SSLP304 - Orthotropic square plate in uniaxial tension out of axes of orthotropy

Summarized:

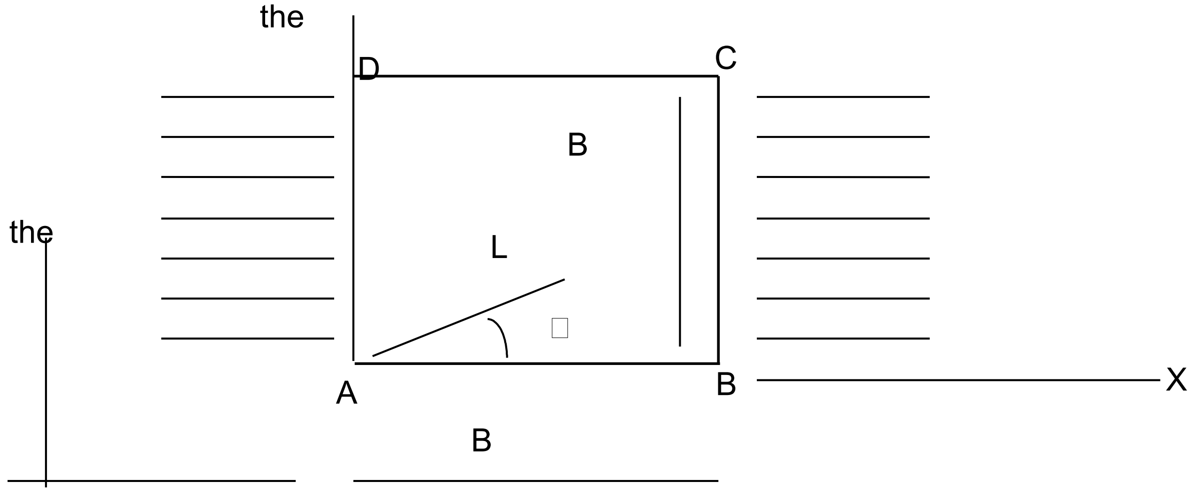
This test represents the static computation of a square plate, out of orthotropic elastic material, whose axes of othotropie are tilted of 30 degrees compared to the basic edge, subjected with a uniaxial tension. It makes it possible to validate the good taking into account of the orthotropic elastic materials and the change of associated reference. 4 modelizations are used: C_PLAN with meshes QUAD8 and TRIA6, in a first reference, C_PLAN in a second reference, COQUE_3D with meshes QUAD9 and TRIA7, in small displacements and COQUE_3D in large displacements. Displacements and the forced obtained are compared with an analytical reference solution.

The first two modelizations of this test result from the validation independent of version 3 of *Code_Aster* (linear static batch).

1 Problem of reference

1.1 Geometry

a square plate, made up of a tilted orthotropic material of 30 degrees compared to the edge AB .



With $b=1\,m$, unspecified thickness (plane stresses), angle of orthotropy: θ = 30 degrees.

1.2 Properties of the materials

the properties of the materials constituting the plate are:

orthotropic elastic:

$$E_L = 4.\mathrm{E}10\,Pa$$
$$E_T = 1.\mathrm{E}10\,Pa$$
$$G_{LT} = 0.45\mathrm{E}10\,Pa$$
$$G_{TN} = 0.35\mathrm{E}10\,Pa$$
$$NU_{LT} = 0.075$$

The axis L is tilted of 30 degrees compared to AB .

1.3 Boundary conditions and loadings

- With point: A $DX=0$, $DY=0$
- With point: B $DX=0$,

- Linear Loading distributed: $Fx=10^4\,Pa$ on BC
- linear Loading distributed: $Fx=-10^4\,Pa$ on DA

1.4 Initial conditions

Without object.

