SSLP321 - The purpose of propagation of a crack X-FEM in a flexbeam 3 points

Summarized

This test is validating the way of crack propagation with $\mathrm{X\text{-}FEM}$ in $\mathrm{2D}$, in the frame of linear elasticity.

This test brings into play a rectangular plate with a crack emerging, and subjected to a bending 3 points as in the article of Mariani and Perego [1].

Four methods to manage the propagation of cracks $\mathrm{X\text{-}FEM}$ are available. Each one of them is the object of a modelization.

- modelization A : method mesh
- modelization B : method simplex
- modelization C : method upwind
- modelization D : geometrical method

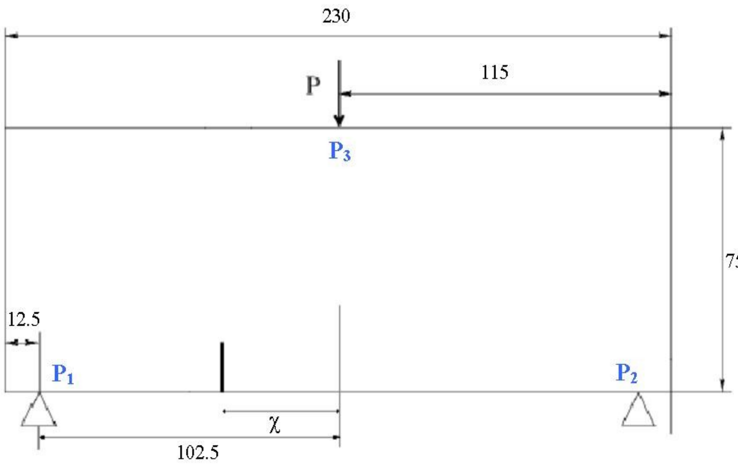
the comparisons are done with the values given by the method mesh.

1 Problem of reference

1.1 Geometry

the geometry, dimensions and the materials are taken identical to those of Mariani and al. **[1]**.
The structure $2D$ is a rectangular plate ($230\,mm\times 75\,mm$), comprising an emerging crack [Appear 1.1-a]. The length of initial crack is $a=19\,mm$.

The noted nodes $P1$, $P2$ and $P3$ are used to impose the boundary conditions, which are clarified in the paragraph [§1.31.3].



Appear 1.1-a: geometry of the fissured plate

1.2 Properties of the material

Modulus Young: $E=31,37\,10^{9}\,Pa$
Poisson's ratio: $\nu=0.2$

1.3 Boundary conditions and loadings

the loading consists in applying a unit nodal force in $P3$.

In order to block the rigid modes, one blocks displacements of the nodes $P1$ and $P2$ as follows:

- $DY^{P1}=DY^{P2}=0$;
- $DX^{P1}=0$.

1.4 Reference solution

the study of this case is based entirely on the article of Mariani and Perego. Three initial crack configurations are selected: $\chi=0,25$ and 50 . In this case test, we chose only $\chi=50$. One thus compares the way of propagation compared to the experimental way of the article **[1]**.

The statements of reference of the stress intensity factors K_I and K_{II} are those of the method mesh. One will thus compare the values of the methods simplex, upwind and geometrical with the values given by the method mesh.

For the propagation of crack, we use the model of Paris:

$\frac{da}{dN}=C\,\Delta K^m$ where has is the length of crack, C and m are constants of the material, ΔK is the difference between two $FICs$ consecutive and N is the number of cycles.

The criterion of bifurcation used is *the maximum hoop stress criterion*:

$$\beta=2\arctan[\frac{1}{4}(\frac{K_I}{K_{II}}-sign(K_{II})\sqrt{(\frac{K_I}{K_{II}})^2+8})]$$

With the numerical values of the test:

No propagation: $0{,}3\,m$
$x_0: 65\,mm$
$y_0: 19\,mm$
Many steps of propagation: 13
$RI: 3\,mm$
$RS: 12\,mm$
$RP: 12\,mm$

