SSLL400 - Non-prismatic beam, subjected to forces specific or distributed

Summarized:

This test is resulting from the validation independent of version 4 of the models of beams.

This test allows the checking of computations of straight beams in the linear static field. (a modelization with beam elements POU_D_E, straight beam of EULER).

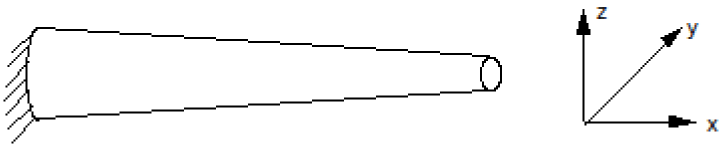
One calculates simultaneously 3 beams of the different sections: section rings, right-angled, and general. These beams are subjected to forces specific or distributed.

The values tested are displacements and rotations, the generalized forces, and the forced.

1 Problem of reference

1.1 Geometry

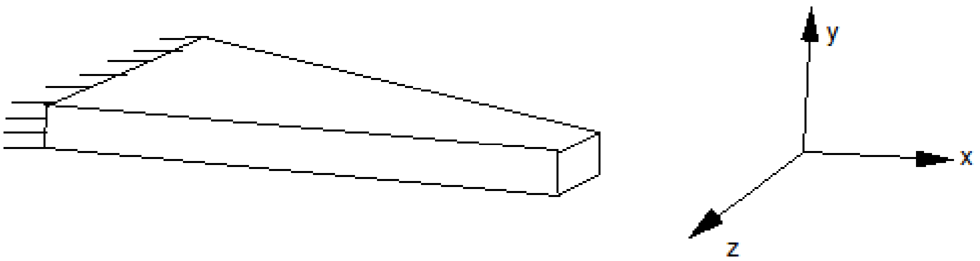
1.1.1 Straight beam of circular section variable



Appears 1.1.1-a : Beam with variable circular section.

Length	: $1\,m$
Radius with the fixed support	: $0{,}1\,m$
Radius at the loose lead	: $0{,}05\,m$

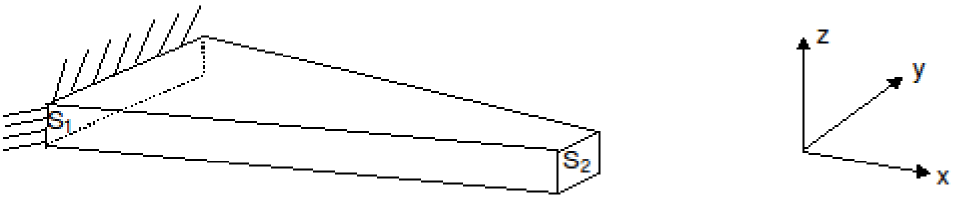
1.1.2 Straight beam of rectangular section variable



Appears 1.1.2-a : Beam with rectangular section variable

Length	:	$1\,m$	
with the fixed support	:	$Hy=0{,}05\,m$	$Hz=0{,}10\,m$
at the loose lead	:	$Hy=0{,}05\,m$	$Hz=0{,}05\,m$

1.1.3 Straight beam of general section variable



Appears 1.1.3-a : Beam with general section variable

Length	:	$1\,m$	
with the fixed support	:	$A=10^{-2}\,m^2$	$Iy=8{,}3333\,10^{-6}\,m^4$
at the loose lead	:	$A=2{,}5 10^{-3}\,m^2$	$Iy=5{,}20833\,10^{-7}\,m^4$

1.2 Properties of the materials

Modulus Young:	$E=2.10^{11}\,Pa$
Poisson's ratio:	$\nu=0{,}3$

Density: $\rho=7800\,kg.m^{-3}$

1.3 Boundary conditions and loading

Boundary condition:
Clamped end: $DX=DY=DZ=DRX=DRY=DRZ=0$

Loading:
To the straight beam of circular section variable and to the straight beam of rectangular section variable, one applies successively:

Natural	loading case
1	a specific force following X at the loose lead, $Fx=100\,N$
2	a specific force following Y at the loose lead, $Fy=100\,N$
3	one specific moment around the axis X at the loose lead, $Mx=100\,m.N$
4	one specific moment around the axis Z at the loose lead, $Mz=100\,m.N$
5	a distributed load on the group of the beam, $fx=100\,N.m^{-1}$
6	a distributed load on the group of the beam, $fy=100\,N.m^{-1}$

On the straight beam of general section variable, one applies:

Natural	loading cases
7	a force of gravity following z with $g=9{,}81\,m.s^{-2}$

2 Reference solutions

2.1 Method of calculating used for the reference solutions

2.1.1 Section circular

2.1.1.1 Beam subjected to a specific tractive effort Fx

the balance equation are:

$$\frac{\partial}{\partial x}\left(EA(x)\frac{\partial u}{dx}\right)=0 \text{ with } A(x)=A_1\left(1+c\frac{x}{L}\right) \text{ and } c=\sqrt{\frac{A_2}{A_1}}-1\text{ , } N(L)=F_x$$

While integrating twice [R3.08.01], we obtain displacements according to the applied force, that is to say:

$$u(x)=\frac{LF_x}{EA_1}\left(\frac{x}{L+cx}\right)$$

and thus at the end L of beam:

$$u(L)=\frac{L}{E\sqrt{A_1 A_2}}F_x$$

The internal forces are given by:

$$N(x)=EA(x)\,\frac{\partial u}{\partial x}(x)=F_x$$

and the forced by:

$$\sigma_{xx}=\frac{N(x)}{A(x)}$$

2.1.1.2 Beam subjected to a specific bending stress Fy

the balance equation, under the assumption of Eulerian, is given by the equation:

$$\frac{\partial^2}{\partial x^2}\left[EI_z(x)\frac{\partial^2 v}{\partial x^2}\right]=0 \text{ with } I_z(x)=I_{z_1}\left(1+c\frac{x}{L}\right)^4 \text{ and } c=\left(\frac{I_{z_2}}{I_{z_1}}\right)^{\frac{1}{4}}-1\text{ , } V_y(L)=F_y$$

We solve the equation by integration by taking account of the amended constitutive law $MF_z=EI_z\frac{\partial^2 v}{\partial x^2}$ and the balance equation $\frac{\partial MF_z}{\partial x}+V_y=0$

four integrations successive, by taking account for the computation of the constants of integration that:

$$\frac{\partial}{\partial x}\left[EI_z(x)\frac{\partial^2 v}{\partial x^2}\right](L)=-V_y(L)=-F_y$$

$$\left(EI_z(L)\frac{\partial^2 v}{\partial x^2}\right)(L)=0$$

$$\frac{\partial v}{\partial x}(0)=0$$

$$v(0)=0$$

lead to the statement of:

$$v(x)=+\frac{F_y L^2}{6\ E\ I_{z_1}}\ \frac{x^2(3\ L\ -\ x\ +\ 2\ cx)}{(L+cx)^2}$$

and to the statement of $\theta_z(x)$

$$\theta_z(x)=+\frac{F_y L^2}{6\ E\ I_{z_1}}\ \frac{x\left(6\ L^2-3\ Lx+6\ L\ cx-cx^2+2\ c^2\ x^2\right)}{(L+cx)^3}$$

the internal forces are given by:

$$V_y(x)=F_y$$
$$MF_z(x)=F_y(L-x)$$

and the forced by:

$$\sigma_{xx}(x)=\left|MF_z(x)\right|\ \frac{R(x)}{I_z(x)}$$

$\sigma_{xy}=\dfrac{V_y(x)}{A(x)}$ no the coefficient of correction of the shears in assumption of Eulerian.