2 Reference solution

2.1 Method of calculating used for the analytical reference solution

Solution, obtained with the assumption of uniaxiality of the stresses:

$$\sigma_{xx}(x,y)=F_x \qquad \sigma_{xy}(x,y)=\sigma_{yy}(x,y)=\sigma_{zz}(x,y)=0$$

maybe in the reference (A,L,T) :

$$\sigma_{LL}(x,y)=c^2F_x,\sigma_{TT}(x,y)=s^2F_x \qquad \sigma_{LT}(x,y)=-csF_x$$

By the orthotropic elastic constitutive law, by means of conventions of *Code_Aster* with regard to NU_{LT}, (cf document of use of the command DEFI_MATERIAU [§3.5.2]), one obtains directly (see for example [bib1]):

$$\varepsilon_{xx}(x,y)=\frac{F_x}{E_x}\ ,\ \varepsilon_{yy}(x,y)=-\frac{\nu_{xy}}{E_x}F_x,\ 2\varepsilon_{xy}(x,y)=\frac{\eta_x}{E_x}F_x$$

with:

$$\frac{1}{E_x(\theta)}=\frac{c^4}{E_L}+\frac{s^4}{E_T}+c^2s^2\left(\frac{1}{G_{LT}}-2\frac{\nu_{LT}}{E_T}\right) \qquad \frac{\nu_{xy}}{E_x(\theta)}=(c^4+s^4)\frac{\nu_{LT}}{E_T}-c^2s^2\left(\frac{1}{E_L}+\frac{1}{E_T}-\frac{1}{G_{LT}}\right)$$

$$\frac{\eta_y}{E_x(\theta)}=-2cs\left\{\left(\frac{c^2}{E_L}-\frac{s^2}{E_T}\right)+\left(c^2-s^2\right)\left(\frac{\nu_{LT}}{E_T}-\frac{1}{2G_{LT}}\right)\right\}$$

avec $c=\cos\theta$
$s=\sin\theta$

As the strains are uniform in the plate one obtains, by integration, displacements in the references (A,x,y) :

$$u_x(x,y)=\varepsilon_{xx}.x$$

$$u_y(x,y)=\varepsilon_{yy}.y+2\varepsilon_{xy}.x$$

2.2 Results of reference

Displacements in the reference (A,x,y) (in m):

Point	B	C	D
u_x	0.	5.917 10-7	5.917 10-7
u_x	– 2.292 10-7	– 5.028 10-7	– 7.319 10-7

Stresses in the reference related to the orthotropy:

$$\sigma_{LL}(x,y)=7500\,Pa \qquad \sigma_{TT}(x,y)=2500\,Pa\,, \qquad \sigma_{LT}(x,y)=4330.127\,Pa$$

