SSLS100 - Plate circular clamped subjected to a uniform pressure

Abstract:

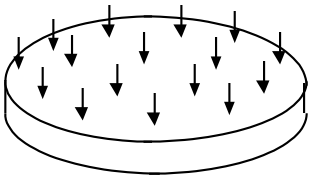
This problem allows a comparison between the solutions obtained and various shell elements in linear elasticity:

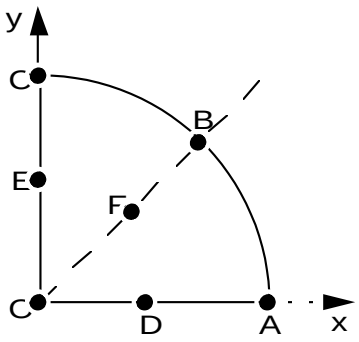
- models of **Coils-Kirchhoff** (plate known as thin):
  - triangular surface mesh (TRIA3) DKT,
  - quadrangular surface mesh (QUAD4) DKQ,
  - linear mesh (SEG3) COQUE_AXIS,
- models of **Mindlin-Reissner** (plate known as thick):
  - triangular surface mesh (TRIA3) DST,
  - quadrangular surface mesh (QUAD4) DSQ,
  - linear mesh (SEG3) COQUE_AXIS,
- models of shells thick: COQUE_3D (QUAD9 and TRIA7).

The same reference solution is treated with three forms of loadings: pressure, gravity and force - shell. The quantities observed are: generalized displacements (translation/rotation), strains and forces.

1 Problem of reference

1.1 Geometry





1/4 de plaque

Rayon R=1 m
Epaisseur t=0.1 m

Coordinated of the points:

	O	A	B	C	D	E	F
x	0.	1.	$1/\sqrt{2}$	0.	0.5	0.	0.4
y	0.	0.	$1/\sqrt{2}$	1.	0.	0.5.0.4	
z	0	0.	0.	0.	0.	0.	0.

1.2 Material properties

$E=1\,Pa$	Modulus Young
$\nu=0.3$	Poisson's ratio
$\rho=1\,kg/m^3$	Density

1.3 Boundary conditions and loadings

Fixed support on edge of the plate:
in all the points P such as $OP=R$: $u=v=w=0$ $\theta_x=\theta_y=\theta_z=0$.

`Uniform`	FORCE_COQUE Pressure	$P=1\,N/m^2$
`FORCE_COQUE`	Charges distributed normal	$F3=-1\,N/m^2$
`PESANTEUR`	$g=10\,m/s^2$ according to Z from where	$FZ=\rho\,g\,t=-1\,N/m^2$

These three loadings lead to the same solution.

2 Reference solution

2.1 Method of calculating used for the reference solution

Two reference solutions are usable, for the computation of the deformed shape:

- the theory of LOVE-KIRCHHOFF, usually used for the plates known as “thin”, that one will A retain for the modelizations , B, C, D, E and I,
- the theory of MINDLIN-REISSNER, including the effects of the shears for the plates known as “thick”, that one will retain for the modelizations F, G, H and J.

In any distant point of r center of the plate ($r \leq R$), the deflection is expressed:

$$w(r) = -\frac{P R^4}{64 D}\left(1-\frac{r^2}{R^2}\right)\left(1-\frac{r^2}{R^2}+\phi\right) \quad \text{avec} \quad D=\frac{E t^3}{12\left(1-\nu^2\right)}$$

avec $\phi = 0$ (LOVE – KIRCHHOFF) ou $\phi = \frac{16}{5}\left(\frac{t}{R}\right)^2 \frac{1}{1-\nu}$ (REISSNER).

For the computation of the moments the two theories lead to the same statements:

$$M_{rr}(r) = \frac{P R^2}{16}\left[(3+\nu)\left(\frac{r}{R}\right)^2-(1+\nu)\right] \quad M_{\theta\theta}(r) = \frac{P R^2}{16}\left[(1+3\nu)\left(\frac{r}{R}\right)^2-(1+\nu)\right]$$

In the center of the plate:

$$w(0) = -\frac{P R^4}{64 D} \text{ (LOVE – KIRCHHOFF) ou } w(0) = -\frac{P R^4}{64 D}(1+\phi) \text{ (REISSNER)}$$

$$M_{rr}(0) = M_{\theta\theta}(0) = -\frac{P R^2}{16}(1+\nu)$$

Note:

Code_Aster and the calculates the moments with the nodes of each finite element in the reference of reference defined by the external norm reference axes defined on the shell (see AFFE_CARA_ELEM).

The value of the moment M_{xx} (or M_{yy}) in a node pertaining to several finite elements can be regarded as being the average of the computed values on the elements which have this joint node. This average can be obtained by procedure POST_RELEVE .

For each node, one a: $\left(M_{rr}+M_{\theta\theta}\right) = \left(M_{xx}+M_{yy}\right) = Sm$

pour le point O	M_{xx}	=	M_{yy}	=	M_{rr}	= $M_{\theta\theta}$
pour les points A et D	M_{xx}	=	M_{rr}	et	M_{yy}	= $M_{\theta\theta}$
pour les points C et E	M_{xx}	=	$M_{\theta\theta}$	et	M_{yy}	= M_{rr}
pour les points B et F	M_{xx}	=	M_{yy}	=	$\left(M_{rr}+M_{\theta\theta}\right)/2$	

2.2 Results of reference

Marks with arrows and moments at the points O, A, B, C, D, E, F. Extraction of the mean values of the components M_{xx} et M_{yy} of field “EFGE_ELNO”.

2.3 Uncertainty on the analytical

solution Solution.

