SSLP320 - the purpose of Propagation of an emerging crack X-FEM solicited in Mode I

Summarized

This test is validating the computation of the stress intensity factors (K_I and K_{II}) and the way of crack propagation with X-FEM in $2\mathrm{D}$, in the frame of linear elasticity.

This test brings into play a rectangular plate with a crack leading, and subjected to a loading of tension to the edges inferior and superior of the plate.

Four methods to manage the propagation of cracks X-FEM are available. Each one of them is the object of a modelization.

Three modelizations are considered:
- modelization a: method MAILLAGE,
- modelization b: method SIMPLEXE,
- modelization C: method UPWIND,
- modelization D: method GEOMETRIQUE.

The relevance of the results is evaluated by comparison of the factors of intensity of the stresses with the analytical values.

One finds a variation enters K_I and K_I theoretical lower than 1,13% for all the methods.

1 Problem of reference

1.1 Geometry

the structure $2D$ is a rectangular $2D$ plate ($LX=10\,m$, $LY=30\,m$), comprising an emerging crack [Figure 1.1-a]. The length of initial crack is $a=5\,m$.

One calls “line lower”, line in $y=0$ and “line higher”, line in $y=Ly$.

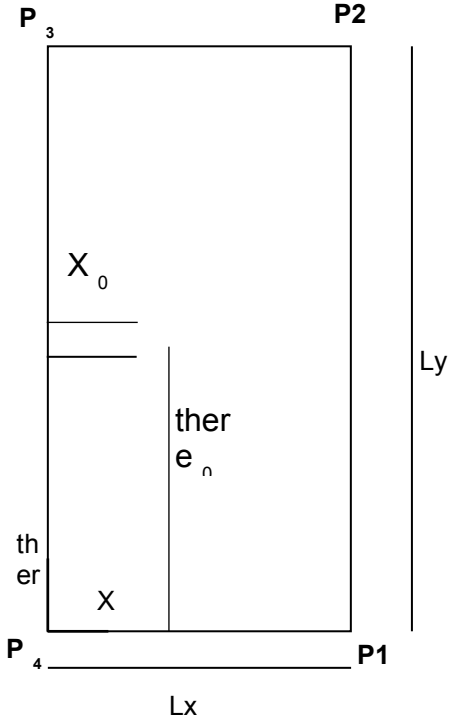


Figure 1.1-a : geometry of the fissured plate

the noted nodes $P1$ and $P4$ on Figure 1.1-a are used to impose the boundary conditions, which are clarified in the paragraph [§1.31.3].

1.2 Properties of the material

Modulus Young: $E=205\,10^{9}\,Pa$
Poisson's ratio: $\nu=0$

1.3 Boundary conditions and loadings

the loading consists in applying a distributed force to the lines lower and higher $p=106\,Pa$ and in the meaning of the norm external to surface.

In order to block the rigid modes, one blocks displacements of the nodes P_1 P_2 , P_3 and P_4 as follows:

- $DY^{P4}=DY^{P1}=0$;
- $DX^{P4}=0$

1.4 Reference solution

the analytical statements of the stress intensity factors K_I and K_{II} are functions of the distributed force p, length of crack a, width of the plate Lx [1]:

$$K_I = p\sqrt{\pi a}\, f\left(\frac{a}{Lx}\right)$$

$$K_{II} = 0$$

where the function f can be given several different ways. We choose that obtained by [1], and which is true for $\frac{a}{Lx} < 0,6$:

$$f\left(\frac{a}{Lx}\right) = 1,12 - 0,231\left(\frac{a}{Lx}\right) + 10,55\left(\frac{a}{Lx}\right)^2 - 21,72\left(\frac{a}{Lx}\right)^3 + 30,39\left(\frac{a}{Lx}\right)^4$$

One advances crack thanks to the model of Paris:

$\frac{da}{dN} = C\Delta K^m$ where a is the length of crack, C and m are constants of the material, ΔK is the difference between two $FICs$ consecutive and N is the number of cycles.

