANALYTIQUE"	– 495.56	0.3%
E	$w(r)$	"ANALYTIQUE"	– 495.56	0.3%
F	$w(r)$	"ANALYTIQUE"	– 441.18	0.3%

Standard		Identification of reference	Values of reference	Tolerance
O	M_{rr}	"ANALYTIQUE"	– 0.20625	0.2%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.20625	0.2%
A	M_{rr}	"ANALYTIQUE"	0.	0.02
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.0875	4.5%
B	M_{rr}	"ANALYTIQUE"	-0.04375	20%
	$M_{\theta\theta}$	"ANALYTIQUE"	-0.04375	20%
	M_{rr}	C'ANALYTIQUE"	– 0.0875	0.02
	$M_{\theta\theta}$	"ANALYTIQUE"	0.	4.5%
D	M_{rr}	"ANALYTIQUE"	– 0.15469	0.25%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	0.25%
E	M_{rr}	"ANALYTIQUE"	– 0.15469	0.25%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	0.25%

F	M_{rr}	"ANALYTIQUE"	– 0.14025	0.35%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.16825	0.35%

11 Modelization I

11.1 Characteristic of the modelization

Modelization: Element of limiting COQUE_3D

11.1.1 MEC3QU9H Conditions

in all the nodes of the arc ABC	DX: 0. , DY: 0. , DZ: 0. DRX: 0. , DRY: 0. , DRZ: 0.
segment $\left\|OA\right\|$	DY: 0. , DRX: 0. , DRZ: 0.
segment $\left\|OC\right\|$	DX: 0. , DRY: 0. , DRZ: 0.
with the node O	DX: 0. , DY: 0. , DRX: 0. , DRY: 0. , DRZ: 0.

Names of the nodes:

Not 0	meshes: $M1$	Not A	meshes: $M21$
Not B	meshes: $M25$	Not C	meshes: $M5$
Not D	meshes: $M11$	Not E	meshes: $M3$

11.2 Characteristics of the mesh

Many nodes: 96
Number of meshes and types: 25 QUAD9

11.3 Quantities tested and Standard

Identification	results of reference	Values of reference Reissner	Tolerance
O $w(r)$	"ANALYTIQUE"	– 703.40	0.5%
A $w(r)$	"ANALYTIQUE"	0.	10-10
B $w(r)$	"ANALYTIQUE"	0.	10-10
$w(r)$	C'ANALYTIQUE"	0.	10-10
D $w(r)$	"ANALYTIQUE"	– 495.56	0.5%
E $w(r)$	"ANALYTIQUE"	– 495.56	0.5%

Standard		Identification of reference	Values of reference	Tolerance
O	M_{rr}	"ANALYTIQUE"	– 0.20625	1.3%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.20625	1.3%
A	M_{rr}	"ANALYTIQUE"	0.	0.002
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.0875	1.3%
	M_{rr}	C'ANALYTIQUE"	0.	0.002
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.0875	2.3%
	M_{rr}	D'ANALYTIQUE"	– 0.15469	1%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	1%

E	M_{rr}	"ANALYTIQUE"	– 0.15469	1%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	1%

11.4 Remarks

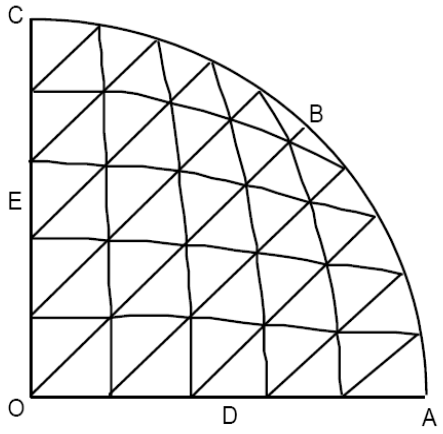
the test of the values is carried out automatically using the functionalities offered by procedure POST_RELEVE :

- extraction on the nodes corresponding to the points observed of the mean values of the components M_{xx} and M_{yy} ; these values are extracted from field "EFGE_ELNO" , and the average is calculated for all the liquid assets on meshes which contain the node observed,
- computation of the variation compared to the value of reference provided by observing the rules of correspondence enters M_{xx} , M_{yy} and M_{rr} , $M_{\theta\theta}$ given page 3.

12 Modelization J

12.1 Characteristic of the modelization

Modelization: Element of limiting COQUE_3D



12.1.1 MEC3TR7H Conditions

in all the nodes of the arc ABC	DX: 0. , DY: 0. , DZ: 0. DRX: 0. , DRY: 0. , DRZ: 0.
segment $]OA]$	DY: 0. , DRX: 0. , DRZ: 0.
segment $]OC]$	DX: 0. , DRY: 0. , DRZ: 0.
with the node O	DX: 0. , DY: 0. , DRX: 0. , DRY: 0. , DRZ: 0.