2.4 Bibliographical references

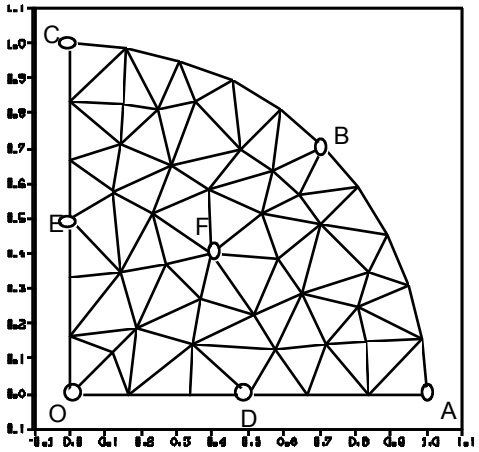
1) TIMOSHENKO and WOINOWSKY-KRIEGER. Plates and shells. Béranger edition, (1961).

2) BATOZ and DHATT. Modelization of structures by finite elements. Shells. Univ presses. Laval, 1992.

3 Modelization A

3.1 Characteristic of the modelization

Shell element DKT (modelization of a quarter of plate)
Number of layer: Limiting COQUE_NCOU =



3 Conditions: DDL_IMPO

in all the nodes of the arc ABC	(GROUP_NO= "ABC", DX= 0. , DY= 0. , DZ= 0.) DRX=0., DRY=0., DRZ=0.)
in all the nodes of segment] OA [	(GROUP_NO= "OA", DY= 0. , DRX=0., DRZ=0.)
in all the nodes of segment] OC [	(GROUP_NO= "OC", DX= 0. , DRY=0., DRZ=0.)
with node O	(GROUP_NO= "O", DX= 0. , DY= 0. , DRX=0., DRY=0., DRZ=0.)

Not O	meshes: M30, M33
Not A	meshes: M76
Not B	meshes: M39, M40, M51
Not C	meshes: M1
Point D	meshes: M55, M56, M65
Not E	meshes: M8, M17, M18
Not F	meshes: M34, M35, M37, M41, M46, M47, M48

3.2 Characteristics of the mesh

Many nodes: 50
Number of meshes and types: 76 TRIA3

3.3 Quantities tested and Standard

Identification	results of reference	Values of reference Coil-Kirchhoff	Tolerance (%)
" $w(r)$	ANALYTIQUE O"	– 170.6251	1.0
$w(r)$	D'ANALYTIQUE"	– 95.9766	0.75
E $w(r)$	"ANALYTIQUE"	– 95.9766	0.75
F $w(r)$	"ANALYTIQUE"	– 78.897	0.5

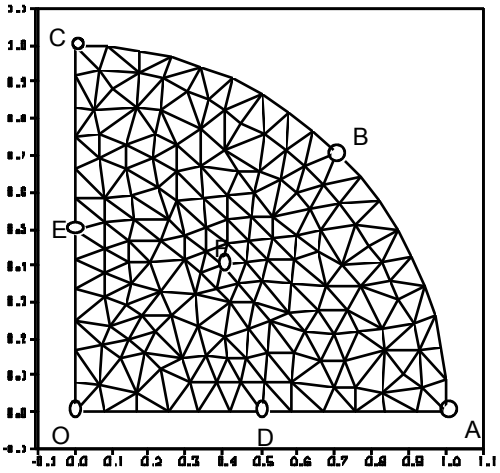
Standard		Identification of reference	Values of reference	Tolerance (%)
O	M_{rr}	"ANALYTIQUE"	– 0.08125	3.0
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.08125	3.0
A	M_{rr}	"ANALYTIQUE"	0.125	2.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	2.0
B	M_{rr}	"ANALYTIQUE"	0.08125	5.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.08125	5.0
	M_{rr}	C'ANALYTIQUE"	0.125	2.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	2.0
	M_{rr}	D'ANALYTIQUE"	– 0.02969	7.0
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.05156	3.5
E	M_{rr}	"ANALYTIQUE"	– 0.02969	7.0
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.05156	3.5
F	M_{rr}	"ANALYTIQUE"	– 0.02925	3.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.02925	4.5

One also tests the continuity of fields EPSI_ELGA and EPSI_ELNO between the layers (tests of NON-regression). Indeed the computed value on level SUP of the layer N is equal to the computed value on level INF of the COUCHE N+1.

4 Modelization B

4.1 Characteristic of the modelization

Shell element DKT (modelization of a quarter of plate)



limiting Conditions:	DDL_IMPO
in all the nodes of the arc ABC	(GROUP_NO= "ABC", DX= 0. , DY= 0. , DZ= 0.) DRX=0., DRY=0., DRZ=0.)
in all the nodes of segment] OA [	(GROUP_NO= "OA", DY= 0. , DRX=0., DRZ=0.)
in all the nodes of segment] OC [	(GROUP_NO= "OC", DX= 0. , DRY=0., DRZ=0.)
with node O	(GROUP_NO= "O", DX= 0. , DY= 0. , DRX=0., DRY=0., DRZ=0.)