Reference (method mesh)			
$x(mm)$	$y(mm)$	$K_I(MPa.m^{0,5})$	$K_{II}(MPa.m^{0,5})$
65	19	2,43961 10-1	4,27722 10-2
66,129	22,313	2,90147 10-1	1,21013 10-4
67,261	25,625	3,30840 10-1	7,10255 10-3
68,533	28,885	3,75984 10-1	1,94683 10-3
69,839	32,132	4,33606 10-1	1,20266 10-3
71,164	35,372	4,96975 10-1	8,82542 10-4
72,5	38,607	5,73785 10-1	-1,23199 10-3
73,821	41,848	6,70222 10-1	-3,54655 10-3
75,109	45,103	7,89716 10-1	-4,54122 10-3
76,359	48,372	9,39463 10-1	-8,18030 10-3
77,552	51,662	1,15201	-1,55772 10-2
78,655	54,984	1,45163	-2,31849 10-2
79,652	58,339	1,91885	-3,52229 10-2

Table 1.4-1 : values of reference de for K_I and K_{II}

1.5 bibliographical References

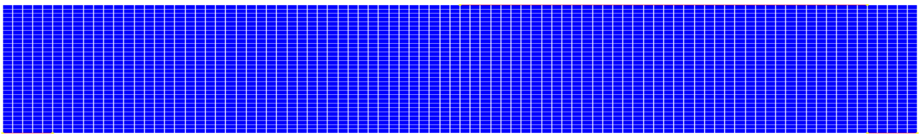
1 **Mariani S, Perego U** – Extended finite element method for quasi-brittle fracture, *International Newspaper for numerical methods in engineering*, 58:103-126 **(2003)**

2 Modelization a: Method mesh

In this modelization, the method mesh is tested for the crack propagation. The level-sets are determined by orthogonal projection on the segments composing crack.

2.1 Characteristics of the mesh

the structure is modelled by a regular mesh composed of 90×30 QUAD4, respectively along the axes x y. The crack is not with a grid.



Appear 2.1-a : mesh of the fissured plate

2.2 Quantities tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G.

2.2.1 Results on K_I :

One carries out a relative test of non regression on K_I with an accuracy of 2.10^{-3} .

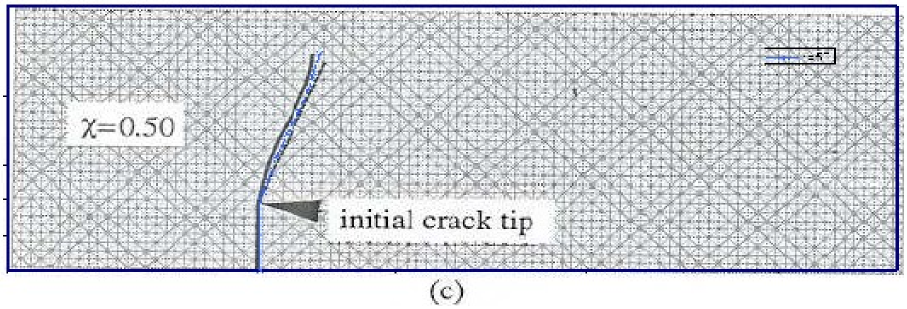
Identification	*Code_Aster*	Reference	difference
CALC_G			
KI_1	2,43961 10-1	2,43961 10-1	-
KI_2	2,90147 10-1	2,90147 10-1	-
KI_3	3,30840 10-1	3,30840 10-1	-
KI_4	3,75984 10-1	3,75984 10-1	-
KI_5	4,33606 10-1	4,33606 10-1	-
KI_6	4,96975 10-1	4,96975 10-1	-
KI_7	5,73785 10-1	5,73785 10-1	-
KI_8	6,70222 10-1	6,70222 10-1	-
KI_9	7,89716 10-1	7,89716 10-1	-
KI_10	9,39463 10-1	9,39463 10-1	-
KI_11	1,15201	1,15201	-
KI_12	1,45163	1,45163	-
KI_13	1,91885	1,91885	-

2.2.2 Results on K_{II} :

For this test, it is wished that K_{II} be such as $K_{II}=K_{IIref}\pm 10^{-2}$.

