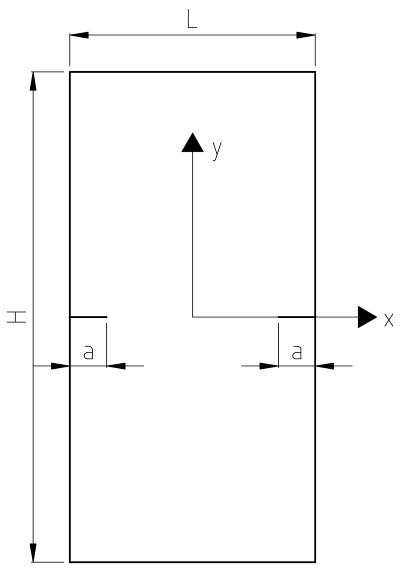
SSLP319 - Propagation of two emerging cracks X-FEM solicited in mode I

Summarized:

The goal of this test is to check that operator PROPA_FISS treats the cases of multi-cracking correctly.
It is about a plate $2D$ containing two cracks, each one made up of only one bottom. Several propagations are calculated by the operator PROPA_FISS. It is checked that the factors of intensity of the stresses of propagated cracks are correct for a propagation in mode I.

1 Problem of reference

1.1 Geometry



Appears 1.1-a: geometry of the fissured plate

geometrical Dimensions of the fissured plate:

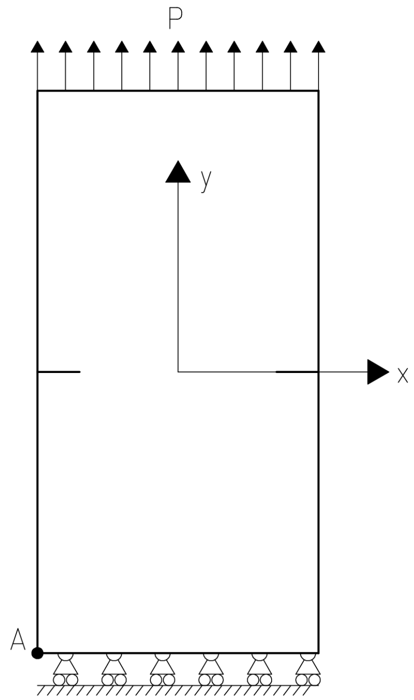
width $L=1000\,mm$
height $H=2000\,mm$

initial Length of cracks: $a_0=300\ mm$.
The cracks are positioned in the middle of the height of the plate ($H/2$).

1.2 Properties of the material

Young's modulus $E=206000\,MPa$
Poisson's ratio $\nu=0.33$

1.3 Boundary conditions and loadings



Appears 1.3-a: boundary conditions and loadings

Boundary conditions:

Point: $A \quad \Delta X = \Delta Y = 0$

Points of the lower end of the plate: $\Delta Y = 0$

Loading:

Pressure applied at the higher end of the plate: $P = 1\,MPa$

Three propagations are calculated by imposing a maximum advance of cracks equalizes with $30\,mm$. As a consequence of the symmetry of the geometry, boundary conditions and loading, the advances of two cracks are always equal in advance imposed maximum.

2 Reference solution

2.1 Method of calculating

Three propagations of crack are calculated. The two cracks always advance same distance and their factors of intensity of the stresses are always equal between them.
One can by means of calculate the factors of intensity of the stresses the following equations [bib1]:

$$K_I = Y \cdot P \cdot \sqrt{a}$$
$$Y = 1.99 + 0.76 \cdot \frac{a}{L} - 8.48 \cdot \left(\frac{a}{L}\right)^2 + 27.36 \cdot \left(\frac{a}{L}\right)^3$$
$$K_{II} = 0$$

2.2 Quantities and results of reference

For the three propagations calculated in the tests, the half-length of crack is the following one:

Propagation	a [mm]
1	330.0
2	360.0
3	390.0

Table 2.1

the value of K_I expected is thus the following one for each propagated bottom:

Propagation	K_I [$Pa\sqrt{mm}$]
1	3.7992E+07
2	4.1791E+07
3	4.6316E+07

Table 2.2

the value of K_{II} expected is always equal to zero.

2.3 Bibliographical references

[1] D.Broek, “Elementary engineering A fractures mechanics”, Martinus Nijhoff Publishers, The La Hague, The Netherlands,

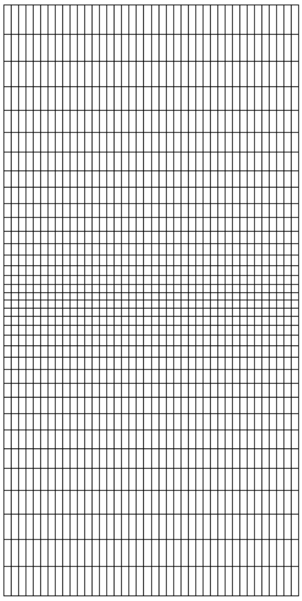
3 1982 Modelization

3.1 Characteristic of the modelization

method `UPWIND` is used by `PROPA_FISS` to solve the equations of propagation of crack.
No auxiliary grid is used. That is possible because the mesh of structure is very regular.

