SSLP311 - Fissure central oblique in a finished rectangular plate, with two materials, subjected to uniform tension

Abstract:

This test is resulting from the validation independent of version 3 in fracture mechanics.

It is about a two-dimensional test in static with bi--material in the presence of an internal crack of interface obliques.

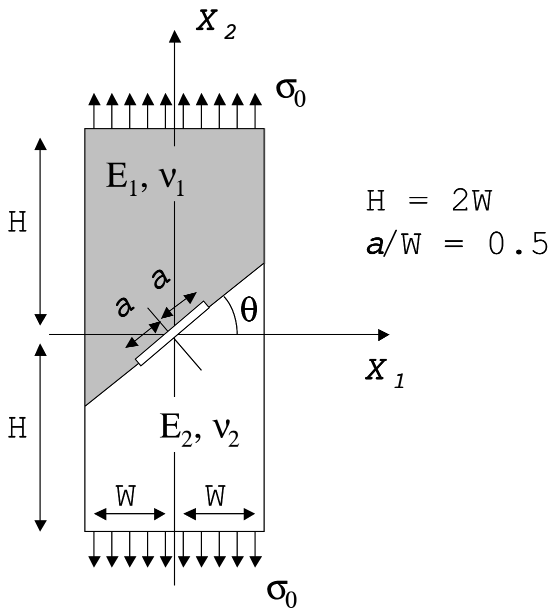
The behavior of the structure (bi--material) is elastic linear isotropic.

The case test understands four modelizations in plane stresses in which the influence of the slope of crack θ is studied (4 cases).

The computation factors of intensity of the stresses is not available for a crack located at the interface of a bi--material; the comparison with the reference solution is thus done on the rate of refund of energy only, calculated with operator CALC_G.

1 Problem of reference

1.1 Geometry



One considers 4 values of the angle θ : 15°, 30°, 45° and 60°.
Other dimensions are selected such as $H=2\mathrm{W}=4\mathrm{a}$.
The value of a is worth $1.\mathrm{E}-3\,m$.

1.2 Properties of the materials

Material n° 1

Elastic, linear, isotropic, Young modulus $E_1=2\mathrm{E}{+}12\,Pa$ and Poisson's ratio $\nu_1=0{,}3$.

Material n° 2

Elastic, linear, isotropic, Young modulus $E_2=2\mathrm{E}{+}11\,Pa$ and Poisson's ratio $\nu_2=0{,}3$.

1.3 Boundary conditions and loading

- the rigid modes are blocked by the following boundary conditions:
  $UX=UY=0$ with the left lower corner of the model.
  $UY=0$ on lower edge.
- Loading: uniform tension $\sigma_{yy}=\sigma_0$ on higher edge.

  The value of σ_0 is $100\,MPa$.

2 Reference solution

2.1 Method of calculating used for the reference solution

Method of the elements of border, with quadratic elements [bib1].

The computation of K_I and K_{II} is carried out by an integral of contour (integral M [bib2]) in which intervene the stresses and displacements calculated in the part, as well as the stresses and displacements deduced from analytically definite solutions asymptotic, in which K_I and K_{II} are alternatively null.

As comparison, the computation of K the east also carried out by the method of virtual extension.

2.2 Results of reference

the results of the reference solution are presented in the table below, for the various values of the angle and the two ends of crack, with $F_j=\frac{K_j}{\sigma_0\sqrt{\pi a}} \quad (j=I,II)$.