Not O meshes: M
Point-to-point
B
Not C
Not D
Not E

4.2 Characteristic of the mesh

Many nodes: 170
Number of meshes and types: 296 TRIA3

4.3 Quantities tested and Standard

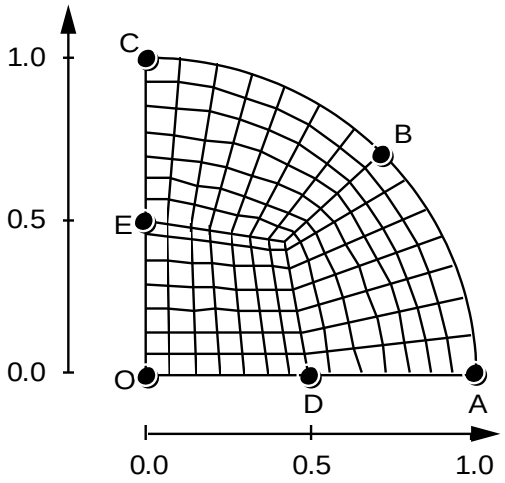
Identification	results of reference	Values of reference Coil-Kirchhoff	Tolerance (%)
" $w(r)$	ANALYTIQUE O"	– 170.6251	0.5
$w(r)$	D'ANALYTIQUE"	– 95.9766	0.5
E $w(r)$	"ANALYTIQUE"	– 95.9766	0.5
F $w(r)$	"ANALYTIQUE"	– 78.897	0.5

Standard		Identification of reference	Values of reference	Tolerance (%)
O	M_{rr}	"ANALYTIQUE"	– 0.08125	1.0
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.08125	1.0
A	M_{rr}	"ANALYTIQUE"	0.125	3.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	9.0
B	M_{rr}	"ANALYTIQUE"	0.08125	3.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.08125	3.0
	M_{rr}	C'ANALYTIQUE"	0.125	3.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	9.0
	M_{rr}	D'ANALYTIQUE"	– 0.02969	2.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.05156	2.0
E	M_{rr}	"ANALYTIQUE"	– 0.02969	2.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.05156	2.5
F	M_{rr}	"ANALYTIQUE"	– 0.02925	2.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.02925	2.5

5 Modelization E

5.1 Characteristic of the modelization

Shell element DKQ (modelization of a quarter of plate)



limiting Conditions:	DDL_IMPO
in all the nodes of the arc ABC	(GROUP_NO= "ABC", DX= 0. , DY= 0. , DZ= 0.) DRX=0., DRY=0., DRZ=0.)
in all the nodes of segment] OA [	(GROUP_NO= "OA", DY= 0. , DRX=0., DRZ=0.)
in all the nodes of segment] OC [	(GROUP_NO= "OC", DX= 0. , DRY=0., DRZ=0.)
with node O	(GROUP_NO= "O", DX= 0. , DY= 0. , DRX=0., DRY=0., DRZ=0.)

Not O	meshes: M1
Point A	meshes: M147
Not B	meshes: M98 M111
Not C	meshes: M14
Not D	meshes: M85 M99
Not E	meshes: M7 M8
Point F	meshes: M91 M92 M105

5.2 Characteristics of the mesh

Many nodes: 169
Number of meshes and types: 147 QUAD4

5.3 Quantities tested and Standard

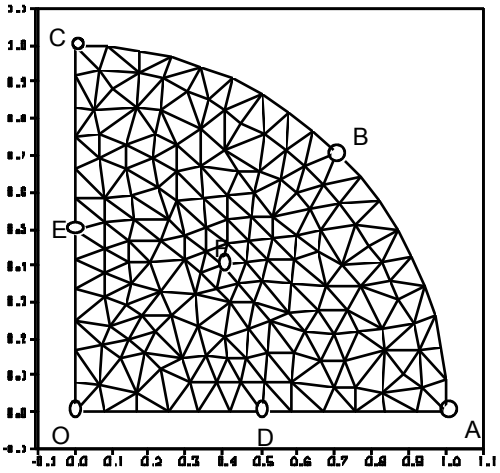
Identification	results of reference	Values of reference Coil-Kirchhoff	Tolerance (%)
" $w(r)$	ANALYTIQUE O"	– 170.6251	0.5
$w(r)$	D'ANALYTIQUE"	– 95.9766	0.5
E $w(r)$	"ANALYTIQUE"	– 95.9766	0.5
F $w(r)$	"ANALYTIQUE"	– 78.897	0.5

Standard		Identification of reference	Values of reference	Tolerance (%)
O	M_{rr}	"ANALYTIQUE"	– 0.08125	0.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.08125	0.5
A	M_{rr}	"ANALYTIQUE"	0.125	0.5
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	0.5
B	M_{rr}	"ANALYTIQUE"	0.08125	0.5
	$M_{\theta\theta}$	"ANALYTIQUE"	0.08125	0.5
	M_{rr}	C'ANALYTIQUE"	0.125	0.5
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	0.5
	M_{rr}	D'ANALYTIQUE"	– 0.02969	2.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.05156	3.5
E	M_{rr}	"ANALYTIQUE"	– 0.02969	2.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.05156	3.5
F	M_{rr}	"ANALYTIQUE"	– 0.02925	1.0
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.02925	1.0

6 Modelization F

6.1 Characteristic of the modelization

Shell element DST (modelization of a quarter of plate)



limiting Conditions:	DDL_IMPO
in all the nodes of the arc ABC	(GROUP_NO= "ABC", DX= 0. , DY= 0. , DZ= 0.) DRX=0., DRY=0., DRZ=0.)
in all the nodes of segment] OA [	(GROUP_NO= "OA", DY= 0. , DRX=0., DRZ=0.)
in all the nodes of segment] OC [	(GROUP_NO= "OC", DX= 0. , DRY=0., DRZ=0.)
with node O	(GROUP_NO= "O", DX= 0. , DY= 0. , DRX=0., DRY=0., DRZ=0.)

