SSLP310 - Crack pressurized in a Summarized unlimited plane

field:

This test is resulting from the validation independent of version 3 in fracture mechanics.

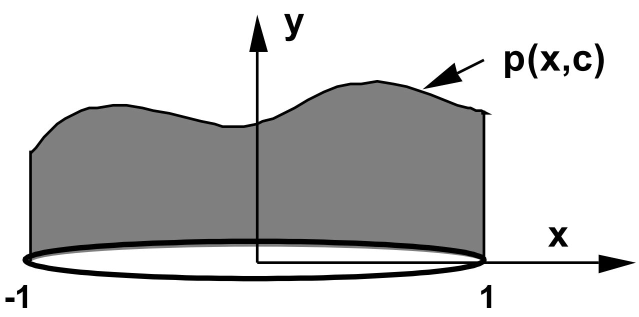
It is about a two-dimensional test in static (strains or plane stresses) which aims at the checking of G and K_I under loading by nonuniform distributed pressure on the lips, in unlimited medium. One also checks the nullity of K_{II} with option `CALC_K_G` of operator `CALC_G`.

The behavior of structure is elastic linear isotropic.

The case test 2D understands only one modelization planes in which one studies the influence of the parameter C intervening in the loading. The computation mechanical is done by call to macro-command `MACRO_ELAS_MULT`.

1 Problem of reference

1.1 Geometry



One considers rectilinear crack $-1 \leq x \leq 1$ in the unlimited plane field.

1.2 Properties of the material

the material is elastic linear homogeneous of Poisson's ratio and Young E modulus ν .
$E=1000\ MPa$, $\nu=0,3$

1.3 Boundary conditions and loadings

Boundary conditions

Relation linear $UX(-1,0)+UX(1,0)=0$

Condition of symmetry $UY=0$ for $x \leq 1$, $x \geq 1$ and $y=0$.

Loading n° 1

$p(x)=1$

Loading n° 2

$p(x,c)=\exp(cx)$ where c is a parameter

Loading n° 3

$p(x,c)=Sh(cx)$ where c is a parameter

Loading n° 4

$p(x,c)=Ch(cx)$ where c is a parameter

Loading n° 5

$p(x,c)=\cos(cx)$ where c is a parameter

2 Reference solution

2.1 Méthode de calcul used for the reference solution

exact Computation symbolic system using software MAPLE V [bib1].

2.2 Results of reference

Loading n° 1

$$K_I(x=1)=\sqrt{\pi}$$

Loading n° 2

$K_I(x=1,c)=\sqrt{\pi}(I_0(c)+I_1(c))$ where I_0 and I_1 are the modified functions of Bessel of first species of indices 0 and 1 [bib2].

Loading n° 3

$$K_I(x=1,c)=\sqrt{\pi}\,I_1(c)$$

Loading n° 4

$$K_I(x=1,c)=\sqrt{\pi}\,I_O(c)$$

Loading n° 5

$K_I(x=1,c)=\sqrt{\pi}\,J_0(c)$ where J0 is the function of Bessel of first species of index 0 [bib2].

In all the cases of loading

$G=\frac{K_I^2}{E}$ in plane stresses

$G=\frac{(1-\nu^2)K_I^2}{E}$ in bibliographical

2.3 plane strains References

[1] There the evaluating of stress intensity factors for is simple ace under parametric loading. Technical notes. N.I. IOKADIMIS and G.T. ANASTASSELOS. Computers and Structures, 51, n°6, 791-794, 1994.

[2] Handbook of mathematical functions, Chapter 9. Mr. ABRAMOWITZ and I.A. STEGUN (Editors). United States Dept. of Commerce, National Office of Standards.

3 Modelization A

3.1 Characteristic of the modelization

The model is restricted with the finished area $-x_{max} \leq x \leq x_{max}$, $-y_{max} \leq y \leq y_{max}$ with $x_{max} = y_{max} = 15$.

It consists of 1156 quadrangles with 8 nodes and 3398 triangles with 6 nodes.
It comprises 10372 nodes.