Integral		method				Left side			
		$\theta=15°$	$\theta=30°$	$\theta=45°$	$\theta=60°$	$\theta=15°$	$\theta=30°$	$\theta=45°$	$\theta=60°$
Right side	F_I	1,0115	0,7868	0,5211	0,2770	1,1266	0,9910	0,7646	0,4919
M	F_{II}	0,4434	0,6244	0,6723	0,5804	0,0862	0,2961	0,4056	0,4057
virtual	F_I	extension	1,0110	0,7864	0,5210	0,2769	1,1260	0,9904	0,7643
0,4919	F_{II}	0,4429	0,6240	0,6720	0,5801	0,0865	0,2960	0,4055	0,4056

the relation between total rate of restitution of energy G and the K_j is written as follows [bib3]:

$$G=\beta(K_I^2+K_{II}^2)$$

with:

$$\kappa_i=\frac{3-\nu_i}{1+\nu_i}$$

$$\beta=\frac{1}{16Ch^2(\alpha\pi)}\left(\frac{1+\kappa_1}{\mu_1}+\frac{1+\kappa_2}{\mu_2}\right) \quad \text{and} \quad \mu_i=\frac{E_i}{2(1+\nu_i)}$$

$$\alpha=\frac{1}{2\pi}\ln\left[\left(\frac{\kappa_1}{\mu_1}+\frac{1}{\mu_2}\right)\left(\frac{\kappa_2}{\mu_2}+\frac{1}{\mu_1}\right)^{-1}\right]$$

2.3 Uncertainty on the solution

Estimated at less than 0,1%. It is noted that the difference between the method of the integrals of contour and the method of virtual extension is generally lower than 0,05%.

2.4 Bibliographical references

1) Stress intensity Factor analysis of interface ace using boundary element method. Of application contour-integral method. N. MIYAZAKI, T. IKEDA, T.SODA and T. MUNAKATA. Engng.Fract.Mechs., 45, n°5, 599-610, 1993. Year analysis of interface

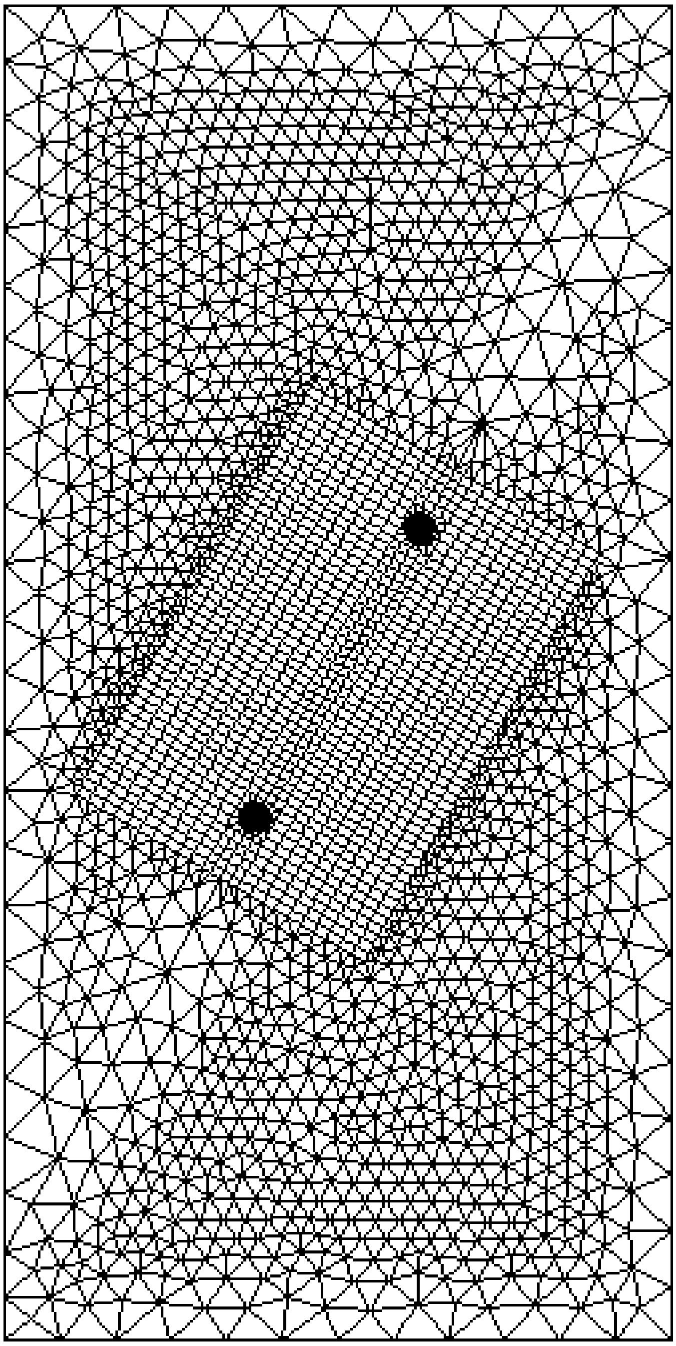
2) aces between dissimilar isotropic materials using conservation integrals in elasticity. J.F. YAU and T. C. CHANG. Engng.Fract.Mechs., 20,423-432, 1984. Adhesive The strength of joints using