Names of the nodes:

Not 0	meshes: $M1$ and $M2$	Not A	meshes: $M41$
Not B	meshes: $M49$ and $M50$	Not C	meshes: $M10$
Not D	meshes: $M21$	Not E	meshes: $M6$

12.2 Characteristics of the mesh

Many nodes: 121
Number of meshes and types: 50 TRIA7

12.3 Quantities tested and Standard

Identification	results of reference	Values of reference Reissner	Tolerance
O $w(r)$	"ANALYTIQUE"	– 703.40	0.5%
A $w(r)$	"ANALYTIQUE"	0.	10-10
B $w(r)$	"ANALYTIQUE"	0.	10-10
$w(r)$	C'ANALYTIQUE"	0.	10-10
$w(r)$	D'ANALYTIQUE"	– 495.56	0.5%
E $w(r)$	"ANALYTIQUE"	– 495.56	0.5%

Standard		Identification of reference	Values of reference	Tolerance
O	M_{rr}	"ANALYTIQUE"	– 0.20625	1.3%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.20625	1.3%
A	M_{rr}	"ANALYTIQUE"	0.	0.002
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.0875	1.5%
	M_{rr}	C'ANALYTIQUE"	– 0.0875	2.3%
	$M_{\theta\theta}$	"ANALYTIQUE"	0.	0.002
	M_{rr}	D'ANALYTIQUE"	– 0.15469	1%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	1%
E	M_{rr}	"ANALYTIQUE"	– 0.15469	1%
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.17656	1%

12.4 Remarks

the test of the values is carried out automatically using the functionalities offered by procedure POST_RELEVE :

• extraction on the nodes corresponding to the points observed of the mean values of the components M_{xx} and M_{yy} ; these values are extracted from field "EFGE_ELNO" , and the average is calculated for all the liquid assets on meshes which contain the node observed,

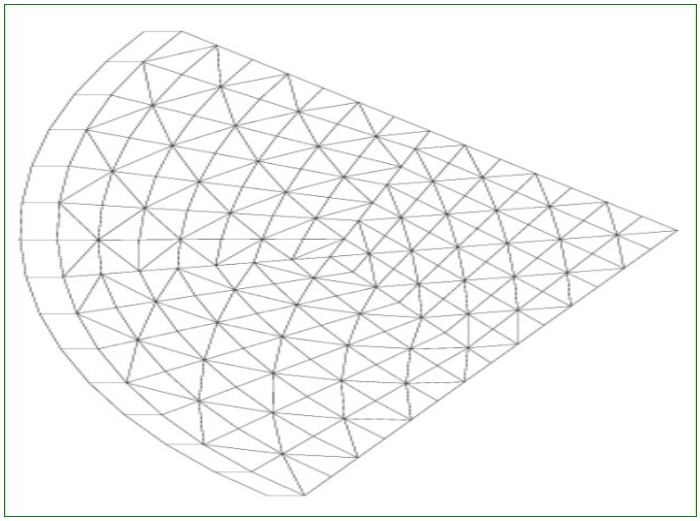
• computation of the variation compared to the value of reference provided by observing the rules of correspondence enters M_{xx} , M_{yy} and M_{rr} , $M_{\theta\theta}$ given page 3.

13 Modelization K

13.1 Characteristic of the modelization

Shell element SHB6 (modelization of a quarter of plate)

13.1.1 limiting Conditions



in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the face $]OA[$: DY: 0. ,
in all the nodes of the face $]OC[$: DX: 0. ,
with the node is outside the field of definition with a right profile of the EXCLU type node: O DX: 0. , DY: 0. ,

13.2 Characteristic of the mesh

Many nodes: 338
Number of meshes and types: 294 SHB6

13.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
O $w(r)$	"ANALYTIQUE"	– 695.6256	0.1%
D $w(r)$	"ANALYTIQUE"	– 489.727	0.1%
E $w(r)$	"ANALYTIQUE"	– 489.727	0.1%
F $w(r)$	"ANALYTIQUE"	– 435.8974	0.1%

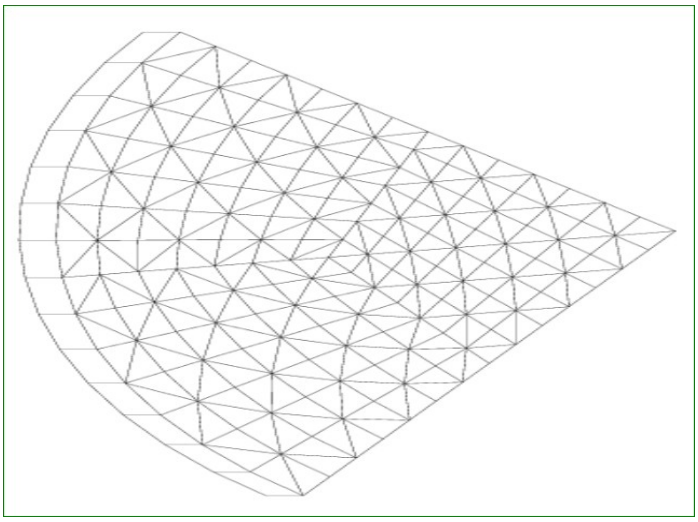
13.4 Remark

When the mesh is refined, the solution does not move practically any more but tends towards a deflection higher from about 5% than the analytical solution. The element thus presents on this test a light numerical blocking, much lower however than that of its counterpart $3D$ the PENTA6 (30% of error).