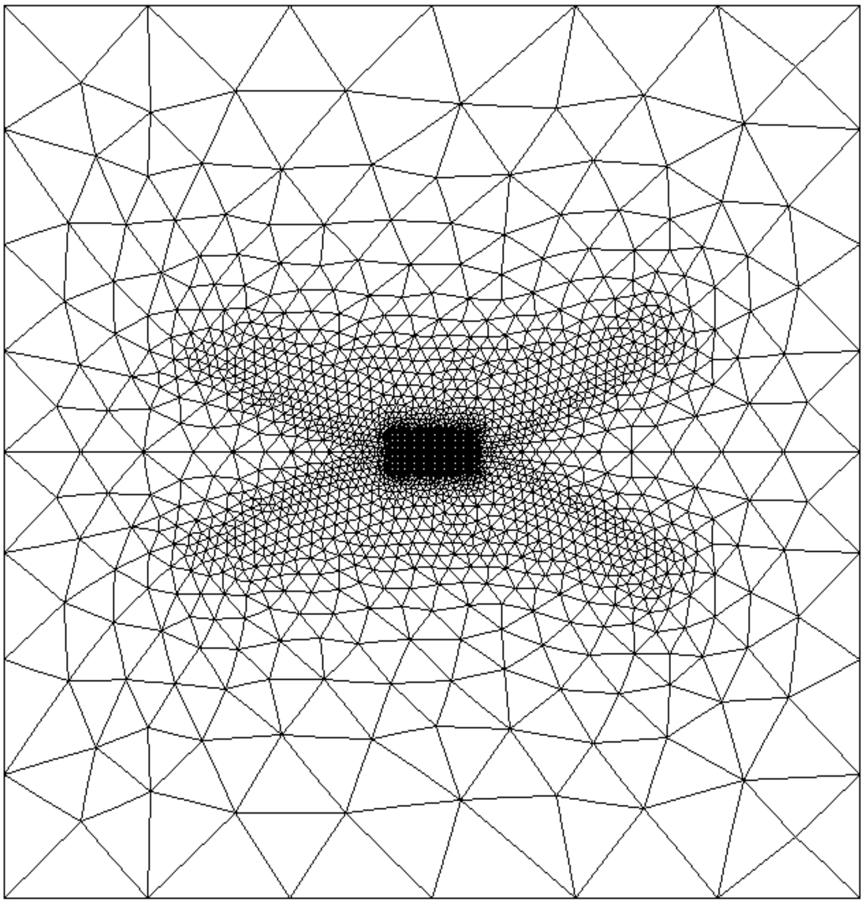
The assumption of the plane stresses is used.

3.2 Characteristics of the mesh

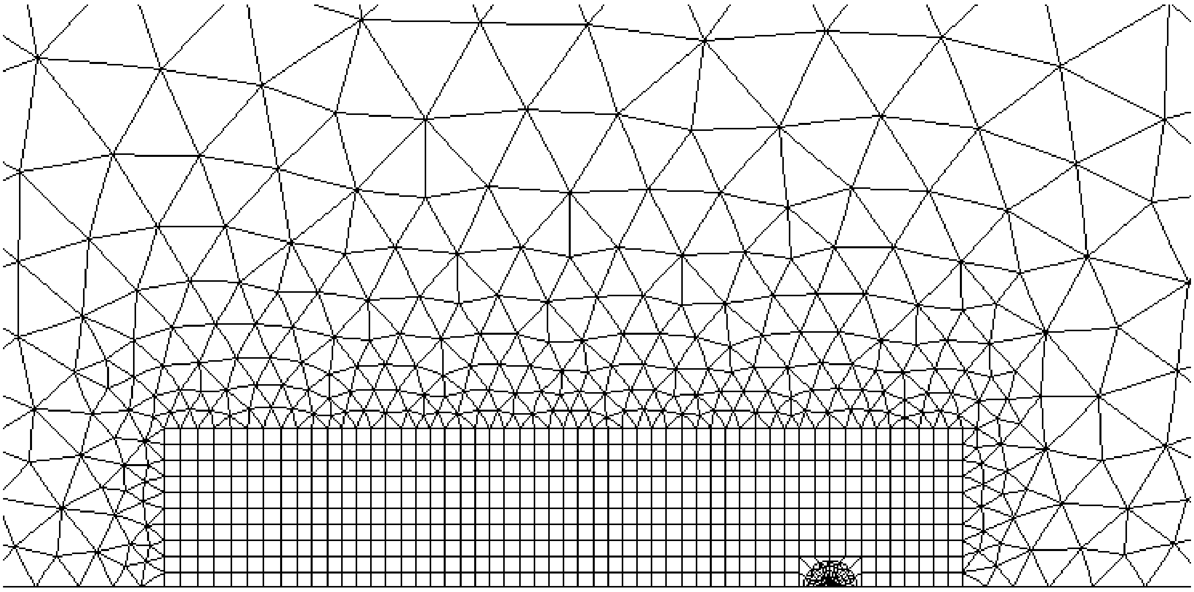
the mesh is generated with *gibi* (use of procedure FISS2D). The topological parameters concerning refinement around the crack tip (torus) are:

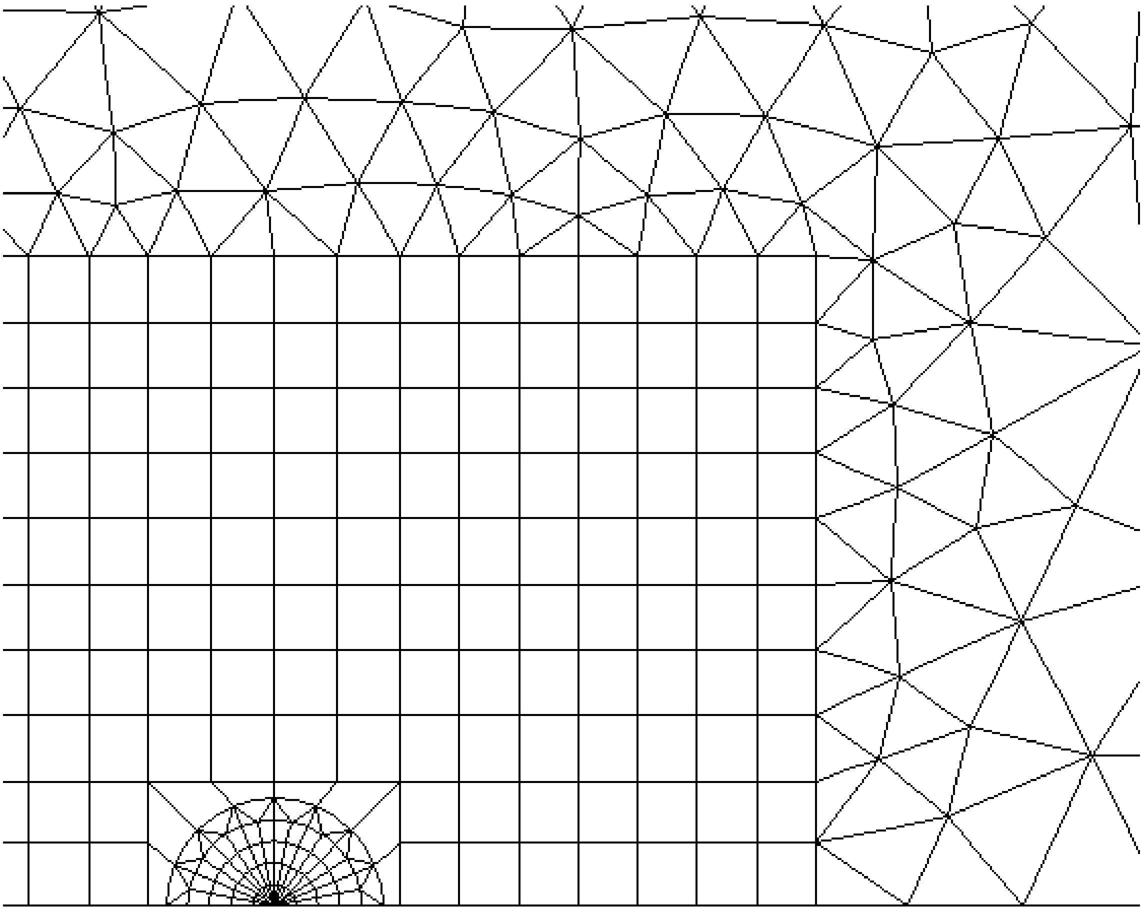
- $nc=4$ (many contours)
- $ns=8$ (many sectors)
- $nt=1$ (many contours of coarsening)

