SSLL404 - Buckling of an arch

Summarized
the scope of application of this test is the analysis of stability of structures. The studied structure is an arch bent by applied moments at the two ends; it is modelled by beam elements rights. The goal is to calculate the breaking values of the moments.

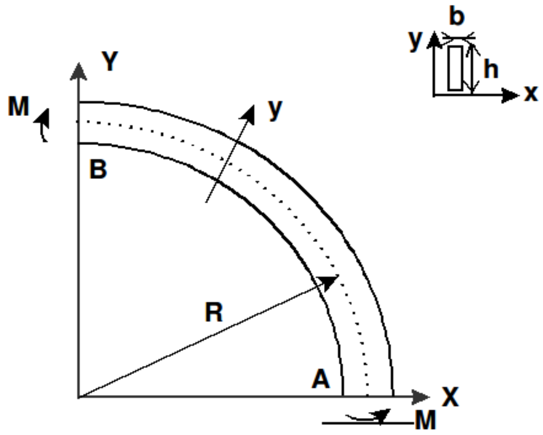
The interest of this test lies in the following aspects:

- computation of a geometrical stiffness matrix for elements POU_D_E.
- test of modal methods MODE_ITER_SIMULT and MODE_ITER_INV of stability
- presence of close eigenvalues

the calculated clean loads are compared with values obtained analytically for a model of beam of Eulerian-Bernoulli.
In this test, one also validates option RAYLEIGH of the command MODE_ITER_INV.

1 Problem of reference

1.1 Geometry



Radius of curvature	$R=0.3\,m$
Height of the profile	$h=0.015\,m$
Width of the profile	$b=0.002\,m$
Section	$S=bh$
the 1st inertia of bending	$I_X=bh^3/12$
the 2nd inertia of bending	$I_Y=hb^3/12$
Inertia of torsion	$J=hb^3/3$

1.2 Properties of

Boundary conditions	$E=7.E\,10\,N/m^2$
the materials	$\nu=0.3$
Young's modulus Poisson's ratio Modulus of rigidity	$G=E/2(1+\nu)$

1.3 and loading

the beam bi--is supported. One prevents the torsion of the section at the ends A and B. To respect the assumptions of the ideal model taken as reference, it is important that the moment is constant and that the normal force is null along the beam. This is why one leaves free displacement u according to X the point B. The boundary conditions are:

- With point: A $u=v=w=0$; $\Phi_Y=0$
- With point: B $v=w=0$; $\Phi_X=0$

The initial stress state which allows to carry out the analysis of stability is obtained by imposing one bending moment around the axis Z, at the points A and B : $M=1\,Nm$

1.4 Initial conditions

Without object in static analysis of stability.

2 Reference solution

2.1 Method of calculating used for the reference solution

the reference solution is obtained analytically for a beam of Eulerian-Bernoulli. The theoretical aspects are developed in the reference [bib1].
By means of the notations of the paragraph [§1], the breaking values are given by the statement:

$$M_{CR}=-\frac{EI_x+GJ}{2R}\pm\sqrt{\left(\frac{EI_x-GJ}{2R}\right)^2+4n^2\frac{EI_x GJ}{R^2}}\quad n=1,2,3,\ldots$$

The plus sign corresponds to positive moments such as they are indicated on the figure of [§1.1].

2.2 Results of reference

the first 5 critical loads are classified by order of increasing modulus.

Moment	**mode criticizes (Nm)**
1	2.86074
2	8.63207
3	– 8.78382
4	14.4147
5	– 14.5551

With *Code_Aster* , one finds the opposites of these critical loads (what is logical compared to the formulation of the problem to solve).

2.3 Uncertainty on the analytical

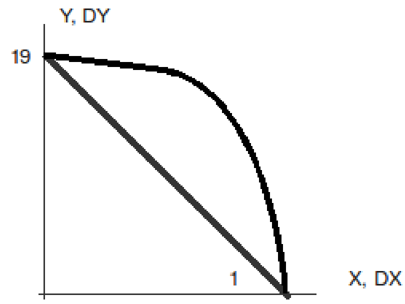
solution Solution

2.4 bibliographical References

[1] TIMOSHENKO Stephen P., MANAGES James Mr., Theory of Elastic Stability, McGraw-Hill, International Edition, 1963, pp. 313-318.

3 Modelization A

3.1 Characteristic of the modelization



the arch is with a grid by means of beam elements right of the type POU_D_E .
Boundary conditions:

- At the point A such as $X=R$ $Y=0$: $DX=DY=DZ=0$ and $RY=0$
- At the point B such as $X=0$ $Y=R$: $DY=DZ=0$ and $RX=0$

For the static analysis, of the unit moments around Z are defined with nodes 1 and 19.

3.2 Characteristics of the mesh

Many nodes: 19
Number of meshes: 18 POU_D_E

3.3 Quantities tested and results

Critical load

3.3.1 MODE_ITER_SIMULT with METHODE = "SORENSEN"

Identification N° critical load	Reference (multiplied by -1)	*Code_Aster*	% difference
1	– 2.86074	– 2.75137	3.823
2	– 8.63207	– 8.30613	3.776
3	8.78382	8.39554	4.420
4	– 14.4147	– 13.93216	3.348
5	14.5551	14.01104	3.738

3.3.2 MODE_ITER_INV with OPTION = "NEAR"

Identification N° critical load	Reference (multiplied by -1)	*Code_Aster*	% difference
1	– 2.86074	– 2.75137	3.823
2	– 8.63207	– 8.30613	3.776
3	8.78382	8.39554	4.420
4	– 14.4147	– 13.93216	3.348
5	14.5551	14.01104	3.738

3.3.3

3.3.4 MODE_ITER_INV with OPTION = "SEPARATE"

Identification N° critical load	Reference (multiplied by -1)	*Code_Aster*	% difference
1	–2.86074	– 2.75137	3.823
2	– 8.63207	– 8.30613	3.776
3	8.78382	8.39554	4.420
4	– 14.4147	– 13.93216	3.348
5	14.5551	14.01104	3.738

3.3.5 MODE_ITER_INV with OPTION = "ADJUST"

Identification N° critical load	Reference (multiplied by -1)	*Code_Aster*	% difference
1	– 2.86074	– 2.75137	3.823
2	– 8.63207	– 8.30613	3.776
3	8.78382	8.39554	4.420
4	– 14.4147	– 13.93216	3.348
5	14.5551	14.01104	3.738

4 Summary of the results

the methods of Sorensen and the inverse iterations (OPTION=' PROCHE' or "SEPARATED" or "ADJUSTS") give identical and satisfactory results since the maximum change with the analytical solution is lower than 4.5% . It is pointed out that the analytical solution takes into account the curvature of structure.
Elements MEPOUCT could not be used in this test because the computation of the geometrical stiffness matrix is not available for this kind of element.