Not O	meshes: M1 m2
Not A	meshes: M248 M255
Not B	meshes: M292 M293 M296
Not C	meshes: M74 M75
Not D	meshes: M76 M108 M109
Not E	meshes: M34 M40 M41
Not F	meshes: M122 M123 M124 M148 M152 M153

6.2 Characteristics of the mesh

Many nodes: 170
Number of meshes and types: 296 TRIA3

6.3 Quantities tested and Standard

Identification	results of reference	Values of reference Reissner	Tolerance (%)
O $w(r)$	"ANALYTIQUE"	– 178.419	1.0
$w(r)$	D'ANALYTIQUE"	– 101.82	1.0
E $w(r)$	"ANALYTIQUE"	– 101.82	1.0
F $w(r)$	"ANALYTIQUE"	– 84.198	1.0

Standard		Identification of reference	Values of reference	Tolerance (%)
O	M_{rr}	"ANALYTIQUE"	– 0.08125	1.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.08125	2.0
A	M_{rr}	"ANALYTIQUE"	0.125	0.5
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	23.0
B	M_{rr}	"ANALYTIQUE"	0.08125	2.5
	$M_{\theta\theta}$	"ANALYTIQUE"	0.08125	2.5
	M_{rr}	C'ANALYTIQUE"	0.125	0.5
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	23.0
	M_{rr}	D'ANALYTIQUE"	-0.05156	1.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	5.0
E	M_{rr}	"ANALYTIQUE"	– 0.02969	5.0
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.05156	1.0
F	M_{rr}	"ANALYTIQUE"	–0.02925	1.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.02925	1.5

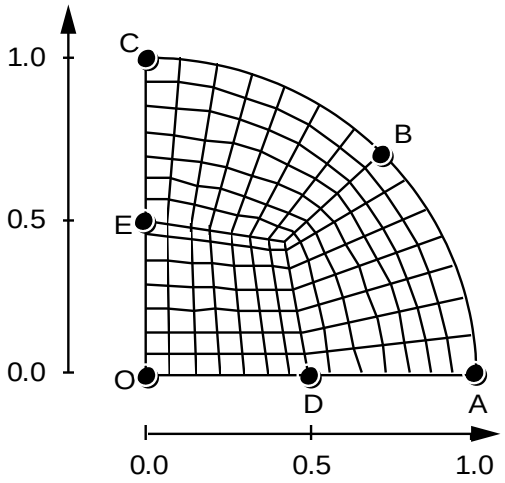
6.4 Contained of the file Values

results at the points of observation of displacements and moments realised.

7 Modelization G

7.1 Characteristic of the modelization

Shell element DSQ (modelization of a quarter of plate)



limiting Conditions:	DDL_IMPO
in all the nodes of the arc ABC	(GROUP_NO= "ABC", DX= 0. , DY= 0. , DZ= 0.) DRX=0., DRY=0., DRZ=0.)
in all the nodes of segment] OA [	(GROUP_NO= "OA", DY= 0. , DRX=0., DRZ=0.)
in all the nodes of segment] OC [	(GROUP_NO= "OC", DX= 0. , DRY=0., DRZ=0.)
with node O	(GROUP_NO= "O", DX= 0. , DY= 0. , DRX=0., DRY=0., DRZ=0.)

Not O	meshes: M1
Point A	meshes: M147
Not B	meshes: M98 M111
Not C	meshes: M14
Not D	meshes: M85 M99
Not E	meshes: M7 M8
Point F	meshes: M91 M92 M105

7.2 Characteristics of the mesh

Many nodes: 169
Number of meshes and types: 147 QUAD4

7.3 Quantities tested and Standard

Identification	results of reference	Values of reference Reissner	Tolerance (%)
O $w(r)$	"ANALYTIQUE"	– 178.419	0.3
$w(r)$	D'ANALYTIQUE"	– 101.82	0.3
E $w(r)$	"ANALYTIQUE"	– 101.82	0.3
F $w(r)$	"ANALYTIQUE"	– 84.198	0.3

Standard		Identification of reference	Values of reference	Tolerance (%)
O	M_{rr}	"ANALYTIQUE"	– 0.08125	0.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.08125	0.5
A	M_{rr}	"ANALYTIQUE"	0.125	2.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	11.0
B	M_{rr}	"ANALYTIQUE"	0.08125	2.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.08125	2.0
	M_{rr}	C'ANALYTIQUE"	0.125	2.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	10.0
	M_{rr}	D'ANALYTIQUE"	– 0.02969	2.5
	$M_{\theta\theta}$	"ANALYTIQUE"	-0.05156	1.5
E	M_{rr}	"ANALYTIQUE"	– 0.02969	2.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.05156	1.5
F	M_{rr}	"ANALYTIQUE"	– 0.02925	18.0
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.02925	18.0

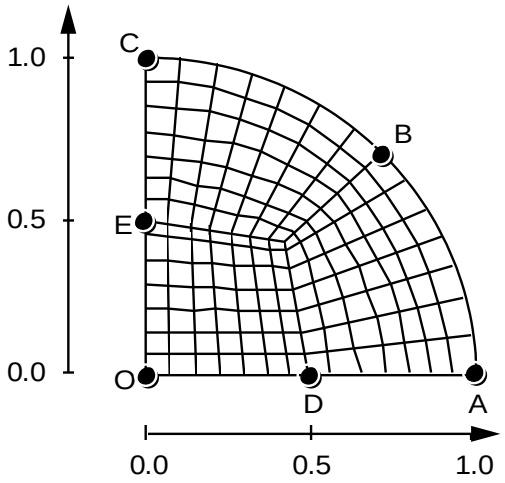
7.4 Contents of the file Values

results at the points of observation of displacements and moments realised.