3.2 Characteristics of the mesh

the structure is modelled by a mesh made up of 1440 elements `QUAD4` (see Appear 3.2-a).



Appear 3.2-a: mesh of structure

The mesh is very coarse to reduce the computing time. It is refined more in the zone of propagation of crack. In this zone, the dimension of the elements is of $25\times25\,mm$. The largest element used has a dimension equalizes with $25\times100\,mm$.

3.3 Quantities tested and results

One tests the values of K_I and K_{II} for two cracks after each propagation. To check if these values are correct, one uses a relative tolerance equal to 5% for the values of K_I . On the other hand, to check if the value of K_{II} is null, one uses an absolute tolerance (threshold value) related to the value of K_I : it is considered that K_{II} is null if its value is lower than 1% value of K_I . Indeed, in this case one can neglect the value of K_{II} .

Propagation	Fissures	K_I reference [$Pa\sqrt{mm}$]	Tolerance
1	left	3.7992E+07	<5%
	right	3.7992E+07	<5%
2	left	4.1791E+07	<5%
	right	4.1791E+07	<5%
3	left	4.6316E+07	<5%
	right	4.6316E+07	<5%

Propagation	Fissures	K_{II} reference [$Pa\sqrt{mm}$]	Tolerance [$Pa\sqrt{mm}$]
1	left	0	< $K_{I\,Aster}/100$
	right	0	< $K_{I\,Aster}/100$
2	left	0	< $K_{I\,Aster}/100$
	right	0	< $K_{I\,Aster}/100$
3	left	0	< $K_{I\,Aster}/100$
	right	0	< $K_{I\,Aster}/100$

3.4 Remarks

All the values tested are in the tolerances used. That means that method UPWIND and the calculates correctly at the same time the position of two cracks level sets.
The error obtained on the values of K_I is almost null and the values of K_{II} are always about 0.01% values of K_I. The got results are thus very satisfactory.

4 Modelization B

4.1 Characteristic of the modelization

method SIMPLEXE is used by PROPA_FISS to solve the equations of propagation of crack. **No auxiliary grid** is used.

4.2 Characteristics of the mesh

One uses the same mesh as that of the modelization A .

4.3 Quantities tested and results

One tests the values of K_I and K_{II} for two cracks after each propagation. To check if these values are correct, one uses a relative tolerance equal to 5% for the values of K_I. On the other hand, to check if the value of K_{II} is null, one uses an absolute tolerance (threshold value) related to the value of K_I : it is considered that K_{II} is null if its value is lower than 1% value of K_I. Indeed, in this case one can neglect the value of K_{II} .

Propagation	Fissures	K_I reference [$Pa\sqrt{mm}$]	Tolerance
1	left	3.7992E+07	<5%
	right	3.7992E+07	<5%
2	left	4.1791E+07	<5%
	right	4.1791E+07	<5%
3	left	4.6316E+07	<5%
	right	4.6316E+07	<5%

Propagation	Fissures	K_{II} reference [$Pa\sqrt{mm}$]	Tolerance [$Pa\sqrt{mm}$]
1	left	0	< $K_{I\,Aster}/100$
	right	0	< $K_{I\,Aster}/100$
2	left	0	< $K_{I\,Aster}/100$
	right	0	< $K_{I\,Aster}/100$
3	left	0	< $K_{I\,Aster}/100$
	right	0	< $K_{I\,Aster}/100$

4.4 Remarks

All the values tested are in the tolerances used. That means that method SIMPLEXE and the calculates correctly at the same time the position of two cracks level sets.
The error obtained on the values of K_I is almost null and the values of K_{II} are always about 0.01% values of K_I. The got results are thus very satisfactory.

5 Modelization C

5.1 Characteristic of the modelization

the method MAILLAGE is used by PROPA_FISS. Operator CALC_G is used for the computation of the factors of intensities of the stresses.

5.2 Characteristics of the mesh

One uses the same mesh as that of the modelization A .

5.3 Quantities tested and results

In this modelization, one does not use key word COMP_LINE of PROPA_FISS, but one gives in entry of PROPA_FISS an array containing several times. One thus tests by means of computer that the computation of the cycle is correct.

One tests the values of K_I and K_{II} for two cracks after each propagation. To check if these values are correct, one uses a relative tolerance equal to 1% for the values of K_I . On the other hand, to check if the value of K_{II} is null, one uses an absolute tolerance (threshold value) related to the value of K_I : it is considered that K_{II} is null if its value is lower than $0{,}1\%$ value of K_I. Indeed, in this case one can neglect the value of K_{II} .