The mesh fine radiant is restricted at the right end of crack.
The partly current density of the mesh of crack is selected in order to be able to discretize the loading suitably $p(x, c)$.



Zooms on the mesh of crack:





3.3 Features tested

all the cases of loading are treated simultaneously in macro-command MACRO_ELAS_MULT. Rate of energy restitution G is calculated in postprocessing by the method theta (operator CALC_G) for each case of loading successively and various integration contours.

3.4 Quantities tested and Values

3.5 results tested

Contour 0 (triangles)

$R_{\text{inf}}=0\,mm$, $R_{\text{sup}}=0{,}02\,mm$

Identification	Reference	*Aster*	% difference
G , loading n°1	3,14158E-3	3,04077E-3	-3,209
K_I , loading n°1	1,77245	1,66	-6,57
G , loading n°2, $c=1$	1,05349E-2	0,01	-3,738
K_I , loading n°2, $c=1$	3,24576	3,03108	-6,61
G , loading n°2, $c=5$	8,356742	7,9065	-5,39
K_I , loading n°2, $c=5$	91,41522	85,4189	-6,56
G , loading n°3, $c=1$	1,00344E-3	9,6006E-4	-4,323
K_I , loading n°3, $c=1$	1,00172	0,93505	-6,66
G , loading n°3, $c=5$	1,86052	1,760148	-5,4
K_I , loading n°3, $c=5$	43,13380	40,33829	-6,48
G , loading n°4, $c=1$	5,03571E-3	4,86064E-3	-3,477
K_I , loading n°4, $c=1$	2,24404	2,09602	-6,6
G , loading n°4, $c=5$	2,331095	2,20566	-5,38
K_I , loading n°4, $c=5$	48,28142	45,08068	-6,63
G , loading n°5, $c=1$	1,839487E-3	1,78707E-3	-2,849
K_I , loading n°5, $c=1$	1,356277	1,267569	-6,54
G , loading n°5, $c=2{,}4048255577$	0	4,1738E-8	-
K_I , loading n°5, $c=2{,}4048255577$	0	2,0383E-3	-

Contour 1 (quadrangles)

$R_{\inf}=0{,}02\,mm$, $R_{\sup}=0{,}04\,mm$

Identification	Reference	*Aster*	% difference
G , loading n°1	3,14158E-3	3,1669E-3	0,807
K_I , loading n°1	1,77245	1,78079	0,471
G , loading n°2, $c=1$	1,05349E-2	1,056655E-2	0,30
K_I , loading n°2, $c=1$	3,24576	3,256597	0,334
G , loading n°2, $c=5$	8,356742	8,25545	-1,212
K_I , loading n°2, $c=5$	91,41522	91,528	0,123
G , loading n°3, $c=1$	1,00344E-3	1,000804E-3	- 0,263
K_I , loading n°3, $c=1$	1,00172	1,003475	0,175
G , loading n°3, $c=5$	1,86052	1,83815	-1,202
K_I , loading n°3, $c=5$	43,13380	43,2091	0,175
G , loading n°4, $c=1$	5,03571E-3	5,06348E-3	0,552
K_I , loading n°4, $c=1$	2,24404	2,25312	0,405
G , loading n°4, $c=5$	2,331095	2,302636	-1,221
K_I , loading n°4, $c=5$	48,28142	48,3188	0,078
G , loading n°5, $c=1$	1,839487E-3	1,86066E-3	1,152
K_I , loading n°5, $c=1$	1,356277	1,363914	0,563
G , loading n°5, $c=2{,}4048255577$	0	3,98377E-8	-
K_I , loading n°5, $c=2{,}4048255577$	0	4,721938E-3	-