8 Modelization H

8.1 Characteristic of the modelization

Shell element Q4G (modelization of a quarter of plate)



limiting Conditions: DDL_IMPO

in all the nodes of the arc ABC	(GROUP_NO= "ABC", DX= 0. , DY= 0. , DZ= 0.) DRX=0., DRY=0., DRZ=0.)
in all the nodes of segment] OA [	(GROUP_NO= "OA", DY= 0. , DRX=0., DRZ=0.)
in all the nodes of segment] OC [	(GROUP_NO= "OC", DX= 0. , DRY=0., DRZ=0.)
with node O	(GROUP_NO= "O" DX= 0. , DY= 0. , DRX=0., DRY=0., DRZ=0.)

Not O	meshes: M1
Point A	meshes: M147
Not B	meshes: M98 M111
Not C	meshes: M14
Not D	meshes: M85 M99
Not E	meshes: M7 M8
Point F	meshes: M91 M92 M105

8.2 Characteristics of the mesh

Many nodes: 169
Number of meshes and types: 147 QUAD4

8.3 Quantities tested and Standard

Identification	results of reference	Values of reference Reissner	Tolerance (%)
O $w(r)$	"ANALYTIQUE"	– 178.419	0.4
$w(r)$	D'ANALYTIQUE"	– 101.82	0.4
E $w(r)$	"ANALYTIQUE"	– 101.82	0.4
F $w(r)$	"ANALYTIQUE"	– 84.198	0.4

Standard		Identification of reference	Values of reference	Tolerance (%)
O	M_{rr}	"ANALYTIQUE"	– 0.08125	0.1
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.08125	0.1
A	M_{rr}	"ANALYTIQUE"	0.125	11.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	11.0
B	M_{rr}	"ANALYTIQUE"	0.08125	11.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.08125	11.0
	M_{rr}	C'ANALYTIQUE"	0.125	11.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	11.0
	M_{rr}	D'ANALYTIQUE"	– 0.02969	0.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.05156	1.5
E	M_{rr}	"ANALYTIQUE"	– 0.02969	0.5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.05156	1.5
F	M_{rr}	"ANALYTIQUE"	– 0.02925	1.0
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.02925	1.0

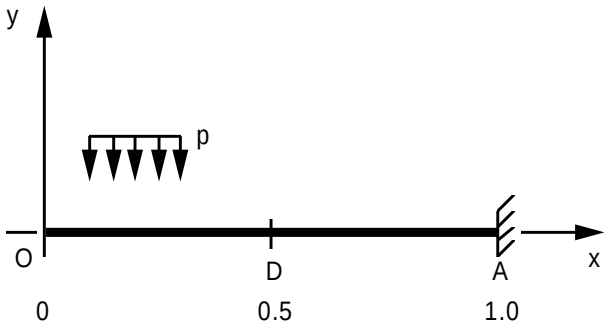
8.4 Contents of the file Values

results at the points of observation of displacements and moments realised.

9 Modelization I

9.1 Characteristic of the axisymmetric

modelization Shell element SEG3, in theory of Coils-Kirchhoff: one does not consider modification of metric, coefficient A_CIS is worth 106.



Limiting conditions:

```
DDL_IMPO= (_F
           (NOEUD= "A", DX: 0. , DY: 0. , DRZ: 0.)
           (NOEUD= "O", DRZ: 0.))
```

9.2 Characteristics of the mesh

Many nodes: 21
Number of meshes and types: 10 SEG3

9.3 Quantities tested and Standard

Identification	results of reference	Values of reference Coil-Kirchhoff	Tolerance (%)
O deflection $w(r)$	"ANALYTIQUE"	– 170.6251	0.6
D deflection $w(r)$	"ANALYTIQUE"	– 95.9765	1.0
D rotation $\beta(r)$	"ANALYTIQUE"	255.940	0.6

Standard			Identification of reference	Values of reference	Tolerance (%)
Not	Component	Mesh			
D	IJK	K^{rr}	"ANALYTIQUE"	170.625	66.
		$K^{\theta\theta}$	"ANALYTIQUE"	511.875	0.5
	KLM	K^{rr}	"ANALYTIQUE"	170.625	60
		$K^{\theta\theta}$	"ANALYTIQUE"	511.875	0.5

Standard			Identification of reference	Values of reference	Tolerance (%)
Not	Component	Mesh			
O	STU	M^{rr}	"ANALYTIQUE"	– 0.08125	0.5
		$M^{\theta\theta}$	"ANALYTIQUE"	– 0.08125	0.5
A	ABC	M^{rr}	"ANALYTIQUE"	0.125	15.0
		$M^{\theta\theta}$	"ANALYTIQUE"	0.0375	15.0
D	IJK	$M^{\theta\theta}$	"ANALYTIQUE"	– 0.05156	6.0
	KLM	$M^{\theta\theta}$	"ANALYTIQUE"	– 0.05156	6.0

Note:

One notes the good performances obtained, except on K^{rr} and M^{rr} , which utilize derivatives of a higher nature less better calculated by the element.

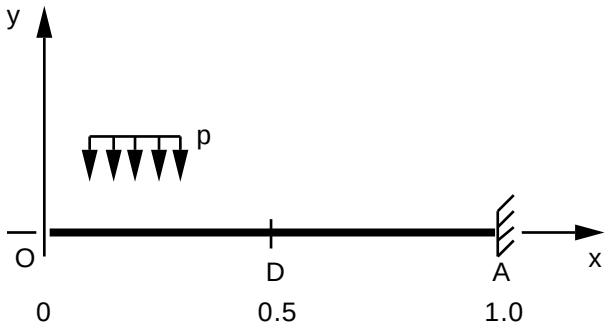
9.4 Contents of the file generalized

Displacements results, strains and forces and nodal stresses.

10 Modelization J

10.1 Characteristic of the axisymmetric

modelization Shell element SEG3, in theory of Mindlin-Reissner: one does not consider modification of metric, coefficient A_CIS is worth $5/6$.