3) the theory of aces. B. Mr. MALYSHEV and R.L. SALGANIK . Int.J.Fract.Mech., 1,114-128, 1965. Modelization A Characteristic of the modelization

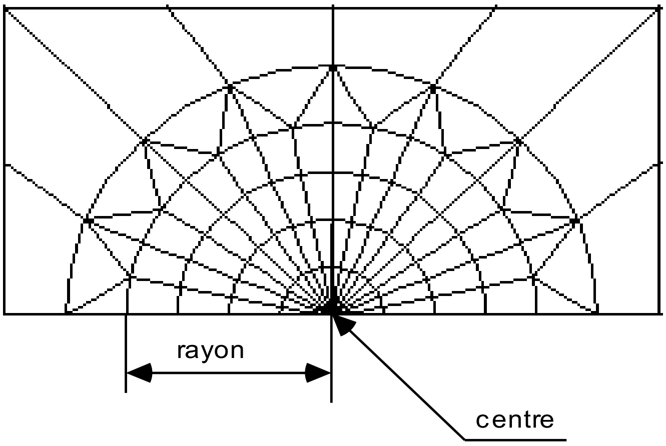
3 the various

3.1 modelizations are identical

except for the slope of crack. Complete mesh for an angle Zoom on



the point of crack the radius is worth $\beta = 60°$



. There are four contours defined

by the command $7.5\mathrm{E}-5\,m$

CALC_THETA: crown 1: crown 2: crown `3:` crown

4: $Rinf=0.$ $R\sup=1.875\mathrm{E}-5\mathrm{m}$
The direction of $R\sup=3.750\mathrm{E}-5\mathrm{m}$
$Rinf=1.875\mathrm{E}-5\mathrm{m}$
propagation $Rinf=3.750\mathrm{E}-5\mathrm{m}$ $R\sup=5.625\mathrm{E}-5\mathrm{m}$
is defined by $Rinf=5.625\mathrm{E}-5\mathrm{m}$ $R\sup=7.500\mathrm{E}-5\mathrm{m}$

: Characteristics of the mesh The mesh $\cos\theta, \sin\theta$

3.2 consists of 10676 nodes and

4584 elements, including 1392 elements QUA8 and 3168 elements TRI6. Features tested The computation of and

3.3 is not valid for

a bimatériau K_I : K_{II} option CALC_K_G cannot be used and only the computation `of` the rate of refund of energy is possible. Quantities tested and Values results tested

3.4 Identification Reference

3.4.1 Aster % left

difference	End, crowns		1 9,67362E+1
9,2428E+1˜ 4,45 $\theta=15°$			
G , contour 2	9,67362E+1	9,6392E+1	˜ 0,356
G , contour 3	9,67362E+1	9,6417E+1	˜ 0,330
G , contour 4	9,67362E+1	9,6421E+1	˜ 0,326
G 5,6694E+6 -	˜ 2,4852	E+6 - -	right
K_I	End	,	crowns
K_{II}	1 1,0125	E+	2

9,6763E+1˜ 4,33, $\theta=15°$			
G contour 2 1,0125	E+2 1,0093	E+2˜	0,315
G , contour 3	1,0125E+2	1,0095E+2	˜ 0,295
G , contour 4	1,0125E+2	1,0095E+2	˜ 0,291
G 6,3145E+6 - -	4,8309E+5	- - Remarks	to obtain
K_I	it	on	
K_{II}	the crack tip		