2.1.1.3 Beam subjected to one specific twisting moment Mx

motion is given by the equation:

$$\frac{\partial}{\partial x}\left[G\ I_p(x)\frac{\partial\theta_x}{\partial x}\right]=0 \text{ with } I_p(x)=I_{p_1}\left(1+c\frac{x}{L}\right)^4 \text{ and } c=\left(\frac{I_{p_2}}{I_{p_1}}\right)^{\frac{1}{4}}-1\ ,\ M_x(L)=M_x$$

After integration, and by taking account owing to the fact that:

$$G\ I_p(L)=\frac{\partial\theta_x}{\partial x}(L)=M_x \text{ , and } \theta_x(0)=0$$

we obtain the statement of $\theta_x(x)$:

$$\theta_x(x)=\frac{L\ M_x}{3\ G\ I_{p_1}}\ \frac{x\ (3\ L^2+3\ L\ cx+c^2\ x^2)}{\left(L+cx\right)^3}$$

We must for the internal forces and the have also forced:

$$M_x(x)=M_x$$
$$\sigma_{xy}(x)=\frac{M_x(x)}{I_p(x)}R_T(x)$$
$$\sigma_{xz}(x)=\frac{M_x(x)}{I_p(x)}R_T(x)$$

2.1.1.4 Beam subjected to one specific bending moment My

the reasoning to find the solution analytical is the same one as previously. We use the constitutive law $M_y(x)=-EI_y(x)\dfrac{\partial^2 w}{\partial x^2}$ and the balance equation $\dfrac{\partial MF_y}{\partial x}-V_z=0$ The computation constants of integration differs: one has $V_z(L)=0$ and $M_{F_y}(L)=M_y$.

One obtains the statement of $w(x)$:

$$w(x)=\frac{L\ M_y}{6\ E\ I_{y_1}}\ \frac{x^2(3\ L+2\,cx)}{(L+cx)^2}\ ,$$

and the statement of $\theta_y(x)$:

$$\sigma_y(x)=\frac{L\,M_y}{3\,E\,I_{y1}}\ \frac{x\left(3\,L^2+3\,L\,cx+c^2\,x^2\right)}{(L+cx)^3}$$

One must for the internal forces and the have also forced:

$$V_z(x)=0$$
$$MF_y(x)=My$$
$$\sigma_{xx}(x)=\left|MF_y(x)\right|\ \frac{R(x)}{I_y(x)}$$

2.1.1.5 Beam subjected to a tractive effort regularly distributed fx

the equilibrium is described by the equation

$$\frac{\partial}{\partial x}\left(EA(x)\ \frac{\partial u}{\partial x}\right)=-f_x \text{ with } A(x)=A_1\left(1+c\frac{x}{L}\right)^2 \text{ and } c=\left(\frac{A_2}{A_1}\right)^{\frac{1}{2}}-1$$

By integrating for the first time this equation, we obtain:

$$E\,A(x)\frac{\partial u}{\partial x}=-f_x\,x+c_1$$

The limiting condition $N(L)=0$ implies $c_1=f_x L$. We thus have:

$$\frac{\partial u}{\partial x}=-f_x\frac{(L-x)}{E\ A(x)}$$

that is to say:

$$u(x)=f_x\int\frac{(L-x)}{E\ A(x)}\,dx+c_2$$

c_2 is given so that $u(0)=0$

Taking everything into account, we have:

$$u(x)=\frac{L^2\,f_x}{E\ A_1\,c^2}\ \frac{c\ x+c^2\,x+(L+c\ x)\log\frac{L}{L+c\ x}}{L+c\ x}\ .$$

The internal forces are deduced from the constitutive law $N(x)=E\ A(x)\frac{\partial u}{\partial x}$:

$$N(x)=f_x(L-x)$$

and the forced are given by:

$$\sigma_{xx}(x)=\frac{N(x)}{A(x)}=\frac{f_x(L-x)}{\left[\sqrt{A_1}+\left(\sqrt{A_2}-\sqrt{A_1}\right)\frac{x}{L}\right]^2}$$