Contour 2 (quadrangles)

$R_{inf}=0{,}04\ mm$, $R_{sup}=0{,}06\ mm$

Identification	Reference	*Aster*	% difference
G , loading n°1	3,14158E-3	3,1678E-3	0,835
K_I , loading n°1	1,77245	1,78075	0,468
G , loading n°2, $c=1$	1,05349E-2	1,056949E-2	0,328
K_I , loading n°2, $c=1$	3,24576	3,256529	0,332
G , loading n°2, $c=5$	8,356742	8,257967	-1,182
K_I , loading n°2, $c=5$	91,41522	9,1527E1	0,123
G , loading n°3, $c=1$	1,00344E-3	1,001087E-3	-0,234
K_I , loading n°3, $c=1$	1,00172	1,0034589	0,174
G , loading n°3, $c=5$	1,86052	1,838717	-1,172
K_I , loading n°3, $c=5$	43,13380	43,2088	0,174
G , loading n°4, $c=1$	5,03571E-3	5,064887E-3	0,579
K_I , loading n°4, $c=1$	2,24404	2,25307	0,402
G , loading n°4, $c=5$	2,331095	2,30333	-1,191
K_I , loading n°4, $c=5$	48,28142	48,31838	0,077
G , loading n°5, $c=1$	1,839487E-3	1,86117E-3	1,179
K_I , loading n°5, $c=1$	1,356277	1,363877	0,560
G , loading n°5, $c=2{,}4048255577$	0	4,0008E-8	-
K_I , loading n°5, $c=2{,}4048255577$	0	4,711869E-3	-

Contour 3 (quadrangles)

$R_{inf}=0{,}06\,mm$, $R_{sup}=0{,}08\,mm$

Identification	Reference	*Aster*	% difference
G , loading n°1	3,14158E-3	3,16794E-3	0,839
K_I , loading n°1	1,77245	1,78078	0,471
G , loading n°2, $c=1$	1,05349E-2	1,05699E-2	0,333
K_I , loading n°2, $c=1$	3,24576	3,2566	0,334
G , loading n°2, $c=5$	8,356742	8,25837	1,177
K_I , loading n°2, $c=5$	91,41522	91,5293	0,125
G , loading n°3, $c=1$	1,00344E-3	1,001132E-3	- 0,230
K_I , loading n°3, $c=1$	1,00172	1,003481	0,176
G , loading n°3, $c=5$	1,86052	1,838809	- 1,167
K_I , loading n°3, $c=5$	43,13380	43,20984	0,176
G , loading n°4, $c=1$	5,03571E-3	5,065103E-3	0,584
K_I , loading n°4, $c=1$	2,24404	2,25312	0,405
G , loading n°4, $c=5$	2,331095	2,303447	- 1,186
K_I , loading n°4, $c=5$	48,28142	48,31948	0,079
G , loading n°5, $c=1$	1,839487E-3	1,86124E-3	1,183
K_I , loading n°5, $c=1$	1,356277	1,363907	0,563
G , loading n°5, $c=2{,}4048255577$	0	4,00631E-8	-
K_I , loading n°5, $c=2{,}4048255577$	0	4,71155E-3	-

3.6 Remarks

contour 0 (surrounding the crack tip and made up by triangles) gives poor results compared to other contours.

The relative variations maxima between contours 1,2 and 3 for G and K_I are given below for the various loadings.

	Loading 1	Loading 2	Loading 3	Loading 4	Loading 5
Variation out of G	0,03%	0,03%	0,03%	0,03%	0,03%
Variation out of K_I	0,002%	0,002%	0,002%	0,002%	0,002%

the variations on G and K_I are negligible.

In all the cases of loading and for all contours, one also checked that K_{II} is null.

4 Summary of the results

Except for the results got on contour 0, computations of K and G are very close to the exact theoretical solution. Indeed, the variations are always lower than 1,2% for the computation of G and lower than 0,6% for the computation of K_I .