Identification	*Code_Aster*	Reference	Difference
CALC_G			
KII_1	4,27722 10-2	4,27722 10-2	-
KII_2	1,21013 10-4	1,21013 10-4	-
KII_3	7,10255 10-3	7,10255 10-3	-
KII_4	1,94683 10-3	1,94683 10-3	-
KII_5	1,20266 10-3	1,20266 10-3	-
KII_6	8,82542 10-4	8,82542 10-4	-
KII_7	-1,23199 10-3	-1,23199 10-3	-
KII_8	-3,54655 10-3	-3,54655 10-3	-
KII_9	-4,54122 10-3	-4,54122 10-3	-
KII_10	-8,18030 10-3	-8,18030 10-3	-
KII_11	-1,55772 10-2	-1,55772 10-2	-
KII_12	-2,31849 10-2	-2,31849 10-2	-
KII_13	-3,52229 10-2	-3,52229 10-2	-

2.3 complementary Results



Appears 2.3-a: Comparison between the way obtained and the method mesh with the ways of the study of Mariani and Perego

On Appears 2.3-a, one can see: in black numerical results of Mariani and Perego, in dotted lines experimental results and blue results of the method mesh of Code_Aster. The method mesh gives results close to the experimental data.

3 Modelization b: Method simplex

In this modelization, the method simplex is tested for the crack propagation.
The level-sets are determined by resolution of the equations of reactualization.
One uses here the same mesh as in the modelization A (§2.1).

3.1 Quantities tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} data by `CALC_G`.

3.1.1 Results on KI:

One carries out a relative test of non regression on K_I compared to $K_{I\,maillage}$ with an accuracy of 5% .

Identification	*Code_Aster*	Reference	Difference ($\%$)
CALC_G			
KI_1	2,43961 10-1	2,43961 10-1	-2,03 10 6%
KI_2	2.92165 10-1	2,90147 10-1	0,7%
KI_3	3.28560 10-1	3,30840 10-1	0,7%
KI_4	3.77477 10-1	3,75984 10-1	0,4%
KI_5	4.30381 10-1	4,33606 10-1	-0,74%
KI_6	4.91730 10-1	4,96975 10-1	-1,05%
KI_7	5.70274 10-1	5,73785 10-1	-0,61%
KI_8	6.72226 10-1	6,70222 10-1	0,3%
KI_9	7.92798 10-1	7,89716 10-1	0,40%
KI_10	9.53757 10-1	9,39463 10-1	1,52%
KI_11	1.18351	1,15201	2,7%
KI_12	1.91525	1,45163	1,43%
KI_13	1.91525	1,91885	-0,2%

3.1.2 Results on K_{II} :

For this test, it is wished that K_{II} be such as $K_{II}=K_{IIref}\pm 5.10^{-2}$. (absolute test)

Identification	*Code_Aster*	Reference	Difference
CALC_G			
KII_1	4,27722 10-2	4,27722 10-2	-3,92 10-8
KII_2	-1.20651 10-2	1,21013 10-4	-1,2 10-2
KII_3	2.50981 10-2	7,10255 10-3	1,8 10-2
KII_4	-1.27415 10-2	1,94683 10-3	-1,5 10-2
KII_5	1.71501 10-2	1,20266 10-3	1,6 10-2
KII_6	-7.31099 10-3	8,82542 10-4	-8 10-3
KII_7	1.17880 10-3	-1,23199 10-3	2 10-3
KII_8	-1.14348 10-2	-3,54655 10-3	-8 10-3
KII_9	-1.65167 10-2	-4,54122 10-3	4 10-3
KII_10	2.30598 10-2	-8,18030 10-3	-1,5 10-2
KII_11	-1.23573 10-2	-1,55772 10-2	3 10-3
KII_12	2.55864 10-2	-2,31849 10-2	4,9 10-2
KII_13	-3.94340 10-2	-3,52229 10-2	-4 10-3

4 Modelization C: Method upwind

In this modelization, the method upwind is tested for the crack propagation. The level-sets are determined by resolution of the equations of reactualization per diagram to the finite differences. One uses here the same mesh as in the modelization A (§2.1).

4.1 Quantities tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G.

4.1.1 Results on K_I :

One carries out a relative test of non regression on K_I compared to $K_{I\,maillage}$ with an accuracy of 3%.