Propagation	Fissures	K_I reference [$Pa\sqrt{mm}$]	Tolerance
1	left	3.80E+007	<1%
	right	3.80E+007	<1%
2	left	4.18E+007	<1%
	right	4.18E+007	<1%
3	left	4.63E+007	<1%
	right	4.63E+007	<1%

Propagation	Fissures	K_{II} reference [$Pa\sqrt{mm}$]	Tolerance [$Pa\sqrt{mm}$]
1	left	0	< $K_{I\,Réf}/1000$
	right	0	< $K_{I\,Réf}/1000$
2	left	0	< $K_{I\,Réf}/1000$
	right	0	< $K_{I\,Réf}/1000$
3	left	0	< $K_{I\,Réf}/1000$
	right	0	< $K_{I\,Réf}/1000$

5.4 Remarks

All the values tested are in the tolerances used. That means that the method MAILLAGE and the calculates correctly at the same time the position of two cracks level sets.
The error obtained on the values of K_I is lower than 1% and the values of K_{II} are always lower than $K_I/1000$. The got results are thus very satisfactory.

6 Modelization D

6.1 Characteristic of the modelization

the method `MAILLAGE` is used by `PROPA_FISS`. Operator `POST_K1_K2_K3` is used for the computation of the factors of intensities of the stresses.

6.2 Characteristics of the mesh

One uses the same mesh as that of the modelization A .

6.3 Quantities tested and results

One tests the values of K_I and K_{II} for two cracks after each propagation. To check if these values are correct, one uses a relative tolerance equal to 13% for the values of K_I . On the other hand, to check if the value of K_{II} is null, one uses an absolute tolerance (threshold value) related to the value of K_I : it is considered that K_{II} is null if its value is lower than 1% value of K_I. Indeed, in this case one can neglect the value of K_{II} .

Propagation	**Fissures**	K_I **reference** [$Pa\sqrt{mm}$]	**Tolerance**
1	left	3.7993E+07	<13%
	right	3.7993E+07	<13%
2	left	4.179E+07	<13%
	right	4.179E+07	<13%
3	left	4.632E+07	<13%
	right	4.632E+07	<13%

Propagation	**Fissures**	K_{II} **reference** [$Pa\sqrt{mm}$]	**Tolerance** [$Pa\sqrt{mm}$]
1	left	0	< $K_{I\,Aster}/100$
	right	0	< $K_{I\,Aster}/100$
2	left	0	< $K_{I\,Aster}/100$
	right	0	< $K_{I\,Aster}/100$
3	left	0	< $K_{I\,Aster}/100$
	right	0	< $K_{I\,Aster}/100$

6.4 Remarks

All the values tested are in the tolerances used. That means that the method `MAILLAGE` and the calculates correctly at the same time the position of two cracks level sets.
The error obtained on the values of K_I is almost null and the values of K_{II} are always about 0.01% values of K_I. The got results are thus very satisfactory.

7 Modelization E

7.1 Characteristic of the modelization

method GEOMETRIQUE is used by PROPA_FISS to update the position of crack. **No auxiliary grid** is used.

7.2 Characteristics of the mesh

One uses the same mesh as that of the modelization A .

7.3 Quantities tested and results

One tests the values of K_I and K_{II} for two cracks after each propagation. To check if these values are correct, one uses a relative tolerance equal to 5% for the values of K_I . On the other hand, to check if the value of K_{II} is null, one uses an absolute tolerance (threshold value) related to the value of K_I : it is considered that K_{II} is null if its value is lower than 1% value of K_I. Indeed, in this case one can neglect the value of K_{II} .

Propagation	Fissures	K_I reference [$Pa\sqrt{mm}$]	Tolerance
1	left	3.7992E+07	<5%
	right	3.7992E+07	<5%
2	left	4.1791E+07	<5%
	right	4.1791E+07	<5%
3	left	4.6316E+07	<5%
	right	4.6316E+07	<5%

Propagation	Fissures	K_{II} reference [$Pa\sqrt{mm}$]	Tolerance [$Pa\sqrt{mm}$]
1	left	0	< $K_{I\,Aster}/100$
	right	0	< $K_{I\,Aster}/100$
2	left	0	< $K_{I\,Aster}/100$
	right	0	< $K_{I\,Aster}/100$
3	left	0	< $K_{I\,Aster}/100$
	right	0	< $K_{I\,Aster}/100$

7.4 Remarks

All the values tested are in the tolerances used. That means that method GEOMETRIQUE and the calculates correctly at the same time the position of two cracks level sets.
The error obtained on the values of K_I is almost null and the values of K_{II} are always about 0.01% values of K_I. The got results are thus very satisfactory.

8 Summary of the results

All the methods of operator PROPA_FISS used (UPWIND, SIMPLEXE, MAILLAGE and GEOMETRIQUE) made it possible to calculate well the position of two existing cracks in the same model and propagating in mode I. The factors of intensity of the stresses were calculated correctly and the methods used calculate correctly the level sets with each propagation.
The got results make it possible to validate the implementation of multi-cracking (case of several cracks with only one crack tip each one) in operator PROPA_FISS.