With the numerical values of the test:

No propagation: $0,25\,m$

Lx : $10\,m$

Reference		
$a(m)$	$K_I(Pa.m^{0,5})$	$K_{II}(Pa.m^{0,5})$
2,5	4,205998 106	0
2,75	4,63286 106	0
3	5,09492 106	0
3,25	5,59908 106	0.3,5
	6,15349 106	0
3,75	6,76776 106	0
4	7,4531 106	0
4,25	8,2224 106	0.4,5
	9,0905 106	0
4,75	1,0074 107	0
5	1,1192 107	0
5,25	1,2465 107	0.5,5
	1,3916 107	0
5,75	1,55716 107	0
6	1,74586 107	0

Table 1.4-1 : values of reference de for K_I and K_{II}

1.5 Bibliography

1.TADA H., PARIS P., IRWIN G.: The stress analysis of aces, Handbook. Del Research Corporation, Hellertown, Pennsylvania, 1973

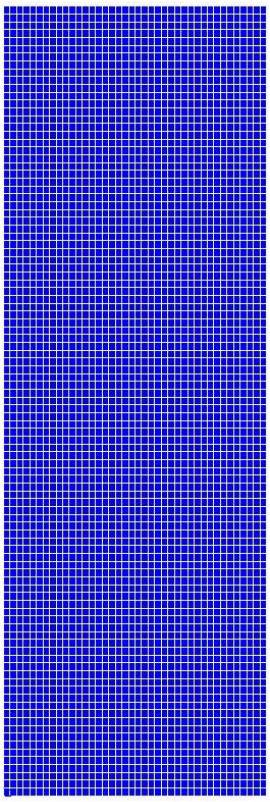
2 Modelization a: Method mesh

2.1 Characteristics of the modelization

In this modelization, the method MAILLAGE is tested for the crack propagation. The level-sets are determined by orthogonal projection on the segments composing crack.

2.2 Characteristics of the mesh

the structure is modelled by a "healthy" mesh regular composed of 40×101 QUAD4, respectively along the axes x , y . The crack is represented by a succession of SEG2, independently of the mesh of structure.



Appear 2.2-a : mesh of the fissured plate

2.3 Quantities tested and results

For each step of propagation ($2,5\,m$), one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G.
One also tests the Y-coordinate of the crack tip given by PROPA_FISS.

2.3.1 Results on K_I :

Identification	Code_Aster	Reference	difference
CALC_G			
KI_1	4,17448 106	4,205998 106	-0,749%
KI_2	4,60197 106	4,63286 106	-0,667%
KI_3	5,0668 106	5,09492 106	-0,552%
KI_4	5,575 106	5,59908 106	-0,43%
KI_5	6,1334 106	6,15349 106	-0,326%
KI_6	6,7499 106	6,76776 106	-0,264%
KI_7	7,4338 106	7,4531 106	-0,259%
KI_8	8,19599 106	8,2224 106	-0,322%
KI_9	9,0497 106	9,0905 106	-0,449%
KI_10	1,0011 107	1,0074 107	-0,627%
KI_11	1,1099 107	1,1192 107	-0,828%
KI_12	1,2339 107	1,2465 107	-1,011%
KI_13	1,37603 107	1,3916 107	-1,121%
KI_14	1,54018 107	1,55716 107	-1,09%
KI_15	1,7313 107	1,74586 107	-0,834%

2.3.2 Results on K_{II} :

For this test, it is wished that K_{II} be lower than $10^{-4} K_I$. Thus, one makes sure that K_{II} is rather close to zero, the value of reference.

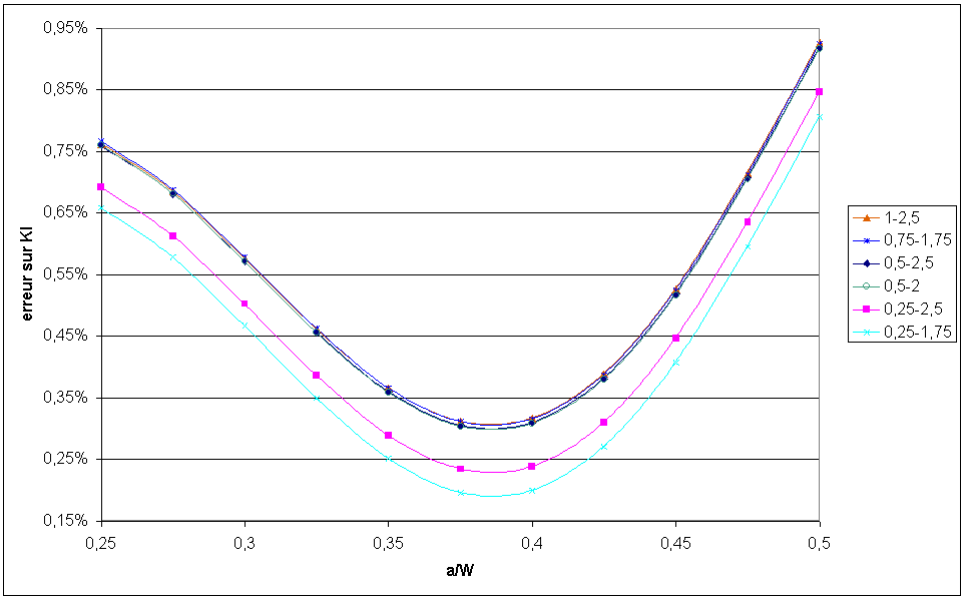
Identification	Code_Aster	Reference
CALC_G		
KII_1	-2,7313 102	0
KII_2	-8,5062 101	0
KII_3	-2,6061 102	0
KII_4	1,5995 102	0
KII_5	-2,7309 102	0
KII_6	-2,3176 102	0
KII_7	-3,1276 102	0
KII_8	3,1327 102	0
KII_9	-3,8393 102	0
KII_10	-4,1916 102	0
KII_11	-4,986 102	0
KII_12	-5,6998 102	0
KII_13	-6,7642 102	0
KII_14	-7,9542 102	0
KII_15	-9,5344 102	0