Identification	*Code_Aster*	Reference	Difference ($\%$)
CALC_G			
KI_1	2,43961 10-1	2,43961 10-1	-2,03 10 $^{-6\%}$
KI_2	2,93562 10-1	2,90147 10-1	1,18%
KI_3	3,30633 10-1	3,30840 10-1	-0,06%
KI_4	3,78989 10-1	3,75984 10-1	0,80%
KI_5	4,35284 10-1	4,33606 10-1	0,39%
KI_6	4,96344 10-1	4,96975 10-1	-0,13%
KI_7	5,76792 10-1	5,73785 10-1	0,52%
KI_8	6,7717 10-1	6,70222 10-1	1,04%
KI_9	8,01923 10-1	7,89716 10-1	1,55%
KI_10	9,5467 10-1	9,39463 10-1	1,62%
KI_11	1,15267	1,15201	0,06%
KI_12	1,49437	1,45163	2,94%
KI_13	1,97462	1,91885	2,91%

4.1.2 Results on K_{II} :

For this test, it is wished that K_{II} be such as $K_{II}=K_{IIref}\pm 3.10^{-2}$. (absolute test)

Identification	*Code_Aster*	Reference	Difference
CALC_G			
KII_1	4,27722 10-2	4,27722 10-2	-3,92 10-8
KII_2	-1,51008 10-2	1,21013 10-4	-1,50 10-2
KII_3	1,9837710-2	7,10255 10-3	1,30 10-2
KII_4	3,96386 10-3	1,94683 10-3	2 10-3
KII_5	-7,55701 10-4	1,20266 10-3	-2 10-3
KII_6	5,18582 10-4	8,82542 10-4	-3,64 10-4
KII_7	-1,29680 10-2	-1,23199 10-3	-1,2 10-2
KII_8	7,94729 10-3	-3,54655 10-3	1,1 10-2
KII_9	7,13366 10-3	-4,54122 10-3	1,2 10-2
KII_10	-1,03437 10-2	-8,18030 10-3	-2 10-3
KII_11	-3,26290 10-2	-1,55772 10-2	-1,7 10-2
KII_12	-7,04819 10-3	-2,31849 10-2	1,6 10-2
KII_13	-4,36313 10-2	-3,52229 10-2	-8 10-3

5 Modelization D: Geometrical method

In this modelization, the geometrical method is tested for the crack propagation. The level-sets are recomputed with each step of propagation. One uses here the same mesh as in the modelization A (§2.1).

5.1 Quantities tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} given by CALC_G.

5.1.1 Results on K_I :

One carries out a relative test of non regression on K_I compared to $K_{I\,maillage}$ with an accuracy of 3% .

Identification	*Code_Aster*	Reference	Difference ($\%$)
CALC_G			
KI_1	2,4396 10-1	2,43961 10-1	-2,03 10 $^{-4\%}$
KI_2	2,9026 10-1	2,90147 10-1	0,04%
KI_3	3,3057 10-1	3,30840 10-1	0,08%
KI_4	3,7665 10-1	3,75984 10-1	0,18%
KI_5	4,3352 10-1	4,33606 10-1	0,01%
KI_6	4,966710-1	4,96975 10-1	0,06%
KI_7	5,7348 10-1	5,73785 10-1	0,05%
KI_8	6,7175 10-1	6,70222 10-1	0,23%
KI_9	7,8989 10-1	7,89716 10-1	0,02%
KI_10	9,3925 10-1	9,39463 10-1	0,02%
KI_11	1,15158	1,15201	0,04%
KI_12	1,45290	1,45163	0,09%
KI_13	1,92063	1,91885	0,09%