Limiting conditions:

```
DDL_IMPO= (_F
       (NOEUD= "A",   DX= 0. , DY= 0. , DRZ= 0.)
       (NOEUD= "O",                    DRZ= 0.))
```

10.2 Characteristics of the mesh

Many nodes: 21
Number of meshes and types: 10 SEG3

10.3 Quantities tested and Standard

Identification	results of reference	Values of reference Coil-Kirchhoff	Tolerance (%)
O deflection $w(r)$	"ANALYTIQUE"	– 178.424	0.5
D deflection $w(r)$	"ANALYTIQUE"	– 101.827	0.5
D rotation $\beta(r)$	"ANALYTIQUE"	255.940	0.5

Standard			Identification of reference	Values of reference	Tolerance (%)
Not	Component	Mesh			
D	IJK	K^{rr}	"ANALYTIQUE"	170.625	5.
		$K^{\theta\theta}$	"ANALYTIQUE"	511.875	0.5
	KLM	K^{rr}	"ANALYTIQUE"	170.625	5.
		$K^{\theta\theta}$	"ANALYTIQUE"	511.875	0.5

Standard			Identification of reference	Values of reference	Tolerance (%)
Not	Component	Mesh			
O	STU	M^{rr}	"ANALYTIQUE"	– 0.08125	1.
		$M^{\theta\theta}$	"ANALYTIQUE"	– 0.08125	1.
A	ABC	M^{rr}	"ANALYTIQUE"	0.125	2.
		$M^{\theta\theta}$	"ANALYTIQUE"	0.0375	2.
A	IJK	$M^{\theta\theta}$	"ANALYTIQUE"	– 0.05156	0.5
	KLM	$M^{\theta\theta}$	"ANALYTIQUE"	– 0.05156	0.7

Note:

One notes the good performances obtained, except on K^{rr} and M^{rr} , which utilize derivatives of a higher nature less better calculated by the element.

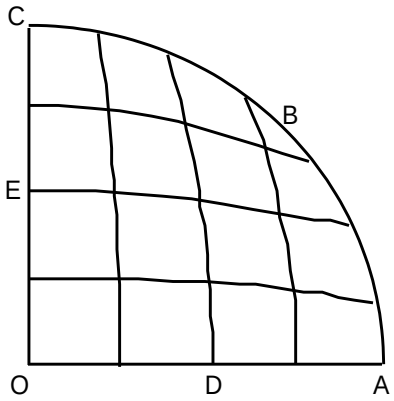
10.4 Contents of the file generalized

Displacements results, strains and forces and nodal stresses.

11 Modelization K

11.1 Characteristic of the modelization

Modelization: Shell element COQUE_3D MEC3QU9H
Number of layer: Limiting COQUE_NCOU =



3 Conditions:

	DDL_IMPO
in all the nodes of L" arc ABC	(GROUP_NO= "ABC", DX= 0. , DY= 0. , DZ= 0.) DRX=0., DRY=0., DRZ=0.)
segment $]OA]$	(GROUP_NO= "OA", DY= 0. , DRX=0., DRZ=0.)
segment $]OC]$	(GROUP_NO= "OC", DX= 0. , DRY=0., DRZ=0.)
with node O	(GROUP_NO= "O", DX= 0. , DY= 0. , DRX=0., DRY=0., DRZ=0.)

Names of the nodes:

Point 0	meshes: M1	Point A	meshes: M21
Not B	meshes: M25	Not C	meshes: M5
Point D	meshes: M11	Not E	meshes: M3

11.2 Characteristic of the mesh

Many nodes: 96
Number of meshes and types: 25 QUAD9

11.3 Quantities tested and Standard

Identification	results of reference	Values of reference REISSNER	Tolerance (%)
O deflection $w(r)$	"ANALYTIQUE"	– 178.419	0.5
D deflection $w(r)$	"ANALYTIQUE"	– 101.82	0.5
E deflection $w(r)$	"ANALYTIQUE"	– 101.82	0.5

Standard		Identification of reference	Values of reference	Tolerance (%)
O	M^{rr}	"ANALYTIQUE"	– 0.08125	2.9
	$M^{\theta\theta}$	"ANALYTIQUE"	– 0.08125	2.9
A	M^{rr}	"ANALYTIQUE"	0.125	2.0
	$M^{\theta\theta}$	"ANALYTIQUE"	0.0375	2.0
	M^{rr}	C'ANALYTIQUE"	+0.125	1.0
	$M^{\theta\theta}$	"ANALYTIQUE"	+0.0375	5.0
	M^{rr}	D'ANALYTIQUE"	– 0.02969	1.0
	$M^{\theta\theta}$	"ANALYTIQUE"	– 0.05156	2.0
E	M^{rr}	"ANALYTIQUE"	– 0.02969	1.0
	$M^{\theta\theta}$	"ANALYTIQUE"	– 0.05156	2.0

Note:

The test of the values is carried out automatically using the features offered by procedure POST_RELEVE :

- *extraction on the nodes corresponding to the points observed of the mean values of the components M_{xx} and M_{yy} ; these values are extracted from field "EFGE_ELNO" , and the average is calculated for all the liquid assets on meshes which contain the node observed,*
- *computation of the variation compared to the value of reference provided by observing the rules of correspondence enters M_{xx} , M_{yy} and M_{rr} , $M_{\theta\theta}$ given page 3.*

Contents of the file Values

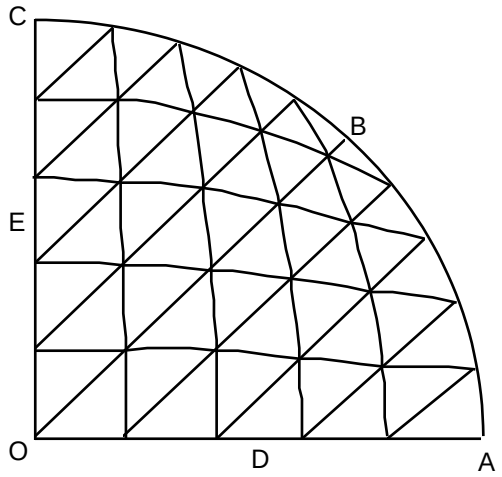
results at the points of observation of displacements and moments realised.