2.3.3 Results on the Y-coordinate of the crack tip:

It is checked that the coordinates in Y-coordinate of the successive crack tips are close to the initial value. This checking gives the same indications as the test on K_{II}.

Identification	Code_Aster	Reference	complementary
Difference			
CALC_G	y_1	15	
15.0%	y_2	15	15 2,18 [10]
4%	y_3	15	15.2,8 [10]
4%	y_4	15	15 4,51 [10]
4%	y_5	15	15 5,47 [10]
4%	y_6	15	15 6,95 [10]
4%	y_7	15	15.8,1 [10]
4%	y_8	15	15.9,5 [10]
4%	y_9	15,0002	15
0,001%	y_10	15,0002	15
0,001%	y_11	15,0002	15
0,001%	y_12	15,0002	15
0,002%	y_13	15,0002	15
0,002%	y_14	15,0003	15
0,002%	y_15	15,0003	15

2.4 0,002% Results



Appears 2.4-a : Influence choice of contours IH and RS on the error on KI

We can see here that the configuration most adapted for the choice of RI and RS (contours lower and higher of the field theta) is: $RI=2*L_0$ and $RS=7*L_0$ where L_0 is the smallest edge of the mesh.

3 Modelization b: Method simplex

3.1 Characteristics of the modelization

In this modelization, method SIMPLEXE is tested for the crack propagation.
The level-sets are determined by resolution of the equations of reactualization.

3.2 Characteristics of the mesh

One uses here the same mesh as in the modelization A (§2.1).

3.3 Quantities tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G.

3.3.1 Results on K_I :

Identification	*Code_Aster*	Reference	difference
CALC_G			
KI_1	4,17448	4,205998 106	-0,749%
KI_2	4,60197	4,63286 106	-0,667%
KI_3	5,06681	5,09492 106	-0,552%
KI_4	5,57501	5,59908 106	-0,43%
KI_5	6,13343	6,15349 106	-0,326%
KI_6	6,74995	6,76776 106	-0,263%
KI_7	7,43384	7,4531 106	-0,258%
KI_8	8,19608	8,2224 106	-0,32%
KI_9	9,04986	9,0905 106	-0,447%
KI_10	1,00111	1,0074 107	-0,625%
KI_11	1,10996	1,1192 107	-0,825%
KI_12	1,23393	1,2465 107	-1,008%
KI_13	1,37608	1,3916 107	-1,117%
KI_14	1,54026	1,55716 107	-1,085%
KI_15	1,73142	1,74586 107	-0,827%

3.3.2 Results on K_{II} :

Identification	Code_Aster	Reference
CALC_G		
KII_1	-2,7313 102	0
KII_2	-2,9582 102	0
KII_3	-3,3848 102	0
KII_4	-3,9204 102	0
KII_5	-4,4895 102	0
KII_6	-5,0302 102	0
KII_7	-5,4802 102	0
KII_8	-5,7767 102	0
KII_9	-5,8608 102	0
KII_10	-5,7002 102	0
KII_11	-5,2708 102	0
KII_12	-4,5292 102	0
KII_13	-3,422 102	0
KII_14	-1,8819 102	0
KII_15	-1,7862 101	0

4 Modelization C: Method upwind

4.1 Characteristic of the modelization

In this modelization, method UPWIND is tested for the crack propagation.
The level-sets are determined by resolution of the equations of reactualization per diagram to the finite differences.

4.2 Characteristics of the mesh

One uses here the same mesh as in the modelization A (§2.1).