2.1.1.6 Beam subjected to a bending stress distributed regularly fy

On the basis of the balance equation:

$$\frac{\partial^2}{\partial x^2}\left[E\ I_z(x)\frac{\partial^2 v}{\partial x^2}\right]=-f_y \text{ with } I_z(x)=I_{z_1}\left(1+c\frac{x}{L}\right)^4 \text{ and } c=\left(\frac{I_{z_2}}{I_{z_1}}\right)^{\frac{1}{4}}-1$$

we carry out four successive integrations. The determination of the constants of integration is made starting from the following limiting conditions:

$$V_y(L)=0$$
$$M_z(L)=0$$
$$\frac{\partial v}{\partial x}(0)=0$$
$$v(0)=0$$

The analytical statement for $v(x)$ and $\theta(z)$ in the presence of a distributed loading is, taking everything into account:

$$v(x)=\frac{-f_y L^3}{12\,EI_{z_1}c^4(L+cx)^2}\left[\begin{array}{c}-6L^2cx+x^2\left(-9Lc^2-3Lc^4\right)+x^3\left(-2c^3+2c^4-2c^5\right)\\ +\log\left(1+c\frac{x}{L}\right)(6L^3+12L^2cx+6Lc^2x^2)\end{array}\right]$$

$$\theta_z(x)=\frac{+L^3 f_y x}{6EI_{z_1}(L+cx)^3}\left[3L^2-3Lx+3Lcx+x^2\left(1-c+c^2\right)\right]$$

The internal forces are given by:

$$V_y(x)=f_y(L-x) \text{ and the } Mf_z(x)=\frac{1}{2}f_y(L-x)^2$$

forced by:

$$\sigma_{xy}(x)=\frac{V_y(x)}{A(x)}$$
$$\sigma_{xx}(x)=\left|Mf_z(x)\right|\frac{R(x)}{I_z(x)}$$

2.1.2 Rectangular section

2.1.2.1 Beam subjected to a specific tractive effort Fx

the balance equation is:

$$\frac{\partial}{\partial x}\left(EA(x)\frac{\partial u}{\partial x}\right)=0 \text{ with } A(x)=A_1+(A_2-A_1)\frac{x}{L},\ N(L)=F_x$$

While integrating twice, and by taking account owing to the fact that:

$$E\ A(L)\frac{\partial u}{\partial x}(L)=F_x,\ u(0)=0$$

for the determination of the constants of integration, we obtain the analytical statement of $u(x)$, that is to say:

$$u(x)=\frac{F_x L}{A_1 Ec}\log\left(1+c\frac{x}{L}\right)$$

For the internal forces and stresses, we have:

$$N(x)=F_x$$

$$\sigma_{xx}=\frac{N(x)}{A(x)}$$

2.1.2.2 Beam subjected to a specific bending stress Fy

motion is given by the equation:

$$\frac{\partial^2}{\partial x^2}\left[E\ I_z(x)\frac{\partial^2 v}{\partial x^2}\right]=0 \text{ with } I_z(x)=I_{z_1}\left(1+c\frac{x}{L}\right)^3 \text{ and } c=\left(\frac{I_{z_2}}{I_{z_1}}\right)^{\frac{1}{3}}-1\ ,\ V_y(L)=F_y$$

the same reasoning that for the circular section leads to result according to:

$$v(x)=-\frac{F_y L^2}{2EI_{z_1}c^3}\ \frac{\left[2Lcx+c^2x^2-c^3x^2+2L(L+cx)\log\left(\frac{L}{L+cx}\right)\right]}{(L+cx)}$$

$$\theta_z(x)=\frac{F_y L^2}{2EI_{z_1}}\frac{x(2L-x+cx)}{(L+cx)^2}$$

We must for the internal forces and the have forced:

$$V_y(x)=F_y$$

$$M_{F_z}(x)=F_y(L-x)$$

$$\sigma_{xx}(x)=\frac{H_y(x)M_{F_x}(x)}{2\mathrm{I}_z(x)}$$

$$\sigma_{xy}(x)=\frac{V_y(x)}{A(x)}$$

2.1.2.3 Beam subjected to one specific twisting moment Mx

motion is given by the equation:

$$\frac{\partial}{\partial x}\left[G\ I_p(x)\frac{\partial\theta_x}{\partial x}\right]=0 \text{ with } I_p(x)=I_{p_1}\left(1+c\frac{x}{L}\right)^3 \text{ and } c=\left(\frac{I_{p_2}}{I_{p_1}}\right)^{\frac{1}{3}}-1\ ,\ M_x(L)=M_x$$

By the same reasoning as the beam with circular section, we obtain the analytical statement of $\theta_x(x)$:

$$\theta_x(x)=\frac{L\ M_x x(2L+cx)}{2\ I_{p_1}G(L+cx)^2}$$

I_{p_1} and I_{p_2} are calculated according to the formulas given in documentation of reference [R3.08.01].
The internal forces and the forced are given by:

$$M_x(x)=M_x$$

$$\sigma_{xy}(x)=\frac{M_x(x)}{I_p(x)}R_T(x)=\sigma_{xz}$$

2.1.2.4 Beam subjected to one specific bending moment My

One takes again the same reasoning that previously, one obtains the following analytical statements for $w(x)$ and $\theta_y(x)$:

$$w(x)=-\frac{L\ M_y x^2}{2\,EI_{y_1}(L+cx)}$$

$$\theta_y(x)=\frac{L\ M_y\,x(2\,L+cx)}{2\ E\ I_{y_1}(L+cx)^2}$$

for the forces:

$$V_z(x)=0$$
$$MF_y(x)=M_y$$

and for the stresses:

$$\sigma_{xx}(x)=\frac{H_z(x)\ \ MF_y(x)}{2\ I_y\ (x)}$$

2.1.2.5 Beam subjected to a tractive effort regularly distributed Fx

the balance equation is:

$$\frac{\partial}{\partial x}\left[EA(x)\frac{\partial u}{\partial x}\right]=-f_x \text{ with } A(x)=A_1\left(1+c\frac{x}{L}\right) \text{ and } c=\left(\frac{A_2}{A_1}\right)-1\ .$$

After two integrations and by taking account owing to the fact that $N(L)=0$ to determine the first constant of integration, and $u(0)=0$ to determine the second, we obtain the analytical statement of $u(x)$:

$$u(x)=\frac{-L\,f_x}{E\ A_1\ c^2}\left[c\ x+(L+L_c)\log\left(\frac{L}{L+c\ x}\right)\right]$$

The internal forces are known by the following statement:

$$N(x)=f_x(L-x)$$

and the forced by:

$$\sigma_{xx}(x)=\frac{f_x(L-x)}{A(x)}$$

2.1.2.6 Beam subjected to a bending stress regularly distributed Fy

the balance equation is:

$$\frac{\partial^2}{\partial x^2}\left[EI_z(x)\frac{\partial^2 v}{\partial x^2}\right]=-f_y \text{ with } I_z(x)=I_{z_1}\left(1+c\frac{x}{L}\right)^3 \text{ and } c=\left(\frac{I_{z_2}}{I_{z_1}}\right)^{\frac{1}{3}}-1$$