2.3 Uncertainty on the analytical

solution Solution

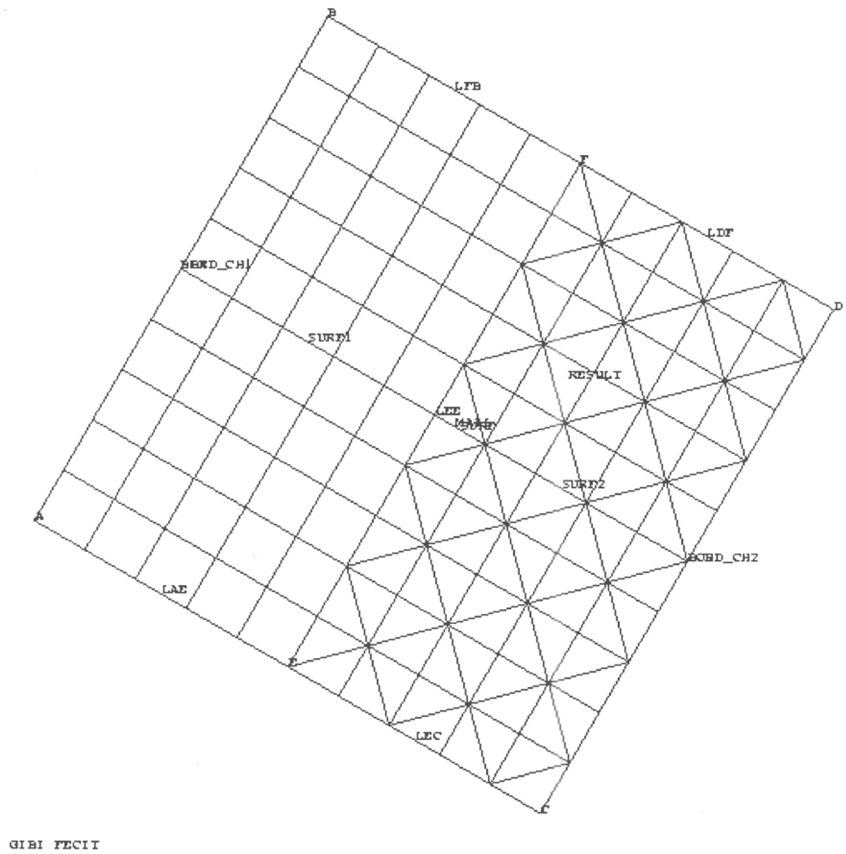
2.4 bibliographical References

1) GAY D: “Composites”; 3rd edition, Hermès

3 Modelization A

3.1 Characteristic of the modelization

Modelization C_PLAN. The plate is turned from -30 degrees around Z, i.e. the total X axis is colinéaire with the axis of orthotropy L. The boundary conditions and loadings, to apply in the reference (A, x, y) related to the plate, are thus projected on the total reference (A, X, Y) (use of LIAISON_DDL in B).