One also tests the continuity of fields EPSI_ELGA and EPSI_ELNO between the layers (tests of NON-regression). Indeed the computed value on level SUP of the layer N is equal to the computed value on level INF of the COUCHE N+1.

12 Modelization L

12.1 Characteristic of the modelization

Modelization: Element of COQUE_3D MEC3TR7H
Number of layer: Limiting COQUE_NCOU =



3 Conditions:

		DDL_IMPO
in all the nodes of the arc ABC	(GROUP_NO= "ABC",	DX= 0. , DY= 0. , DZ= 0.) DRX=0., DRY=0., DRZ=0.)
segment] OA]	(GROUP_NO= "OA",	DY= 0. , DRX=0., DRZ=0.)
segment] OC]	(GROUP_NO= "OC",	DX= 0. , DRY=0., DRZ=0.)
with node O	(GROUP_NO= "O"	DX= 0. , DY= 0. , DRX=0., DRY=0., DRZ=0.)

Names of the nodes:

Point 0	meshes: M1 and m2	Not A	meshes: M41
Not B	meshes: M49 and M50	Not C	meshes: M10
Not D	meshes: M21	Not E	meshes: M6

12.2 Caractéristiques of the mesh

Many nodes: 121
Number of meshes and types: 50 TRIA7

12.3 Quantities tested and Standard

Identification	results of reference	Values of reference REISSNER	Tolerance (%)
O deflection $w(r)$	"ANALYTIQUE"	– 178.419	0.5
D deflection $w(r)$	"ANALYTIQUE"	– 101.82	0.5
E deflection $w(r)$	"ANALYTIQUE"	– 101.82	0.5

Standard		Identification of reference	Values of reference	Tolerance (%)
O	M^{rr}	"ANALYTIQUE"	– 0.08125	2.9
	$M^{\theta\theta}$	"ANALYTIQUE"	– 0.08125	2.9
A	M^{rr}	"ANALYTIQUE"	0.125	1.7
	$M^{\theta\theta}$	"ANALYTIQUE"	0.0375	0.3
	M^{rr}	C'ANALYTIQUE"	+0.125	2.0
	$M^{\theta\theta}$	"ANALYTIQUE"	+0.0375	2.0
	M^{rr}	D'ANALYTIQUE"	– 0.02969	3.0
	$M^{\theta\theta}$	"ANALYTIQUE"	– 0.05156	1.0
E	M^{rr}	"ANALYTIQUE"	– 0.02969	3.0
	$M^{\theta\theta}$	"ANALYTIQUE"	– 0.05156	1.0

Note:

The test of the values is carried out automatically using the features offered by procedure POST_RELEVE :

- *extraction on the nodes corresponding to the points observed of the mean values of the components M_{xx} and M_{yy} ; these values are extracted from field "EFGE_ELNO" , and the average is calculated for all the liquid assets on meshes which contain the node observed,*
- *computation of the variation compared to the value of reference provided by observing the rules of correspondence enters M_{xx} , M_{yy} and M_{rr} , $M_{\theta\theta}$ given page 3.*

Contents of the file Values

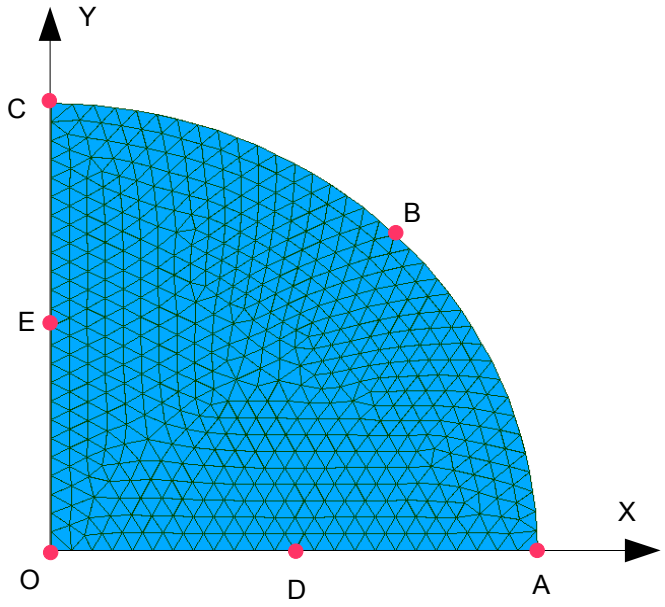
results at the points of observation of displacements and moments realised.

One also tests the continuity of fields EPSI_ELGA and EPSI_ELNO between the layers (tests of NON-regression). Indeed the computed value on level SUP of the layer N is equal to the computed value on level INF of the COUCHE N+1.

