SSLP103 - Computation of the coefficients of intensity of stresses *KI* and *KII* for a circular plate fissured in linear elasticity

Summarized

It acts of a test of fracture mechanics in static linear elasticity for a two-dimensional problem. One considers a fissured circular plate (with a tilted crack of 30 degrees compared to the x-axis) for which one calculates:

- coefficients of intensity of stresses K_I and K_{II},
- rate of energy restitution G from the formula of IRWIN.

The interest of the test is to know the analytical solution which gives the coefficients of intensity of stresses and to have a tilted crack.

Two modelizations are:
- Modelization a: FEM for elements C_PLAN, D_PLAN
- Modelization b: FEM for elements D_PLAN_INCO, D_PLAN_INCO_UP

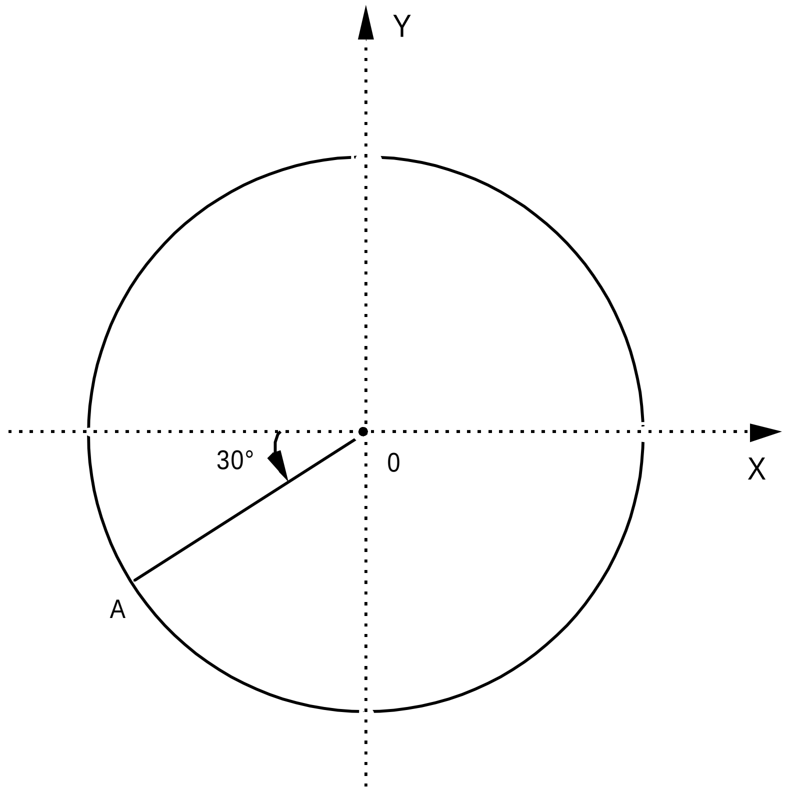
This test understands a modelization which and the treats successively the plane strains plane stresses (elements of continuums).

The numerical results do not deviate more 1% with 2% values of reference.

1 Problem of reference

1.1 Geometry

It acts of a circular plate of radius $OA=100\,mm$, with a tilted crack of 30 degrees compared to the x-axis.



1.2 Material properties

the characteristics of the material are the following ones:

$E=200\,000\,MPa$

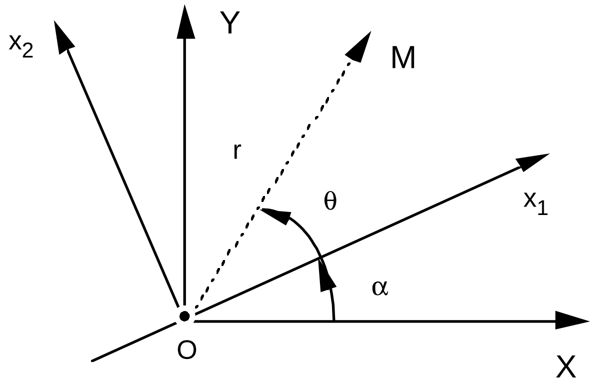
$\nu=0.3$

1.3 Boundary conditions and loadings

displacements are imposed on the contour of the plate. They result from the singular analytical solution in mixed mode (with $K_I=2.$ and $K_{II}=1.$).

2 Reference solution

2.1 Method of calculating used for the reference solution



In plane strains or plane stresses, the distribution of displacements is given in this reference $(0, x_{1,} x_2)$ by:

$$\begin{cases} u_1 = \dfrac{1+\nu}{E}\sqrt{\dfrac{r}{2\pi}}\left(K_I \cos\dfrac{\theta}{2}(k-\cos\theta)+K_{II}\sin\left(\dfrac{\theta}{2}\right)(k-\cos\theta+2)\right) \\ u_2 = \dfrac{1+\nu}{E}\sqrt{\dfrac{r}{2\pi}}\left(K_I \sin\dfrac{\theta}{2}(k-\cos\theta)-K_{II}\cos\left(\dfrac{\theta}{2}\right)(k+\cos\theta-2)\right) \end{cases}$$

with $k=3-4\nu$ in plane strains

$k=\dfrac{3-\nu}{1+\nu}$ in plane stresses

or in the reference (O, X, Y) by: $\begin{cases} u_x = \cos\alpha\, u_1 - \sin\alpha\, u_2 \\ u_y = \sin\alpha\, u_1 + \cos\alpha\, u_2 \end{cases}$

On the contour of the plate, one a: $r=OA=100\,mm$.