3.4.2 , one

calculates rate of energy restitution G using the relation between and the [bib3]: Modelization B Values tested G Identification K_j

$$\kappa_1=\kappa_2=2.076923$$
$$\mu_1=7.6923\,E+11$$
$$\mu_2=7.6923\,E+10$$
$$\alpha=-9.37742\,E-2$$
$$\beta=2.524488\,E-12$$
$$G=\beta(K_I^2+K_{II}^2)$$

4 Reference

4.1 Aster % left

difference	End, crowns		1 8,0017E+1
7,6431E+1˜ 4,48, $\theta=30°$			
G contour 2 8,0017	E+1 7,9707	E+1˜	0,387
G , contour 3	8,0017E+1	7,9730E+1	˜ 0,358
G , contour 4	8,0017E+1	7,9734E+1	˜ 0,353
G 4,4100E+6 - -	˜ 3,499	E+6 - - right	
K_I	End,	crowns	
K_{II}	1 8,48417	E	+1
8,1080E+1˜ 4,433 $\theta=30°$			
G , contour 2	8,48417E+1	8,4583E+1	˜ 0,305
G , contour 3	8,48417E+1	8,4602E+1	˜ 0,282
G , contour 4	8,48417E+1	8,4602E+1	˜ 0,282
G 5,5545E+6 -	- 1,6596E+6	- - Modelization	C
K_I	Values tested		
K_{II}	Identification		

5 Reference

5.1 Aster % left

difference	End, crowns		1 5,73826E+1
5,48161E+1˜ 4,473 $\theta=45°$			
G , contour	2 5,73826E+	1 5,71687E+	1˜ 0,373
G , contour	3 5,73826E+	1 5,71865E+	1˜ 0,342
G , contour	4 5,73826E+	1 5,7189E+1	˜ 0,337
G 2,92076E+6 -	-˜ 3,7682	E+6 - -	right
K_I	End	,	crowns
K_{II}	1 5,94122	E	+1
5,7039E+1˜ 3,994 $\theta=45°$			
G , contour 2	5,94122E+1	5,9505E+1	0,157 , contour
G 3 5,94122	E+1 5,9516	E+1 0,175	, contour
G 4 5,94122	E+1 5,9518	E+1 0,179	4,28557
G E+6 - - 2,27338	E+6 -	- Modelization	
K_I	D Values tested		
K_{II}	Identification		

6 Reference

6.1 Aster % left

difference	End, crowns		1 3,28015E+1
3,10680E+1˜ 5,285 $\theta=60°$			
G , contour	2 3,28015E+	1 3,24037E+	1˜ 1,213
G , contour	3 3,28015E+	1 3,24140E+	1˜ 1,181
G , contour	4 3,28015E+	1 3,24156E+	1˜ 1,177
G 1,55258E+6	- -˜ 3,2531	E+6 - -	right
K_I	End	,	crowns
K_{II}	1 3,22436	E	+1
3,11825E+1˜ 3,291 $\theta=60°$			
G , contour	2 3,22436E+	1 3,25321E+	1 0,895 , contour
G 3 3,22436	E+1 3,25383	E+1 0,914	, contour
G 4 3,22436	E+1 3,25398	E+1 0,919	2,75709
G E+6 - - 2,27394	E+6 -	- Summary	of
K_I	the results	The computation	
K_{II}	of and		N

7 “is not available for

a crack K_I located K_{II} at L” interfaces of a bimatériau, and the comparison is thus done directly on the rate of refund of energy. The computation of is not precise on the first G

contour G in all the cases of slope of crack, which confirms that it is necessary to avoid taking a null radius. With regard to other contours $Rinf$, the variations are about 0,4%. In the case of slope the variation exceeds 1%. As a whole, $\theta=60°$ the results are satisfactory for. G