13 Modelization M

13.1 Characteristic of the modelization

Shell element T3G (modelization of a quarter of plate)



limiting Conditions:

in all the nodes of the arc ABC	DDL_IMPO (GROUP_NO= "ABC", DX= 0. , DY= 0. , DZ= 0. DRX=0., DRY=0., DRZ=0.)
in all the nodes of segment] OA [	(GROUP_NO= "OA", DY= 0. , DRX=0., DRZ=0.)
in all the nodes of segment] OC [	(GROUP_NO= "OC", DX= 0. , DRY=0., DRZ=0.)
with node O	(GROUP_NO= "O", DX= 0. , DY= 0. , DRX=0., DRY=0., DRZ=0.)

Not O	meshes: M31
Not A	meshes: M19
Not B	meshes: M10 M100 M54
Not C	meshes: M1
Point D	meshes: M71 M113 M25
Not E	meshes: M37 M123 M86

13.2 Characteristics of the mesh

Many nodes: 561
Number of meshes and types: 1036 TRIA3

13.3 Quantities tested and Standard

Identification	results of reference	Values of reference Reissner	Tolerance (%)
O $w(r)$	"ANALYTIQUE"	– 178.419	0.5
D $w(r)$	"ANALYTIQUE"	– 101.82	1.0
E $w(r)$	"ANALYTIQUE"	– 101.82	1.0

Standard		Identification of reference	Values of reference	Toléranceformule (%)
O	M_{rr}	"ANALYTIQUE"	– 0.08125	1.0
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.08125	1.0
A	M_{rr}	"ANALYTIQUE"	0.125	7.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	7.0
B	M_{rr}	"ANALYTIQUE"	0.08125	8.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.08125	8.0
C	M_{rr}	"ANALYTIQUE"	0.125	6.0
	$M_{\theta\theta}$	"ANALYTIQUE"	0.0375	6.5
	M_{rr}	D'ANALYTIQUE"	– 0.02969	1.0
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.05156	0.5
E	M_{rr}	"ANALYTIQUE"	– 0.02969	0,5
	$M_{\theta\theta}$	"ANALYTIQUE"	– 0.05156	1.

14 Summary of the %

results of the differences compared to the reference solutions

Mode.	DKT A Coil Kirchhoff B		DKQ E	DST F	DSQ G	Q4G H
Point	50 nodes 76 TRIA3	170 nodes 296 TRIA3	Coils-Kirchhoff 169 nodes 147 QUAD4	Reissner 170 nodes 296 TRIA3	Reissner 169 nodes 147 QUAD4	Reissner 169 nodes 147 QUAD4
O $w(r)$	– 0.76	+0.12	+0.22	+0.74	+0.19	– 0.08
D $w(r)$	– 0.23	+0.18	+0.23	+0.77	+0.19	– 0.28
E $w(r)$	– 0.25	+0.24	+0.23	+0.84	+0.19	– 0.28
F $w(r)$	– 0.32	+0.22	+0.20	+0.75	+0.14	– 0.34

Mode.	Q4G M Reissner
Point	561 nodes - 1036 TRIA3
O $w(r)$	+0.176
D $w(r)$	+0.225
E $w(r)$	+0.225
F $w(r)$	-

Mode.	COQU_AXIS I Coil-Kirchhoff	Reissner J	MEC3QU9H K	MEC3TR7H L
Point	21 nodes 10 SEG3		96 nodes 25 QUAD9	121 nodes 50 TRIA7
O $w(r)$	+0.51	0.03	– 0.16	– 0.13
D $w(r)$	+0.28	0.05	– 0.029	– 0.35
E $w(r)$	-	-	– 0.029	– 0.35
F $w(r)$	-	+0.22	-	-

Mode.	DKT A Coil Kirchhoff B		DKQ E	DST F	DSQ G	Q4G H
Point	50 nodes 76 TRIA3	170 nodes 296 TRIA3	Coils-Kirchhoff 169 nodes 147 QUAD4	Reissner 170 nodes 296 TRIA3	Reissner 169 nodes 147 QUAD4	Reissner 169 nodes 147 QUAD4
O $Sm/2$	– 1.15	+0.19	+0.46	+1.04	– 0.33	– 0.07
A $Sm/2$	+0.81	+4.02	+0.49	+5.26	+3.79	– 10.73
B $Sm/2$	+4.58	+2.64	+0.20	+2.02	+1.69	– 10.95
Channel + $Sm/2$	0.75	+4.13	+0.45	+5.34	+3.64	– 10.69
D $Sm/2$	+4.55	+1.99	+2.71	+2.07	+0.40	+0.74
E $Sm/2$	+4.55	+2.19	+2.71	+2.29	+0.40	+0.74
F $Sm/2$	+1.71	+2.05	– 0.79	+1.19	+17.80	– 0.94

Mode.	Q4G M Reissner
Point	561 nodes - 1036 TRIA3
O $Sm/2$	+0.265
A $Sm/2$	+1.65
B $Sm/2$	+4.95
Channel + $Sm/2$	1.73
D $Sm/2$	+5.18
E $Sm/2$	+5.50
F $Sm/2$	+4.07

	COQU_AXIS		MEC3QU9H	MEC3TR7H
Mode.	I	Reissner	K	L
	Coil-Kirchhoff	J		
Point	21 nodes 10 SEG3		96 nodes 25 QUAD9	121 nodes 50 TRIA7
O $Sm/2$	+0.18	+0.62	2.67	2.89
A $Sm/2$	+14.2	– 1.01	– 1.45	– 1.33
B $Sm/2$	-	-	-	-
C $Sm/2$	-	-	– 1.46	– 1.32
D $Sm/2$	+0.84	– 0.85	1.08	– 1.23
E $Sm/2$	-	-	0.95	– 1.23
F $Sm/2$	-	-	-	-

Note::

Concerning the forces, direct computation with the nodes leads to variations in several nodes, in particular at the point F in DSQ and on edge ABC in Q4G .