14 Modelization L

14.1 Characteristic of the modelization

Shell element SHB15 (modelization of a quarter of plate)



14.1.1 limiting Conditions

in all the nodes of the arc ABC : DX: 0. , DY: 0. , DZ: 0.
in all the nodes of the face $]OA[$: DY: 0. ,
in all the nodes of the face $]OC[$: DX: 0. ,
with the node is outside the field of definition with a right profile of the EXCLU type node: O DX: 0. , DY: 0. ,

14.2 Characteristic of the mesh

Many nodes: 1431
Number of meshes and types: 294 SHB15

14.3 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
O $w(r)$	"ANALYTIQUE"	– 695.6256	1.2%
D $w(r)$	"ANALYTIQUE"	– 489.727	1.3%
E $w(r)$	"ANALYTIQUE"	– 489.727	1.3%
F $w(r)$	"ANALYTIQUE"	– 435.8974	1.5%

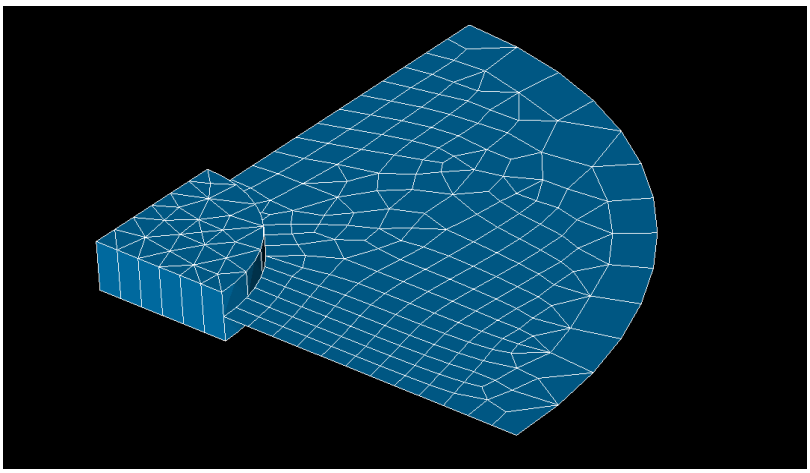
14.4 Notices

This element does not present numerical blocking and converges well towards the analytical solution.

15 Modelization M

15.1 Characteristic of the modelization

Connection between elements 3D and shell elements (DKT)



15.1.1 limiting Conditions

circular Edge : simple bearing
right sides : conditions of symmetry

15.1.2 Characteristic of mesh

MODELISATION	ELEMENT FINI	TYPE NETS	NOMBRE
DKT	MEDKTR3	TRIA3	32
DKT	MEDKQU4	QUAD4	196
3D	MECA_PENTA15	PENTA15	48

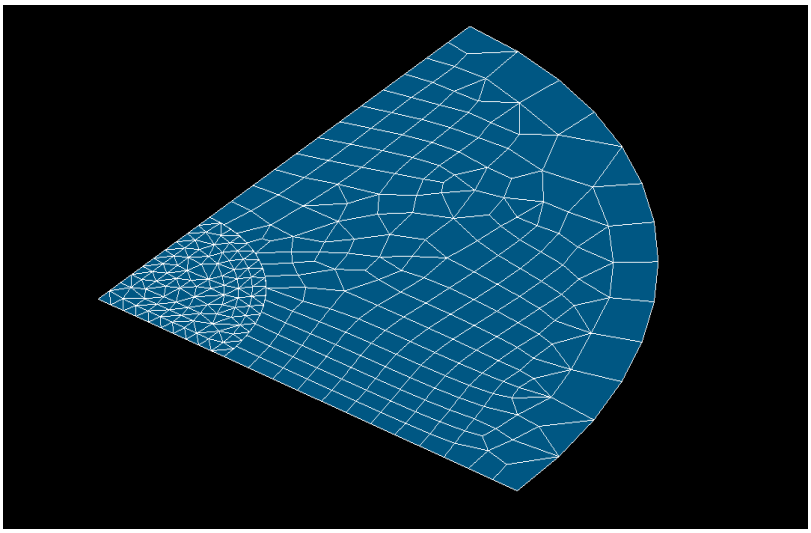
15.2 Quantities tested and Standard

Identification		results of reference	Values of reference	Tolerance
Not	TYPE_RACCORD			
O $w(r)$	COQUE_MASSIF	"ANALYTIQUE"	– 695.6256	0.3%
O $w(r)$	MASSIF_COQUE	"ANALYTIQUE"	– 695.6256	1%