3.2 Characteristics of the mesh

Many nodes: 391

Number of meshes and types: 50 QUAD8, 100 TRIA6

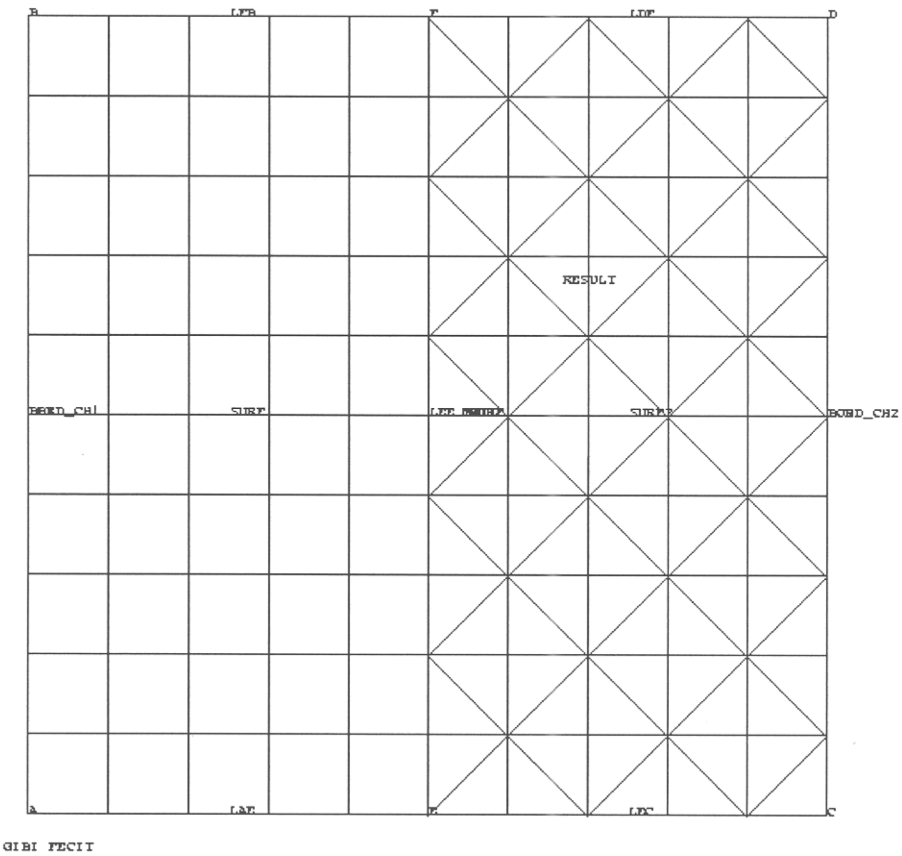
3.3 Values tested

Value	Identification	Reference
$Ux(c)=Ux(D)$	$DX(C)$	$5.917\ 10^{-7}$
$Uy(B)$	$DY(B)$	$-2.292\ 10^{-7}$
$Uy(C)$	$DY(C)$	$-5.028\ 10^{-7}$
$Uy(D)$	$DY(D)$	$-7.319\ 10^{-7}$
$Sigma\,\mathrm{LL}$	$SIXX$ (any point)	7500
$Sigma\,\mathrm{TT}$	$SIYY$ (any point)	2500
$Sigma\,\mathrm{LL}$	$SIXY$ (any point)	4300.127

4 Modelization B

4.1 Characteristic of the modelization

Modelization C_PLAN. The plate is parallel to the total axes, i.e. the total X axis is colinéaire with the axis x. It is thus the axis of orthotropy L which is to be directed (using key word MASSIF of AFFE_CARA_ELEM).



4.2 Characteristics of the mesh

Many nodes: 391

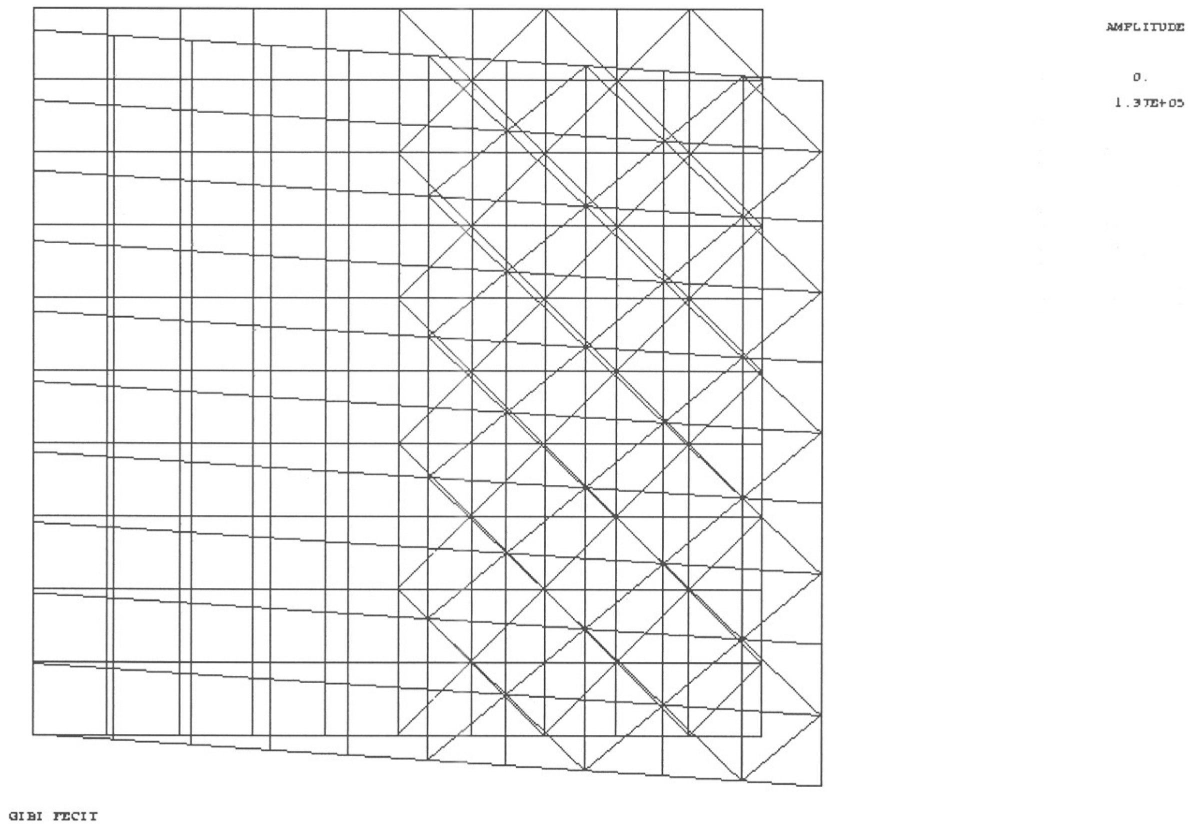
Number of meshes and types: 50 QUAD8, 100 TRIA6