One chooses to take $K_I=2.$ and $K_{II}=1.$ to impose displacements on the contour of the circular plate.

2.2 Results of reference

$K_I=2.$
$K_{II}=1.$
$G=2.275\,10^{-5}$ in plane strains
$G=2.5\,10^{-5}$ in plane stresses

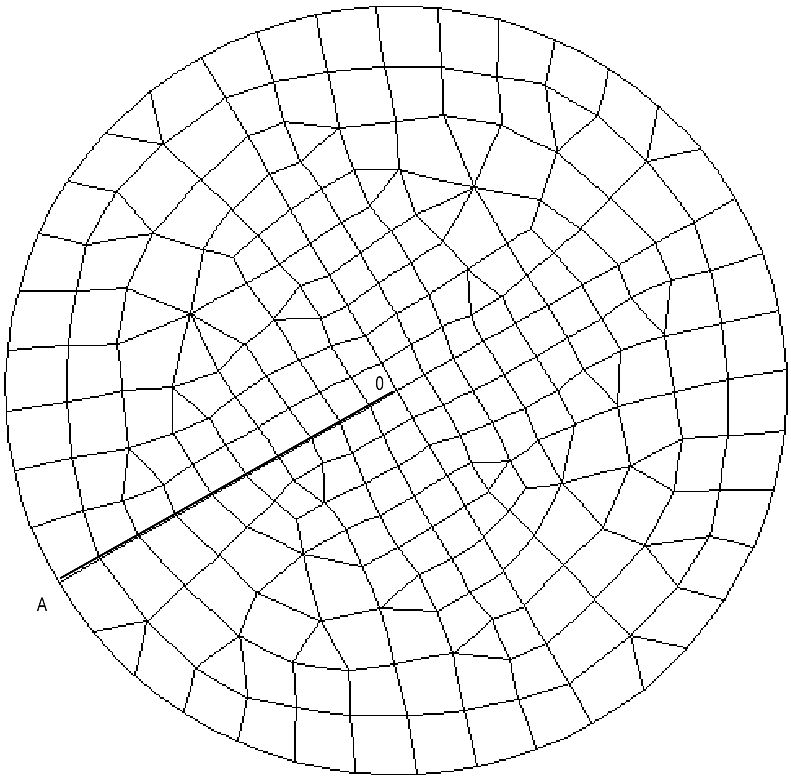
2.3 bibliographical References

1) H.D. BUI Fracture mechanics Brittle - ED. Masson 1978

3 Modelization a: FEM for elements `D_PLAN` and `C_PLAN`

3.1 Characteristics of the modelization

The computation is carried out in plane stresses (`C_PLAN`) then in plane strains (`D_PLAN`).



3.2 Characteristics of the mesh

Many nodes: 737

Number of meshes and types: 204 meshes `QUAD8`, 30 meshes `TRIA6`

3.3 Quantities tested and results

the values tested are the coefficients of intensity of stresses K_I and K_{II} the rate of energy restitution G calculated by the formula of IRWIN:

Identification	**Reference**	***Aster***	$\%$ **difference**
Plane stresses			
K_I	2.0	2.0067	0.33
K_{II}	1.0	0.9877	1.23
G	$2.5\ 10^{-5}$	$2.5213\ 10^{-5}$	0.85
Plane strains			

K_I	2.0	2.0030	0.15
K_{II}	1.0	0.9960	0.39
G	2.275 10 $^{-5}$	2.2968 10 $^{-5}$	0.96

3.4 Remarks

the formula of IRWIN gives: $G=\frac{(1-\nu^2)}{E}(K_I^2+K_{II}^2)$ in plane strains

and $G=\frac{1}{E}(K_I^2+K_{II}^2)$ plane stresses.

Computations are carried out with a lower integration contour of radius 10.0 and higher radius 20.0 .

4 Modelization b: FEM for elements D_PLAN_INCO, D_PLAN_INCO_UP

4.1 Characteristics of the modelization

Identical to the modelization A except the use of elements D_PLAN_INCO, D_PLAN_INCO_UP.

4.2 Characteristics of the mesh

Identical to the modelization A

4.3 Quantities tested and results

the values tested are the coefficients of intensity of stresses K_I and K_{II} and rate of energy restitution G calculated by the formula of IRWIN:

Identification	Standard	Method	Reference of reference	% tolerance
G	CALC_G	2.275E-5	ANALYTIQUE	2
$K1$	CALC_K_G	2	ANALYTIQUE	2
$K2$	CALC_K_G	1	ANALYTIQUE	7
G	CALC_K_G	2.275E-5	ANALYTIQUE	2
G_{IRWIN}	CALC_K_G	2.275E-5	ANALYTIQUE	3

5 Summaries of the results

the numerical values of the coefficients of intensity of stresses and from the rate of refund of energy do not deviate more than 1 with 2% values of reference, which is satisfactory.

The mesh could be improved, in particular in the vicinity of the crack tip.