We integrate successively four times this equation. The constants of integration are calculated by taking account owing to the fact that:

$$V_y(L)=0$$
$$MF_z(L)=0$$
$$\frac{\partial v}{\partial x}(0)=0$$
$$v(0)=0$$

The result analytical one for the deflection and rotation L is the following:

$$v(x)=\frac{L^3 f_y}{4\ E\ I_{z_1}\ c^4(L+c\ x)}\left[x\left(6\ Lc+4\ Lc^2\right)+x^2\left(5c^2+2c^3-c^4\right)\right]$$
$$+\left(6L^2+4L^2c+8Lcx+4Lc^2x+2c^2x^2\right)\log\left(\frac{L}{L+cx}\right)$$
$$\theta_z(x)=\frac{L^3 f_y}{4\,EI_{z_1}\,c^3(L+cx)^2}\left[x\left(2\,Lc+2\,Lc^3\right)+x^2\left(3\,c^2+2\,c^3-c^4\right)\right]$$
$$+\left(2\,L^2+Lcx+2\,c^2\ x^2\right)\log\left(\frac{L}{L+cx}\right)$$

The internal forces are given by the following statements:

$$V_y(x)=f_y(L-x)\ \ ,\ \ Mf_z(x)=\frac{1}{2}f_y(L-x)^2$$

stresses by:

$$\sigma_{xy}(x)=\frac{V_y(x)}{A(x)}$$
$$\sigma_{xx}(x)=\left|\frac{Mf_z(x)\,h_y}{I_z(x)2}\right|$$

2.1.3 General section

2.1.3.1 Beam subjected to the forces of gravity

the forces of gravity are applied along the axis z. The motion of the beam induced by these forces is thus a motion of bending in the plane $(x\,o\,z)$.
The balance equation is given by the statement:

$$\frac{\partial^2}{\partial x^2}\left(E\ I_y(x)\frac{\partial^2 w}{\partial x^2}\right)=\underbrace{\rho\,A(x)\ g}_{poids\,linéique}$$

with $A(x)=A_1\left(1+c\dfrac{x}{L}\right)^2$, $c=\left(\dfrac{A_2}{A_1}\right)^{\frac{1}{2}}-1$ and $I_y(x)=I_{y1}\left(1+d\dfrac{x}{L}\right)^4$ $d=\left(\dfrac{I_{y2}}{I_{y1}}\right)^{\frac{1}{4}}-1$

While integrating for the first time, we obtain the internal shears:

$$V_z(x)=-\int\ \rho\ A(x)\ g\ dx+C_1$$

C_1 is given so that $V_z(L)=0$.

We obtain:

$$V_z(x)=\frac{L A_1 \rho g}{3c}\left[-\left(1+c\frac{x}{L}\right)^3+(1+c)^3\right].$$

While integrating for the second time, we obtain the internal bending moment:

$$M_y(x)=\int V_z(x)\,dx+c_2$$

C_2 is calculated so that $M_y(L)=0$

We obtain:

$$M_y(x)=\frac{A_1 \rho g}{12 L^2}(L-x)^2\left(6L^2+8L^2c+3L^2c^2+4Lcx+2Lcx+\frac{2Lc^2x}{+c^2x^2}\right)$$

We calculate then rotation from the constitutive law $M_y(x)=EI_y(x)\frac{\partial \theta_y}{\partial x}$

We thus have $\theta_y(x)=\int \frac{M_y(x)}{EI_y(x)}dx+C_3$ with $\theta_y(0)=0$

the deflection $w(x)$ is determined from the relation of Eulerian: $\theta_y=-\frac{\partial w}{\partial x}$

We calculate $w(x)$ by integration of $\theta_y(x)$: $w(x)=-\int \theta_y(x)+C_4$

with C_4 such as $w(0)=0$.

The analytical statements of $\theta_y(x)$ and $w(x)$ are not retranscribed here because they are too much heavy. They were calculated, like the preceding ones, by the formal computation software MATHEMATICA.

2.2 Results of reference

- Displacements and rotations at the loose lead
- Internal forces at the two ends
- Forced at the two Uncertainty

2.3 ends on the analytical

solution Solution.

2.4 Bibliographical references

[1] Ratio n° 2314/A of the Institute Aerotechnics “Proposal and realization for new cases tests missing to the validation of beams ASTER”

3 Modelization A

3.1 Characteristic of the modelization

The model is composed of 10 elements straight beam of Eulerian.