5.1.2 Results on K_{II} :

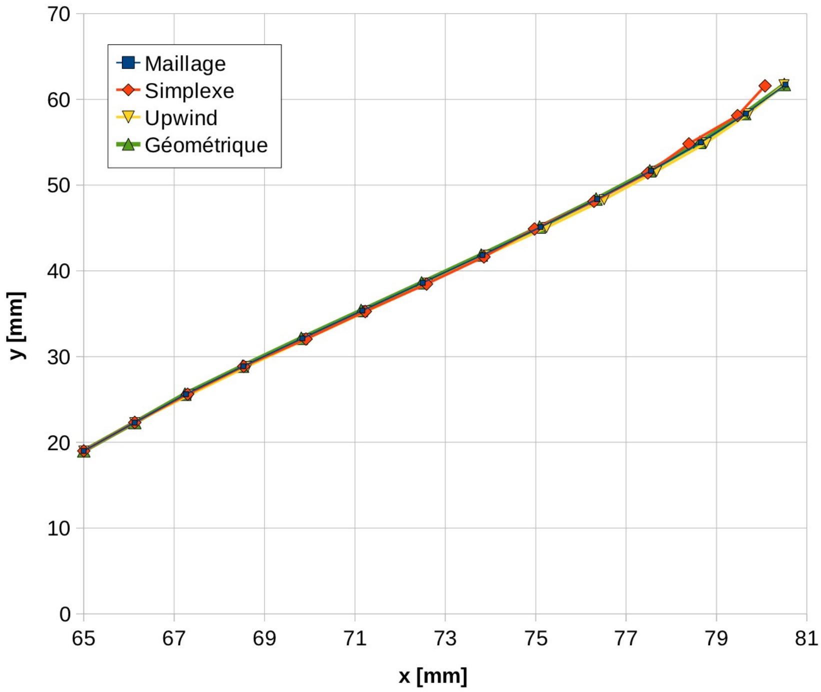
For this test, it is wished that K_{II} be such as $K_{II}=K_{IIref}\pm 3.10^{-2}$ (test in absolute).

Identification	*Code_Aster*	Reference	Difference %
CALC_G			
KII_1	4,27721 10-2	4,27722 10-2	3,92 10-8
KII_2	5,49728 10-5	1,21013 10-4	6,6 10-5
KII_3	8,31292 10-3	7,10255 10-3	1,21 10-3
KII_4	1,41042 10-3	1,94683 10-3	5,36 10-4
KII_5	1,93652 10-4	1,20266 10-3	7,34 10-4
KII_6	7,04563 10-4	8,82542 10-4	1,78 10-4
KII_7	0,0	-1,23199 10-3	1,23 10-3
KII_8	0,0	-3,54655 10-3	3,55 10-3
KII_9	0,0	-4,54122 10-3	4,54 10-3
KII_10	0,0	-8,18030 10-3	8,18 10-3
KII_11	0,0	-1,55772 10-2	1,56
KII_12	0,0	-2,31849 10-2	2,32
KII_13	0,0	-3,52229 10-2	3,52

6 Summaries of the Comparison

6.1 results of the methods

One can compare the ways which the four methods give (mesh, geometrical simplex, upwind and):



Appear 6.1-a: Comparison between the ways of the four methods and Code_Aster

the four methods give all the same path of propagation, which is very close with the experimental data (Appears 2.3-a).

Only the solution calculated by the method simplex shows a small variation towards the end of simulation compared to the others three solutions.

6.2 Performance of the methods

One can compare the computing time for the same number of steps of propagation (13) of the four methods. For the methods simplex and upwind, one checked the performance by means of or not the restriction of the field of computation (respectively `ZONE_MAJ=' TORE'` or `ZONE_MAJ=' TOUT'`). It is noticed that the restriction of the field makes it possible to strongly reduce the computing time of these two methods and to return the performance of the methods mesh, comparable simplex and upwind. However, the geometrical method is undoubtedly the most powerful method.

Mesh	Method	Time (S)
quadrangles	Mesh	145
	Simplex	280 (`ZONE_MAJ=' TOUT'`)
		131 (`ZONE_MAJ=' TORE'`)

	Upwind	200 (ZONE_MAJ=' TOUT')
		150 (ZONE_MAJ=' TORE')
	Geometrical	24

6.3 Conclusion

the purposes of this test are reached:

- To validate on a case simple the computation of the stress intensity factors in mode I for the elements X-FEM for the four methods mesh, simplex, upwind and geometrical.
- If the restriction of the field is used, the performance of the three methods mesh, simplex and upwind is comparable. However, in all the cases, the geometrical method is much more powerful.