4.3 Values tested

Value	Identification	Reference
$Ux(c)=Ux(D)$	$DX(C)$	$5.917\ 10^{-7}$
$Uy(B)$	$DY(B)$	$-2.292\ 10^{-7}$
$Uy(C)$	$DY(C)$	$-5.028\ 10^{-7}$
$Uy(D)$	$DY(D)$	$-7.319\ 10^{-7}$
$Sigma$ LL	$SIXX$ (any point)	7500
$Sigma$ TT	$SIYY$ (any point)	2500
$Sigma$ LL	$SIXY$ (any point)	4300.127

4.4 Remarks

Pace of the deformed shape: asymmetric because of the orthotropy.



5 Modelization C

5.1 Characteristic of the modelization

Modelization COQUE_3D. The plate is parallel to the total axes, i.e. the total X axis is colinéaire with the axis x. It is thus the axis of orthotropy L which is to be directed (using key word MASSIF of AFFE_CARA_ELEM). The mesh is identical to that of the modelization B.

5.2 Characteristic of the mesh

Many nodes: 541

Number of meshes and types: 50 QUAD9, 100 TRIA7

5.3 Values tested

Value	Identification	Reference
$Ux(c)=Ux(D)$	$DX(C)$	$5.917\ 10^{-7}$
$Uy(B)$	$DY(B)$	$-2.292\ 10^{-7}$
$Uy(C)$	$DY(C)$	$-5.028\ 10^{-7}$
$Uy(D)$	$DY(D)$	$-7.319\ 10^{-7}$
$Sigma$ LL	$SIXX$ (any point)	7500
$Sigma$ TT	$SIYY$ (any point)	2500
$Sigma$ LL	$SIXY$ (any point)	4300.127

6 Modelization D

6.1 Characteristic of the modelization

Modelization COQUE_3D in large displacements. The plate is parallel to the total axes, it is - with - to say that the total X axis is colinéaire with the axis x. It is thus the axis of orthotropy L which is to be directed (using key word MASSIF of AFFE_CARA_ELEM). The mesh is identical to that of the modelization B.

6.2 Characteristic of the mesh

Many nodes: 541

Number of meshes and types: 50 QUAD9, 100 TRIA7

6.3 Features tested

6.4 Values tested

Value	Identification	Reference
$Ux(c)=Ux(D)$	$DX(C)$	$5.917\ 10^{-7}$
$Uy(B)$	$DY(B)$	$-2.292\ 10^{-7}$
$Uy(C)$	$DY(C)$	$-5.028\ 10^{-7}$
$Uy(D)$	$DY(D)$	$-7.319\ 10^{-7}$
$Sigma$ LL	$SIXX$ (any point)	7500
$Sigma$ TT	$SIYY$ (any point)	2500
$Sigma$ LL	$SIXY$ (any point)	4300.127

7 Summary of the results

the results of the four modelizations are very close to the analytical solution: to the maximum 0.015% of variation for the 4 modelizations.

This test thus validates the taking into account of orthotropic elasticity.