16 Modelization N

16.1 Characteristic of the modelization

Connection between 2 meshes of shell (DKT) incompatible



16.1.1 limiting Conditions

circular Edge : simple bearing
right sides : conditions of symmetry

16.1.2 Characteristic of mesh

MODELISATION	ELEMENT FINI	TYPE NETS	NOMBRE
DKT	MEDKTR3	TRIA3	213
DKT	MEDKQU4	QUAD4	196

16.2 Quantities tested and Standard

Identification	results of reference	Values of reference	Tolerance
o $w(r)$	"ANALYTIQUE"	– 695.6256	0.2%

17 Summary of the %

results of the differences compared to the reference solutions

	DKT A Coils-Kirchhoff 50 nodes 76 TRIA3	B Coils-Kirchhoff 170 nodes 296 TRIA3	**DKQ** E Coils-Kirchhoff 169 nodes 147 QUAD4	**DST** F Reissner 170 nodes 296 TRIA3	**DSQ** G Reissner 169 nodes 147 QUAD4	**Q4G** H Reissner 169 nodes 147 QUAD4
O $w(r)$	– 1.10	– 0.09	– 0.09	+0.12	– 0.11	– 0.15
D $w(r)$	– 1.01	– 0.1	– 0.11	+0.08	– 0.13	– 0.20
E $w(r)$	– 1.03	–0.09	– 0.12	+0.09	– 0.13	– 0.20
F $w(r)$	– 1.05	– 0.09	– 0.09	+0.07	– 0.15	– 0.21
	MEC3QU9H I 96 nodes 25 QUAD9	**MEC3TR7H** J 121 nodes 50 TRIA7	**SHB8** C 338 nodes 147 HEXA8	**SHB20** D 1137 nodes 147 HEXA20	**SHB6** K 338 nodes 294 PENTA6	**SHB15** L 1431 nodes 294 PENTA15
O $w(r)$	1.42 10^{-3}	– 0.03	0.4.1.1.3.7			1.1
D2 $w(r)$	. 10^{-3}	– 0.07	0.3.1.3.4.1			1.2
E $w(r)$	2. 10^{-3}	– 0.07	0.3.1.3.4.0			1.2
F $w(r)$	-	-	0.2.1.3.3.8			1.3

Concerning displacements:
The elements plates and shells give good performances on rather coarse meshes.
The elements 3D-shells SHB give less good results, especially the SHB6 which presents a light numerical blocking.

	DKT A Coils-Kirchhoff 50 nodes 76 TRIA3	B Coils-Kirchhoff 170 nodes 296 TRIA3	**DKQ** E Coils-Kirchhoff 169 nodes 147 QUAD4	**DST** F Reissner 170 nodes 296 TRIA3	**DSQ** G Reissner 169 nodes 147 QUAD4	**Q4G** H Reissner 169 nodes 147 QUAD4
O Sm/2	– 1.19	+0.02	+0.07	+0.07	– 0.76	– 0.14
A Sm/2	+5.79	– 0.06	– 0.49	– 4.40	– 9.80	+17.80
B Sm/2	– 13.100	– 5.53	+1.00	– 9.10	– 7.12	+19.70
C Sm/2	+5.73	– 0.06	– 0.46	– 4.41	– 9.44	+17.90
D Sm/2	+0.20	+0.35	+0.50	+0.43	+0.49	+0.05
E Sm/2	+0.19	+0.42	+0.50	+0.49	+0.50	+0.05
F Sm/2	– 0.66	+0.25	– 0.30	+0.15	+19.00	– 0.33

	MEC3QU9H I 96 nodes 25 QUAD9	**MEC3TR7H** J 121 nodes 50 TRIA7
O Sm/2	1.05	1.14
A Sm/2	2.9	0.25
B Sm/2	-	-
C Sm/2	2.9	0.25
D Sm/2	0.28	– 0.28
E Sm/2	0.28	– 0.28
F Sm/2	-	-

Concerning the forces:

- on supported edge, one notes important errors (going up to 20%) compared to the analytical solutions. The error is marked on modelization H (Q4G).

- by refining the mesh of each modelization one observes the convergence of the forces, i.e. that the error tends towards 0. Nevertheless the order of convergence is lower for the modelization H : element Q4G indeed requires to net very finely in the directions requested in bending (it uses a bilinear approximation of rotations whereas modelization DST leans on a quadratic approximation).