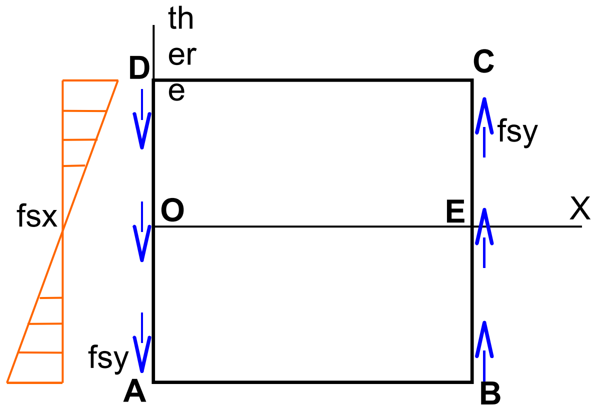
# SSLP106 – Solid mass rectangular in pure bending (test of elements QUAD4 under integrated)

**Summarized:**
One tests the finite elements under integrated into a Gauss point stabilized by the method *assumed strain* on a computation of pure bending in plane strain.

# 1 Problem of reference

## 1.1 Geometry

the geometry is a square on side $L=100\,\mathrm{mm}$ .




## 1.2 Properties of the material

the material is elastic incompressible and has as properties:

$E=100\,\mathrm{MPa}$

$\nu=0.4999$

## 1.3 Boundary conditions and loadings

Taking into account the skew-symmetric nature of the problem, one models only half of the solid mass with the following boundary conditions:

On $OE$ :

$DX(OE)=0$

On $OD$ :

$DX(O)=DY(O)=0$

$DX(D)=0$

$$fsx=\frac{8y}{L}.\sigma_d$$

$$fsy=-\left(1-\frac{4y^2}{L^2}\right).\sigma_d$$

On $BC$ :

$$fsy=+\left(1-\frac{4y^2}{L^2}\right).\sigma_d$$

With $\sigma_d$ a given stress, which one will take equal to 1 in the test. $\sigma_d=1\,\mathrm{MPa}$

# 2 Reference solution

## 2.1 Method of calculating

the reference solution comes from an analytical solution of [Bib1]:

$$u_x(x,y)=\frac{4(1-\nu^2)}{EL^2}.\left(y.(x^2-2.\mathrm{Lx})+\frac{2+\nu(1-\nu)}{3}.y.(\frac{L^2}{4}-y^2)\right).\sigma_d \quad (1)$$

And following $y$ :

$$u_y(x,y)=\frac{4(1-\nu^2)}{EL^2}\left(Lx^2-\frac{x^3}{3}-\nu.(1-\nu).y^2.(x-L)+\frac{4+5\nu.(1-\nu)}{12}.xL^2)\right).\sigma_d \quad (2$$

## 2.2 Quantities and results of reference

By applying 1, one finds displacement following $x$ with point: $C$
$u_x(L,L/2)=-1.5\,\mathrm{mm}$
And by applying 2, one finds displacement following $y$ with point: $C$ $u_y(L,L/2)=4.25\,\mathrm{mm}$

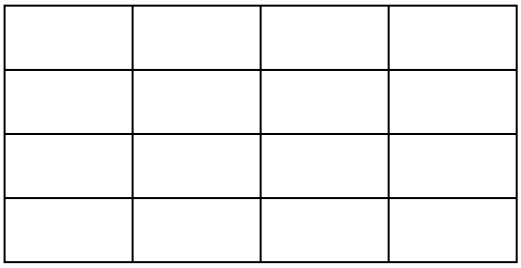
## 2.3 Bibliographical references

[Bib1] Timoshenko & Woinowsky-Krieger, “Theory of punts and shells”, McGrawHill, 1964.

# 3 Modelization A

## 3.1 Characteristic of the modelization

Modelization in plane strains on the following $2D$ mesh:



## 3.2 Characteristics of the mesh

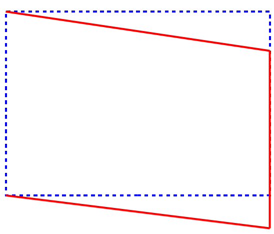
Many nodes: 25
Numbers and types of meshes: 16 SEG2, 16 QUAD4 with elements D_PLAN_SI.

## 3.3 Quantities tested and Value

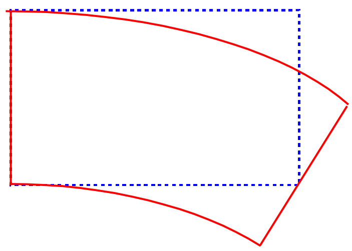
| results tested | Standard | Reference | *Variation* |
|---|---|---|---|
| Displacement $DX$ in $C$ | -1,25 | Analytical | 1.7% |
| Displacement $DY$ in $C$ | 4,25 | Analytical | 0,3% |

## 3.4 Remarks

the request is said to dominant bending. Through this computation, one shows the difficulty for the QUAD4 even under-integrated to represent the modes of strain in bending in plane strain and for a coefficient $\nu$ close to 0,5. This results in an excessive stiffness of the element due under the terms of shears of the operator discretized gradient: it is about a numerical blocking.



**QUAD4**
(important shears)



**QUAD8**

# 4 Modelization B

## 4.1 Characteristic of the modelization

One take again the preceding mesh which one passes in quadratic elements with an aim of using modelization D_PLAN_INCO (elements adapted to the incompressible problems).

## 4.2 Characteristics of the mesh

Many nodes: 65
Numbers and types of meshes: 16 SEG3, 16 QUAD8 with elements D_PLAN_INCO.

## 4.3 Quantities tested and Value

| results tested | Standard | Reference | *Variation* |
|---|---|---|---|
| Displacement $DX$ in $C$ | -1,25 | Analytical | <0.01% |
| Displacement $DY$ in $C$ | 4,25 | Analytical | <0.01% |

## 4.4 Remarks

These elements adapted to the incompressible problems give result identical to the analytical solution.

# 5 Summary of the results

the poor quality of result of elements QUAD4 under-integrated is explained by the numerical phenomenon of blocking which makes the element very rigid. Moreover, its convergence towards the analytical solution is very slow. This phenomenon appears of course also for the completely integrated element. The computation using incompressible quadratic elements allows to obtain an exact solution.