S1 section: variable circular section

with the fixed support,	$R1=0.1\,m$ (full section)
the loose lead,	$R2=0.05\,m$ (full section)

S2 Section: variable rectangular section

with the fixed support,	$H_{y1}=0.05\,m$	$H_{z1}=0.10\,m$
the loose lead,	$H_{y2}=0.05\,m$	$H_{z2}=0.05\,m$

S3 Section: variable general section

with the fixed support,	$A_1=10^2\,m^4$	$I_{y1}=8.3333\,10^6\,m^4$
the loose lead,	$A_2=2.5\,10^3\,m^2$	$I_{y2}=5.20833\,10^7\,m^4$

3.2 Characteristic of mesh

3 sections $\times$ 10 elements `POU_D_E`

3.3 Quantities tested and results

Loading case	Section	Identification	Reference	Tolerance %
1	$S1$	$u(l)$	3.1831E-08	1.00E-05
		$n(0)$	1.0000E+02	1.00E-05
		$n(l)$	1.0000E+02	1.00E-05
		$\sigma_{xx}(0)$	3.1831E+03	1.00E-05
		$\sigma_{xx}(l)$	1.2732E+04	1.00E-05
2	$S1$	$v(l)$	4.2441E-06	1.00E-05
		$\theta_z(l)$	8.4882E-06	1.00E-05
		$v_y(0)$	1.0000E+02	1.00E-05
		$v_y(l)$	1.0000E+02	1.00E-05
		$mf_z(0)$	1.0000E+02	1.00E-05
		$mf_z(l)$	0.0000E+00	1.00E-05
		$\sigma_{xx}(0)$	1.2732E+05	1.00E-05
		$\sigma_{xx}(l)$	0.0000E+00	1.00E-05
		$\sigma_{xy}(0)$	3.1831E+03	1.00E-05
		$\sigma_{xy}(l)$	1.2732E+04	1.00E-05
3	$S1$	$\theta_x(l)$	3.8621E-05	1.00E-05
		$m_x(0)$	1.0000E+02	1.00E-05
		$m_x(l)$	1.0000E+02	1.00E-05
		$\sigma_{xy}(0)$	6.3661E+04	1.00E-05
		$\sigma_{xy}(l)$	5.0929E+05	1.00E-05
		$\sigma_{xz}(0)$	6.3661E+04	1.00E-05

		$\sigma_{xz}(l)$	5.0929E+05	1.00E-05
4	*S1*	$w(l)$	– 8.4882E-06	1.00E-05
		$\theta_y(l)$	2.9708E-05	1.00E-05
		$v_z(0)$	0.0000E+00	1.00E-05
		$v_z(l)$	0.0000E+00	1.00E-05
		$mf_y(0)$	1.0000E+02	1.00E-05
		$mf_y(l)$	1.0000E+02	1.00E-05
		$\sigma_{xx}(0)$	1.2732E+05	1.00E-05
		$\sigma_{xx}(l)$	1.0185E+06	1.00E-05
5	*S1*	$u(l)$	1.2296E-08	1.00E-02
		$n(0)$	1.0000E+02	1.00E-05
		$n(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xx}(0)$	3.1831E+03	1.00E-05
		$\sigma_{xx}(l)$	0.0000E+00	1.00E-03 (absolute)
6	*S1*	$v(l)$	1.3486E-06	1.00E-02
		$\theta_z(l)$	2.1220E-06	1.00E-02 (absolute)
		$v_y(0)$	1.0000E+02	1.00E-05
		$v_y(l)$	0.0000E+00	1.00E-03 (absolute)
		$mf_z(0)$	5.0000E+01	1.00E-02
		$mf_z(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xx}(0)$	6.3662E+04	1.00E-05
		$\sigma_{xy}(0)$	3.1831E+03	1.00E-05