4.3 Quantities tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G.

4.3.1 Results on KI:

Identification	*Code_Aster*	Reference	difference
CALC_G			
KI_1	4.17448E+06	4,205998 106	-0,749%
KI_2	4.60197E+06	4,63286 106	-0,667%
KI_3	5.06680E+06	5,09492 106	-0,552%
KI_4	5.57500E+06	5,59908 106	-0,43%
KI_5	6.13340E+06	6,15349 106	-0,326%
KI_6	6.74991E+06	6,76776 106	-0,264%
KI_7	7.43378E+06	7,4531 106	-0,259%
KI_8	8.19599E+06	8,2224 106	-0,322%
KI_9	9.04972E+06	9,0905 106	-0,449%
KI_10	1.00110E+07	1,0074 107	-0,627%
KI_11	1.10993E+07	1,1192 107	-0,828%
KI_12	1.23389E+07	1,2465 107	-1,011%
KI_13	1.37603E+07	1,3916 107	-1,122%
KI_14	1.54018E+07	1,55716 107	-1,09%
KI_15	1.73130E+07	1,74586 107	-0,834%

4.3.2 Results on KII:

Identification	Code_Aster	Reference
CALC_G		
KII_1	-2,7313 102	0
KII_2	-2,8454 102	0
KII_3	-3,0026 102	0
KII_4	-3,2069 102	0
KII_5	-3,4643 102	0
KII_6	-3,7828 102	0
KII_7	-4,1724 102	0
KII_8	-4,6462 102	0
KII_9	-5,2209 102	0
KII_10	-5,9177 102	0
KII_11	-6,7643 102	0
KII_12	-7,7966 102	0
KII_13	-9,0621 102	0
KII_14	-1,0624 103	0
KII_15	-1,2568 103	0

5 Modelization D: Geometrical method

5.1 Characteristics of the modelization

In this modelization, method GEOMETRIQUE is tested for the crack propagation.
The level-sets are recomputed with each step of propagation.

5.2 Characteristics of the mesh

One uses here the same mesh as in the modelization A (§2.1).

5.3 Quantities tested and results

For each step of propagation, one tests the value of the stress intensity factors K_I and K_{II} data by CALC_G.

5.3.1 Results on KI:

Identification	*Code_Aster*	Reference	Difference
CALC_G			
KI_1	4,174476 106	4,205998 106	-0,749%
KI_2	4,601963 106	4,63286 106	-0,667%
KI_3	5,066799 106	5,09492 106	-0,522%
KI_4	5,574998 106	5,59908 106	-0,430%
KI_5	6,133404 106	6,15349 106	-0,326%
KI_6	6,749911 106	6,76776 106	-0,264%
KI_7	7,433777 106	7,4531 106	-0,259%
KI_8	8,195987 106	8,2224 106	-0,322%
KI_9	9,049722 106	9,0905 106	-0,449%
KI_10	1,001096 107	1,0074 107	-0,627%
KI_11	1,109928 107	1,1192 107	-0,828%
KI_12	1,233893 107	1,2465 107	-1,01%
KI_13	1,376027 107	1,3916 107	-1,12%
KI_14	1,540181 107	1,55716 107	-1,09%
KI_15	1,731302 107	1,74586 107	-0,834%

5.3.2 Results on KII:

Identification	Code_Aster	Reference
CALC_G		
KII_1	0,0	0
KII_2	1,30 X 102	0
KII_3	0,0	0
KII_4	2,48 X 101	0
KII_5	0,0	0
KII_6	0,0	0
KII_7	0,0	0
KII_8	0,0	0
KII_9	0,0	0
KII_10	0,0	0
KII_11	0,0	0
KII_12	0,0	0
KII_13	0,0	0
KII_14	0,0	0
KII_15	0,0	0

6 Summaries of the Performance

6.1 results of the methods

One can compare the computing time for the same number of steps of propagation (15) of the four methods. It is noticed here that method GEOMETRIQUE is definitely faster than the three other methods for the same discretization.

Mesh	Method	Time (s)
40×101	MAILLAGE	129
	SIMPLEXE	290
	UPWIND	244
	GEOMETRIQUE	30

6.2 Conclusion

the purposes of this test are reached:

• MAILLAGE to validate on a simple case the computation of the stress intensity factors in I mode for elements X-FEM for the four methods, SIMPLEXE, UPWIND and GEOMETRIQUE.

• to distinguish the best method for this case. For equal results, the geometrical method is most powerful. One thus recommends here to choose this method for this case.