Loading case	Section	Identification	Reference	Variation %
1	*S2*	$u(l)$	1.3862E-07	1.00E-05
		$n(0)$	1.0000E+02	1.00E-05
		$n(l)$	1.0000E+02	1.00E-05
		$\sigma_{xx}(0)$	2.0000E+04	1.00E-05
		$\sigma_{xx}(l)$	4.0000E+04	1.00E-05
2	*S2*	$v(l)$	1.8969E-04	2.30E-02
		$\theta_z(l)$	3.0238E-04	2.70E-02
		$v_y(0)$	1.0000E+02	1.00E-05
		$v_y(l)$	1.0000E+02	1.00E-05
		$mf_z(0)$	1.0000E+02	1.00E-05
		$mf_z(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xx}(0)$	2.4000E+06	1.00E-05
		$\sigma_{xx}(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xy}(0)$	2.0000E+04	1.00E-05
		$\sigma_{xy}(l)$	4.0000E+04	1.00E-05
3	*S2*	$\theta_x(l)$	8.3506E-04	5.70E-02
		$m_x(0)$	1.0000E+02	1.00E-05
		$m_x(l)$	1.0000E+02	1.00E-05
		$\sigma_{xy}(0)$	1.5600E+06	1.00E-05
		$\sigma_{xy}(l)$	4.0371E+06	5.00E-02

		$\sigma_{xz}(0)$	1.5600E+06	1.00E-05
		$\sigma_{xz}(l)$	4.0371E+06	5.00E-02
4	*S2*	$w(l)$	– 1.2000E-04	3.00E-03
		$\theta_y(l)$	3.600E-04	4.00E-03
		$v_z(0)$	0.0000E+00	1.00E-03 (absolute)
		$v_z(l)$	0.0000E+00	1.00E-03 (absolute)
		$mf_y(0)$	1.0000E+02	1.00E-05
		$mf_y(l)$	1.0000E+02	1.00E-05
		$\sigma_{xx}(0)$	1.2000E+06	1.00E-05
		$\sigma_{xx}(l)$	4.8000E+06	1.00E-05
5	*S2*	$u(l)$	6.1370E-08	1.00E-05
		$n(0)$	1.0000E+02	1.00E-05
		$n(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xx}(0)$	2.0000E+04	1.00E-05
		$\sigma_{xx}(l/2)$	1.3333E+04	1.00E-05
		$\sigma_{xx}(l)$	0.0000E+00	1.00E-03 (absolute)
6	*S2*	$v(l)$	6.8626E-05	2.00E-02
		$\theta_z(l)$	9.4847E-05	2.40E-02
		$v_y(0)$	1.0000E+02	1.00E-05
		$v_y(l)$	0.0000E+00	1.00E-03 (absolute)
		$mf_z(0)$	5.0000E+01	1.00E-02
		$mf_z(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xx}(0)$	1.2000E+06	1.00E-05
		$\sigma_{xx}(l)$	0.0000E+00	1.00E-03 (absolute)
		$\sigma_{xy}(0)$	2.0000E+04	1.00E-02
		$\sigma_{xy}(l)$	0.0000E+00	1.00E-03 (absolute)
7	*S3*	$w(l)$	– 3.8259E-05	1.00E-02
		$\theta_y(l)$	5.7388E-05	1.00E-02
		$v_z(0)$	– 4.4633E+02	1.00E-03
		$mf_y(0)$	1.7535E+02	1.00E-02

3.4 Remarks

The modelization being made out of beams of Eulerian, the shear coefficients is $ky=kz=1$.

4 Summary of the results

the got results confirm that elements POU_D_E with variable section present a good degree of reliability.
For the circular section, the results all are exact with the nodes (one finds the properties of the element with constant section) except for the forces distributed where the effect of the smoothness of discretization is felt.
For a rectangular section and a general section, it is necessary to discretize finely to have a correct solution.